MATH 27
Exam #3
Spring 2018

Name: _____

Roll #: _____ Score: _____/100

Work each problem in the space provided on the exam page. Full or partial credit will be given only if all work has been shown and the answers are clearly indicated. Each numbered problem is worth 10 points.

1. Answer **T** if always true, **F** if otherwise.

   For parts a and b, refer to Figure 1 where $\beta > \theta$.

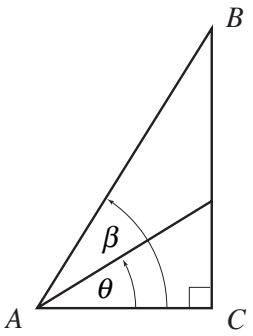
   

   Figure 1

   _____ a) $\sin\beta < \sin\theta$

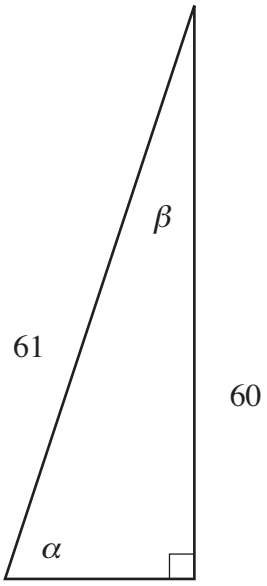
   _____ b) $\beta = \cos^{-1}\left(\frac{AC}{AB}\right)$

   _____ c) $\tan\left(\tan^{-1}\left(\frac{\pi}{2}\right)\right) = \frac{\pi}{2}$

   _____ d) The amplitude of $y = -4\cos(4x - 4)$ is –4.

   _____ e) There are infinitely many solutions to the equation $\sin x = \frac{\pi}{2}$.

2. Determine the following values for the triangle shown on the right.
   Leave answers in reduced fraction form if possible.

   a) Find the length of the missing side.

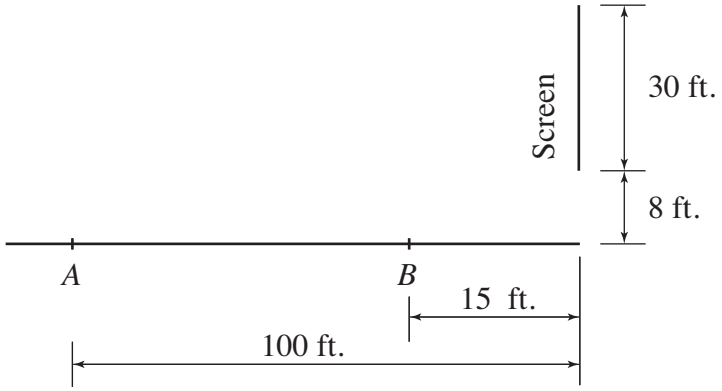
   b) Find the **exact** value of $\cot\alpha$.

   c) Find the **exact** value of $\cos\beta$.

   d) Find the measure of β to the nearest degree.

   e) Find the **exact** value of $\csc^2\beta - \cot^2\beta$.

3. An outdoor movie is being shown at a park. The base of the screen is 8 feet off the ground and the screen is 30 feet high.



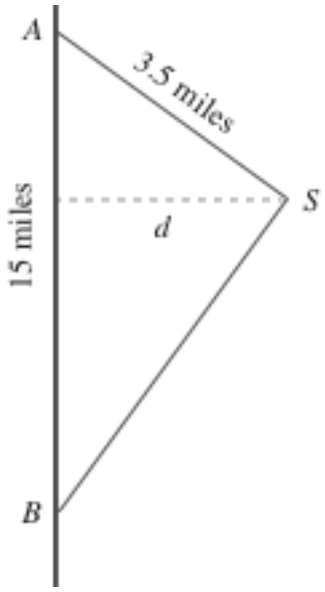
   a) Find the angles of elevation to the top of the screen from points *A* and *B*. (To the nearest tenth of a degree.)

   b) You are lying on the ground and the angle of elevation to the top of the screen is 42°. How far are you from a point on the ground directly below the screen? (To the nearest tenth of a foot.)

4. A submarine (*S*) is near two coastal towns, *A* and *B*. Town *B* is 15 miles due south of town *A*. The submarine's bearing from *A* is S48°20'E and its distance from *A* is 3.5 miles.

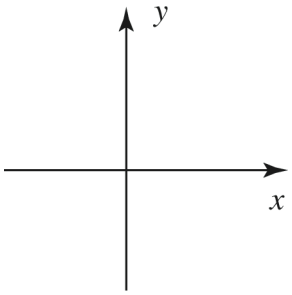
   a) What is the distance (*d*) from the submarine to the coast? (To the nearest tenth of a mile.)

   b) What is the bearing from town *B* to the submarine? (To the nearest ten minutes.)



5. Draw each angle in standard position and give the **exact** value of the trigonometric functions that are listed.

a) $\theta = -240°$

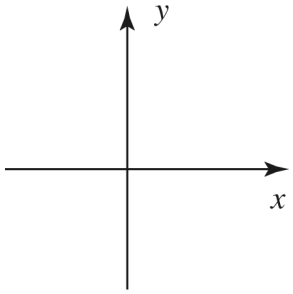


$\sin\theta =$

$\sec\theta =$

$\tan\theta =$

b) $\theta = \frac{5\pi}{4}$



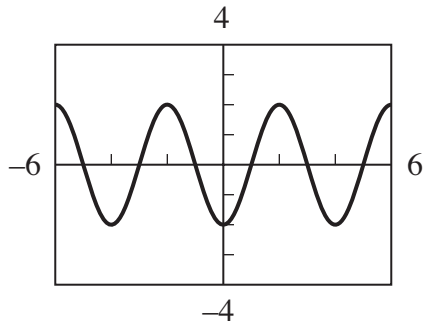
$\cos\theta =$

$\csc\theta =$

$\cot\theta =$

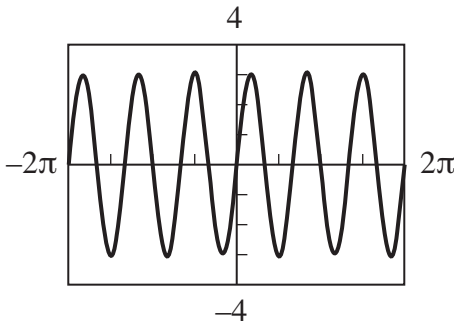
6. a) For the given graph, write the function in the form $f(x) = a\cos(bx)$.

$f(x) =$ _____



b) For the given graph, write the function in the form $f(x) = a\sin(bx)$.

$f(x) =$ _____



7. Sunset times in Indianapolis, Indiana, can be roughly approximated using the function

$$f(t) = 89\sin\left(\frac{2\pi}{365}t + \frac{3\pi}{2}\right) + 409$$

where t is the day of the year ($t = 1$ corresponds to January 1) and $f(t)$ is the corresponding sunset time, in minutes, after 12:00 noon. Determine the following features to two-decimal place accuracy.

a) Amplitude =

b) Period =

c) Phase Shift =

d) Vertical Shift =

e) Use your calculator to obtain a graph of one full period of the function $f(t)$. Sketch the graph on the grid below. Clearly label the axes and indicate the scale.

8. Find the **exact** number values, in radians, without using a calculator.

a) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

b) $\arctan(-1)$

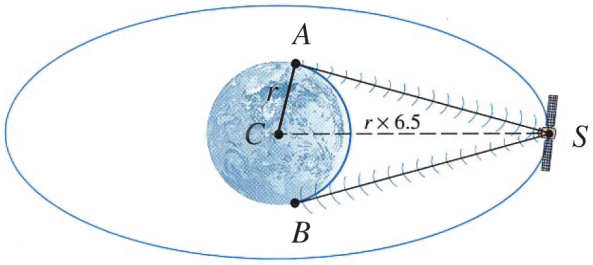
9. Find the **exact** value of each of the following expressions, without using a calculator.

a) $\tan\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$

b) $\cos^{-1}\left(\cos\frac{7\pi}{4}\right)$

10. A communications satellite (S) is placed into a synchronous orbit with the earth. The radius of the orbit of such a satellite is 6.5 times the radius of the earth (3950 miles). The satellite is used to relay a signal from one point on the earth to another point on the earth. The sender (A) and receiver (B) of a signal must be in a line of sight with the satellite, as shown in the figure.

a) Find the measure of $\angle ACS$ to three decimal places. [*Hint*: A radius drawn to the point of tangency is perpendicular to the tangent line at that point.]



b) Find the distance along the surface of the earth from point A to point B, to the nearest mile.