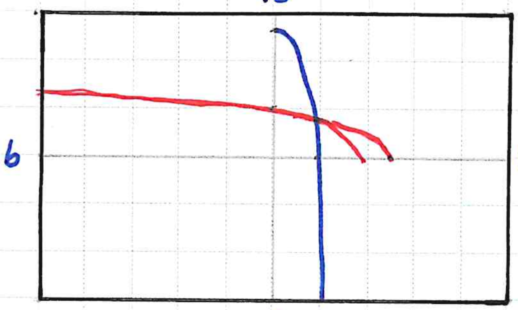
Math 27, HW #9

Pg. 67, #22 $f(x) = 9 - x^2 \ (x \geq 0)$, $g(x) = \sqrt{9-x}$

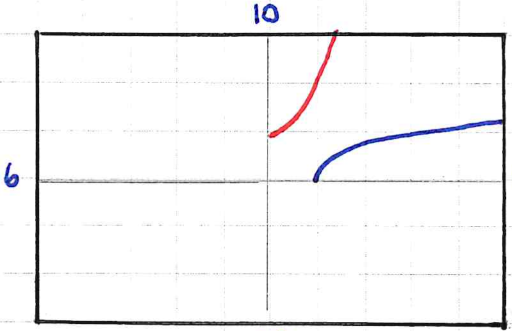


Now, $(f \circ g)(x) = f(g(x)) = f(\sqrt{9-x}) = 9 - (\sqrt{9-x})^2 = x$

and $(g \circ f)(x) = g(f(x)) = g(9-x^2) = \sqrt{9-(9-x^2)} = x$

So $f^{-1}(x) = g(x)$ and $g^{-1}(x) = f(x)$

Pg. 67, #30 $f(x) = \sqrt[4]{3x-10}$, $g(x) = (x^4+10)/3 \ (x \geq 0)$



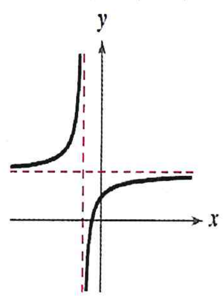
Now, $(f \circ g)(x) = f((x^4+10)/3) = \sqrt[4]{x^4} = x$

and $(g \circ f)(x) = g(\sqrt[4]{3x-10}) = (3x-10+10)/3 = x$

Thus f and g are inverse functions.

Pg. 68, #43

43.



The graph passes the VLT and the HLT so it is the graph of a one to one function.

Bonus Problem

$f(x) = \dfrac{x+3}{x-2}$ Find f^{-1} algebraically.

① $y = \dfrac{x+3}{x-2}$

② $x = \dfrac{y+3}{y-2}$

③ $x(y-2) = y+3$

$xy - 2x = y + 3 \longrightarrow -3 - 2x = y - xy$

$xy - y = 2x + 3$ $\quad -3 - 2x = y(1-x)$

$y(x-1) = 2x + 3$ $\quad \dfrac{-3-2x}{1-x} = y$

$y = \dfrac{2x+3}{x-1}$

④ $\boxed{f^{-1}(x) = \dfrac{2x+3}{x-1}}$