Guess-and-Check (Trial-and-Error) through multiplication

$6x^2-7x-5=?$

$(6x-5)(x+1),\ b=1,\quad \text{no}$
$(3x+1)(2x-5),\ b=-13,\quad \text{no}$
$\cdots$
$(2x+1)(3x-5),\ b=-7,\quad \text{yes}$

Note: if you give students tips for how to organize their work in guess-and-check, please include them in the survey.

Factor by Grouping

$$\begin{aligned}6x^2-7x-5 &= 6x^2-10x+3x-5\\ &= 2x\cdot(3x-5)+1\cdot(3x-5)\\ &= (2x+1)(3x-5)\end{aligned}$$

Note: if you ask students to multiply $6x^2$ *and* -5 *first in order to find* $-10x$ *and* $3x$*, please choose one of the methods below.*

AC Method (Box Method)

Multiply A and C:

$a\cdot c=-30$

List all pairs of factors of -30:

$(1,-30),\ (-1,30),\ (2,-15),\ (-2,15),\cdots$

Until you find a pair that add to $b=-7$:

$(-10,3)$

Use these two numbers in grouping:

$$\begin{aligned} 6x^2-7x-5 &= 6x^2-10x+3x-5 \\ &= 2x\cdot(3x-5)+1\cdot(3x-5) \\ &= (2x+1)(3x-5) \end{aligned}$$

As an alternative, some instructions ask students to express the coefficient of x as a sum of the two factors first:

$$\begin{aligned} 6x^2-7x-5 &= 6x^2+(3-10)x-5 \\ &= 6x^2+3x-10x-5 \\ &= 3x(2x+1)-5(2x+1) \\ &= (3x-5)(2x+1) \end{aligned}$$

The AC Method can be combined with the visual presentation of Algebra Tiles, thus resulting in the so-called "Box Method"

	$6x^2$	
		-5

Factor $a\cdot c=-30$ into a pair that add to $b=-7$: $(-10,3)$

Fill the remaining boxes with the two linear terms, with the coefficients found above. The positions of these two terms are interchangeable.

	$6x^2$	$3x$
	$-10x$	-5

Find greatest common factors for each row and column (including the negative sign if both are negative)

	$2x$	$+1$
$3x$	$6x^2$	$3x$
-5	$-10x$	-5

Note: if you give students tips about how to arrange all the pairs of factors of AC, please include them in the survey.

AC Method with Substitution

There are two main variations of this technique. To begin with, we will multiply A and C, like in the AC method:

$a \cdot c = -30$

Next, we will list all pairs of factors of -30:

$(1,-30),\ (-1,30),\ (2,-15),\ (-2,15),\cdots$

Until you find a fair (k_1, k_2) that add to $b = -7$:

$(-10, 3)$

Variation 1:

Put these two numbers into $(ax + k_1)(ax + k_2)$ (which is a factoring of $(a^2)x^2 + (ab)x + (ac)$)

$(6x - 10)(6x + 3)$

Divide the result by A and simplify by canceling common factors:

$$\frac{(6x-10)(6x+3)}{6} = \frac{(3x-5)(6x+3)}{3}$$
$$= (3x-5)(2x+1)$$

Variation 2:

Divide each of the two factors by 6, and reduce to lowest terms:

$$-\frac{10}{6} = -\frac{5}{3}$$

$$\frac{3}{6} = \frac{1}{2}$$

Use these two reduced fractions in the following factored form:

$$(x+\frac{5}{3})(x-\frac{1}{2})$$

Multiply the result by 6 and simplify:

$$(3x+5)(2x-1)$$

Note: this method is named "AC Method with Substitution" because both variations rely on the fact that $ax^2+bx+c = \frac{u^2+bu+ac}{a}$, *where* $u = 6x$.

Quadratic Formula

Use quadratic formula to find the two roots:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$= \frac{7 \pm \sqrt{49-4\cdot 6\cdot(-5)}}{2\cdot 6}$$
$$= \frac{7 \pm 13}{12}$$

Express the solution as reduced fractions: $x = \frac{5}{3}$ or $x = -\frac{1}{2}$. So the original trinomial can be factored into:

$$6\left(x-\frac{5}{3}\right)\left(x+\frac{1}{2}\right)$$

Multiply 6 with the two factors, we get $(3x-5)(2x+1)$ as the result.

Tic-Tac-Toe Method

Arrange the three terms of trinomials as follows:

$6x^2$	-5	
		$-7x$

Multiply the two terms in the first row:

$6x^2$	-5	$-30x^2$
		$-7x$

Find a pair of factors of $-30x^2$ that will add to the term on the bottom:

$6x^2$	-5	$-30x^2$
		$-10x$
		$3x$

	$-7x$

Fill the four empty cells so that the product of each row/column is equal to the term occupying the same row/column.

$6x^2$	-5	$-30x^2$
$2x$	-5	$-10x$
$3x$	1	$3x$
		$-7x$

Check: $2x\cdot(-5)=-10x$, , $3x\cdot 1=3x$, $2x\cdot 3x=6x^2$, $-5\cdot 1=-5$

Solution (pair the 4 terms in red to the diagonal):

$(2x+1)(3x-5)$