ELEC 88.81 Computer Hardware

Binary Numbers

Computers use the **binary number system** to store and manipulate numeric data.

The **binary number system,** also referred to as base 2, uses two digits:

0 and 1

The **decimal number system**, sometimes referred to as base 10, uses ten digits:

0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

Counting with the Decimal System

When you count using the decimal number system, you get the next number by adding 1. For example, you begin with 1, then add 1+1 to get the next number, 2.

```
  1
+ 1
---
  2
```

How do you get past 9? To count past 9, you need to add 9+1. Since you have run out of digits, you must write a zero and "carry" a 1 to the next column.

```
  9
+ 1
---
 10
```

The decimal number system is **positional --** the value of a digit is determined by its position. For example, the digit 1 by itself is worth 1, but the digit 1 followed by a 0 is worth 10.

The value for a column is referred to as its **place value**. In the decimal number system, the column on the right has a place value of 1. The column on the left has a place value of 10.

Ten's Place	One's Place
1	0

Counting with the Binary Number System

Counting with the binary number system is similar to counting with decimal, but you can use only two digits, 0 and 1. You begin counting with 1. To get to the next number, you must add 1.

```
  1
 +1
---
 10
```

However, in binary 1+1 cannot equal 2 because there is no 2 digit. So, just as in the decimal system, when you run out of digits, you "carry" a 1 to the next column.

To summarize, the binary number system is positional, but uses only two digits, 1 and 0.

In binary, the digit 1 by itself is worth 1. The digit followed by a 0 is worth two, not ten as in the decimal system.

Two's Place	**One's Place**
1	**0**

Binary Digits

The binary number system uses **binary digits** 0 and 1. Usually referred to as **bits**, the binary digits **0** and **1** correspond to electrical states **off** and **on**, respectively.

Binary Place Values

You know that when you want to count past 1 in the binary number system, you must carry a 1 to the next column. This column has a **place value** of 2. We can call this column the 2's place.

Power of Two	2^1	2^0
Place Value	**2**	1
Binary Digit	**1**	**0**
Decimal Equivalent		**2**

A 1 bit has been placed in the 2's column. This bit has a decimal value of 2. The binary number 10 is equivalent to the decimal number 2. The decimal equivalent is the sum of the place value multiplied by the binary digit:

2 1
x 1 x 0

2 + 0 = 2

In the binary number system the place values are powers of 2. As you move left, you simply multiply by 2 to get the next place value.

Power of Two	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Place Value	128	64	32	16	8	4	2	1

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Converting Binary to Decimal

Suppose you only have a pencil and paper and you want to convert the binary number **10001100** to its decimal equivalent.

The first step is to set up a table of binary place values like the one shown. Next, write the binary digit in each column as shown below. Now, multiply the place value by each binary digit. This produces the decimal value for each column. For the final step, find the sum of the decimal values for each column. This produces the decimal equivalent of the binary number.

Place Value	128	64	32	16	8	4	2	1
Binary Digit	**1**	**0**	**0**	**0**	**1**	**1**	**0**	**0**
Decimal Value	128	0	0	0	8	4	0	0

The decimal equivalent: 128 + 8 + 4 = **140**

Try converting the binary number **10101110** into its decimal equivalent.

Place Value	128	64	32	16	8	4	2	1
Binary Digit								
Decimal Value								

Converting Decimal to Binary

Now suppose you want to determine the binary equivalent of a decimal number such as **73.**

1. Write down the binary place values in a table as shown.
2. Next, write the decimal number you want to convert.
3. Now, find the largest binary place value that you can subtract from the decimal number. In this case, it is **64**.
4. Perform the subtraction, then place a 1 in the 64's column of your table.
5. Now you have a remainder. What is the largest place value you can subtract from this remainder? It is **8**.
6. Perform the subtraction, then place a 1 in the 8's column of your table.
7. You now have another remainder. What is the largest binary place value that you can subtract from this remainder? It is **1**.
8. Perform the subtraction, and put a 1 in the 1's column of your table.
9. You now have no remainder. That means the conversion is complete. Fill in the rest of your table with 0s. The binary equivalent is the sequence of 1s and 0s you have written in your table.

Place Value	128	64	32	16	8	4	2	1
Binary Digit	**0**	**1**	**0**	**0**	**1**	**0**	**0**	**1**

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Here's a summary of the decimal to binary conversion process:

1. Find the largest place value that will fit into the decimal number.
2. Subtract that place value from the decimal number.
3. Put a 1 in the column for that place value.
4. Repeat the process until there is no remainder.

Convert the decimal number 18 into its binary equivalent.

Place Value	128	64	32	16	8	4	2	1
Binary Digit								

Working with Binary and Hexadecimal Numbers

Powers of 2	Decimal Value
2^0	1
2^1	2
2^2	4
2^3	8
2^4	16
2^5	32
2^6	64
2^7	128
2^8	256
2^9	512
2^{10}	1,024
2^{11}	2,048
2^{12}	4,096
2^{13}	8,192
2^{14}	16,384
2^{15}	32,768
2^{16}	65,536

Powers of 16	Decimal Value
16^0	1
16^1	16
16^2	256
16^3	4,096
16^4	65,536
16^5	1,048,576
16^6	16,777,216
16^7	268,435,456
16^8	4,294,967,296

Decimal with Hexadecimal and Binary Equivalents

Decimal	Hexadecimal	Binary
0	0	0000 0000
1	1	0000 0001
2	2	0000 0010
3	3	0000 0011
4	4	0000 0100
5	5	0000 0101
6	6	0000 0110
7	7	0000 0111
8	8	0000 1000
9	9	0000 1001
10	A	0000 1010
11	B	0000 1011
12	C	0000 1100
13	D	0000 1101
14	E	0000 1110
15	F	0000 1111
16	10	0001 0000
32	20	0010 0000
64	40	0100 0000
128	80	1000 0000
255	FF	1111 1111

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Use the chart below to aid in converting Hexadecimal numbers to their Decimal equivalent.

Hexadecimal to Decimal Conversion Chart								
Power of 16	**16^7**	**16^6**	**16^5**	**16^4**	**16^3**	**16^2**	**16^1**	**16^0**
Decimal Place Value	**268,435,456**	**16,777,216**	**1,048,576**	**65,536**	**4,096**	**256**	**16**	**1**
Hex Multiplier								
1	268,435,456	16,777,216	1,048,576	65,536	4,096	256	16	1
2	536,870,912	33,554,432	2,097,152	131,072	8,192	512	32	2
3	805,306,368	50,331,648	3,145,728	196,608	12,288	768	48	3
4	1,073,741,824	67,108,864	4,194,304	262,144	16,384	1,024	64	4
5	1,342,177,280	83,886,080	5,242,880	327,680	20,480	1,280	80	5
6	1,610,612,736	100,663,296	6,291,456	393,216	24,576	1,536	96	6
7	1,879,048,192	117,440,512	7,340,032	458,752	28,672	1,792	112	7
8	2,147,483,648	134,217,728	8,388,608	524,288	32,768	2,048	128	8
9	2,415,919,104	150,994,944	9,437,184	589,824	36,864	2,304	144	9
A	2,684,354,560	167,772,160	10,485,760	655,360	40,960	2,560	160	10
B	2,952,790,016	184,549,376	11,534,336	720,896	45,056	2,816	176	11
C	3,221,225,472	201,326,592	12,582,912	786,432	49,152	3,072	192	12
D	3,489,660,928	218,103,808	13,631,488	851,968	53,248	3,328	208	13
E	3,758,096,384	234,881,024	14,680,064	917,504	57,344	3,584	224	14
F	4,026,531,840	251,658,240	15,728,640	983,040	61,440	3,840	240	15

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Converting from Hexadecimal to Decimal

Problem: Convert the hex number F1CA into its decimal equivalent

Solution: To convert from Hex to Decimal, multiply each hex digit by its place value and then add the values together.

For example:

Place Value	**16^3**	**16^2**	**16^1**	**16^0**
Decimal Value	4,096	256	16	1
Hex number	F	1	C	A

F1CA = (F x 4096) + (1 x 256) + (C x 16) + (A x 1)
= 61,440 + 256 + 192 + 10
= 61,898

Hex to Decimal Conversion				
Place Value	**16^3**	**16^2**	**16^1**	**16^0**
Decimal Value	**4,096**	**256**	**16**	**1**
Hex Multiplier				
1	4,096	256	16	1
2	8,192	512	32	2
3	12,288	768	48	3
4	16,384	1,024	64	4
5	20,480	1,280	80	5
6	24,576	1,536	96	6
7	28,672	1,792	112	7
8	32,768	2,048	128	8
9	36,864	2,304	144	9
A	40,960	2,560	160	10
B	45,056	2,816	176	11
C	49,152	3,072	192	12
D	53,248	3,328	208	13
E	57,344	3,584	224	14
F	61,440	3,840	240	15