Math 27, HW #18 Selected Problems

Using a Compound Interest Formula **In Exercises 13–20, complete the table for a savings account in which interest is compounded continuously.**

For interest compounded continuously, use: $A = Pe^{rt}$.

	Initial Investment	Annual % Rate	Time to Double	Amount After 10 Years
13.	$10,000	4%		

For the time to double, solve: $2\cancel{0,000} = 1\cancel{0,000}\, e^{.04t} \Rightarrow 2 = e^{.04t}$

$$\Rightarrow \ln 2 = \ln e^{.04t} = .04t \Rightarrow t = \frac{\ln 2}{.04} = \boxed{17.3 \text{ years}}$$

In 10 years: $A = 10{,}000\, e^{.04(10)} = \boxed{\$14{,}918.25}$

	Initial Investment	Annual % Rate	Time to Double	Amount After 10 Years
16.	$1000		12 years	

If the time to double is 12 years then $2\cancel{000} = 1\cancel{000}\, e^{12r}$

$$\Rightarrow 2 = e^{12r} \Rightarrow \ln 2 = 12r \Rightarrow r = (\ln 2)/12 = 0.0578 = \boxed{5.78\%}$$

In 10 years: $A = 1000\, e^{(.0578)\cdot 10} = \boxed{\$1{,}782.64}$

	Isotope	Half-Life (years)	Initial Quantity	Amount After 1000 Years
25.	^{226}Ra	1600	10 g	

For radioactive decay, use:

$$A = A_0 e^{rt}$$

To find r, solve: $5 = 10e^{1600r}$ $\Rightarrow$ $1/2 = e^{1600r}$

$\Rightarrow$ $\ln(1/2) = 1600r$ $\Rightarrow$ $r = \ln(1/2)/1600 = -0.000433$

In 1000 years: $A = 10e^{-.000433(1000)} = \boxed{6.49\text{ g}}$

35. Demography The populations P (in thousands) of Antioch, California, from 2006 through 2012 can be modeled by $P = 90e^{0.013t}$, where t is the year, with $t = 6$ corresponding to 2006. (Source: U.S. Census Bureau)

(a) According to the model, was the population of Antioch increasing or decreasing from 2006 through 2012? Explain your reasoning.

(b) What were the populations of Antioch in 2006, 2009, and 2012?

(c) According to the model, when will the population of Antioch be approximately 116,000?

a) Increasing, since the exponent is postive.

b) 2006: $P = 90e^{0.013(6)} = 97.3$ thousands

2009:

2012:

c) Solve for t: $116 = 90e^{0.013t}$ $\Rightarrow$ $(116/90) = e^{0.013t}$

$\Rightarrow$ $\ln(116/90) = 0.013t$ $\Rightarrow$ $t = \ln(116/90)/0.013 = 19.5$ years

$\boxed{2019}$

Pg. 230, #46

46. **Biology** The number Y of yeast organisms in a culture is given by the model

$$Y = \frac{663}{1 + 72e^{-0.547t}}$$

where t represents the time (in hours).

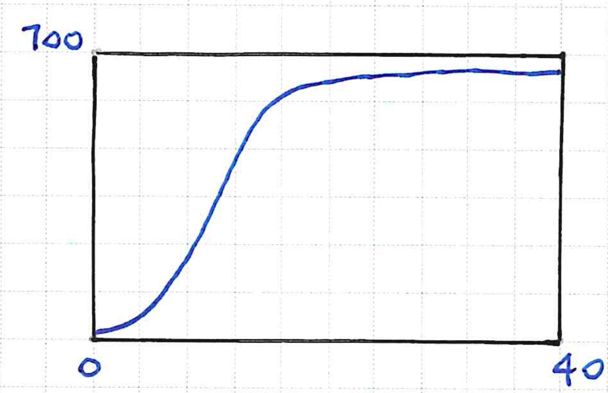
(a) Use a graphing utility to graph the model.

(b) Use the model to predict the populations for the 19th hour and the 30th hour.

(c) According to this model, what is the limiting value of the population?

(d) Why do you think this population of yeast follows a logistic growth model instead of an exponential growth model?

a) $Y1 = 663/(1 + 72e^{-0.547x})$



b) $$Y(19) = \frac{663}{1 + 72e^{-0.547(19)}} = 661$$

$$Y(30) = \frac{633}{1 + 72e^{-0.547(30)}} = 663$$

c) Limiting value = 663