MATH 27
Exam #2
Spring 2018

Name: Key

Roll #: _____ Score: _______/100

Work each problem in the space provided on the exam page. Full or partial credit will be given only if all work has been shown and the answers are clearly indicated. Each numbered problem is worth 10 points.

1. Answer **T** if always true, **F** if otherwise.

T a) The graph of a polynomial function of degree 30 may have at most 30 x-intercepts.

T b) The graph of a rational function may cross its horizontal asymptote.

F c) $\ln(3A) = 3\ln(A)$

F d) Any two isosceles triangles are similar.

T e) If the legs of an isosceles right triangle have length 4 cm, then its hypotenuse will have length $4\sqrt{2}$ cm.

2. For each of the following functions, determine the **right hand** end behavior of the graph of the function. Circle the appropriate response.

a)	$f(x) = \frac{1}{2}x^{20} - 2x^{10} + 50$	RISES	FALLS	NEITHER
b)	$g(x) = -2x^{23} + 3x^{32}$	RISES	FALLS	NEITHER
c)	$h(x) = (\pi - \sqrt{10})x^{19}$	RISES	FALLS	NEITHER
d)	$m(x) = (2x+3)^3(2-x)$	RISES	FALLS	NEITHER
e)	$r(x) = \dfrac{8}{8x^2 + bx + c}$	RISES	FALLS	NEITHER

3. Consider the function: $f(x) = \dfrac{8x - 2x^2}{x^2 + x - 6}$

a) Determine **algebraically**, the horizontal and vertical asymptotes of f, if any.

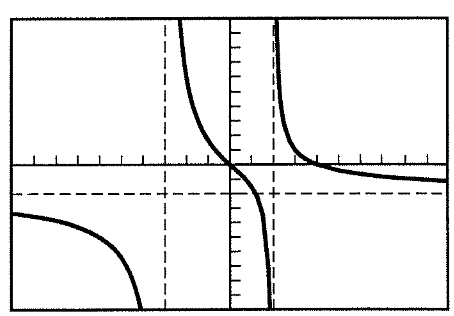
VA: $x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$\boxed{x = -3}$ or $\boxed{x = 2}$

HA: $y = \dfrac{-2x^2}{x^2} = -2$

b) Use your graphing calculator to draw the graph of f in the viewing window: $[-12, 12] \times [-9, 9]$ Sketch the graph in the space provided. Include any asymptotes.

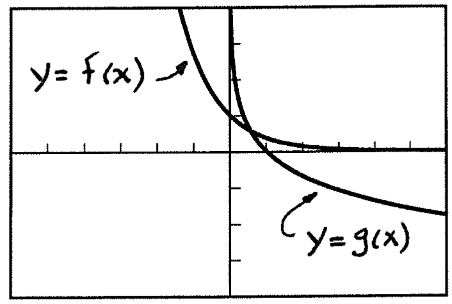


4. Use your graphing calculator to draw the graphs of the functions

$$f(x)=\left(\tfrac{1}{e}\right)^x \text{ and } g(x)=\log_{1/e} x$$

in the viewing window: [–6, 6] x [–4, 4]

Sketch the graphs in the space provided. **Clearly label each one.**



Note: $g(x) = \log_{1/e} x = \dfrac{\log x}{\log(1/e)}$

5. Find the **exact** value of each logarithm expression. (A calculator is not necessary.)

-2 a) $\log_a a^{-2}$

1 b) $\log_b\left(\frac{1}{b}\right)-\log_b\left(\frac{1}{b^2}\right)$

$1/2$ c) $\log_c \sqrt{c}$

7 d) $\log_d\left(d^4\cdot d^3\right)$

$\sqrt{2}$ e) $\left(\sqrt{2}\right)^{\log_{\sqrt{2}}\sqrt{2}}$

6. Solve the equation $\log(x+1)+\log(x-1)=\log(x+5)$ in two ways.

a) **Algebraic Solution**

$$\log[(x+1)(x-1)] = \log(x+5)$$

$$x^2-1 = x+5$$

$$x^2-x-6=0$$

$$(x-3)(x+2)=0$$

$\boxed{x=3}$ or $\cancel{x=-2}$

Reject!

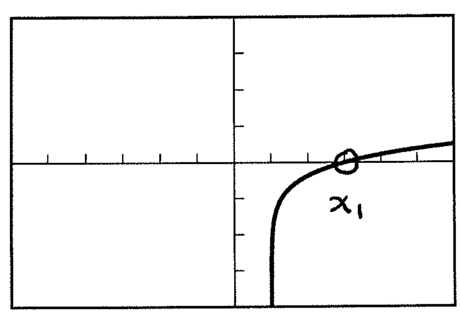
b) **Graphical Solution**

Write the equation in Y = form:

Y1 = $\log(x+1)+\log(x-1)-\log(x+5)$

Viewing window: $[-6,6]\times[-4,4]$

Label any solutions on the graph.



$\boxed{x_1 = 3}$

7. A pizza is removed from an oven at a temperature of 450°. Its temperature, t minutes after it has been removed, is modeled by the function:

$$f(t) = 25 + 325e^{-0.1005t}$$

a) What will the temperature of the pizza be 10 minutes after it has been removed from the oven? (To the nearest degree.)

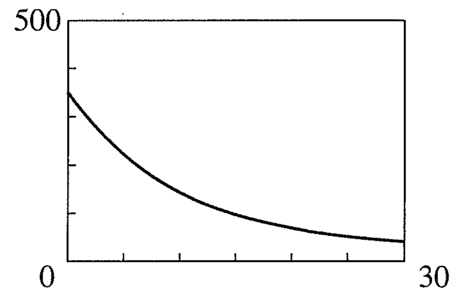
$$f(10) = 25 + 325\,e^{-0.1005(10)} \approx \boxed{144^\circ}$$

b) Use your graphing calculator to draw the graph of the function $f(t)$ in the viewing window: $[0,30]_5 \times [0,500]_{100}$

Sketch the graph in the space provided.

Estimate the equation of the horizontal asymptote.

$y = 25$



c) Solve the following problem **algebraically:** If the pizza must cool to 100° before you can eat it, then how long (to the nearest minute) must you wait?

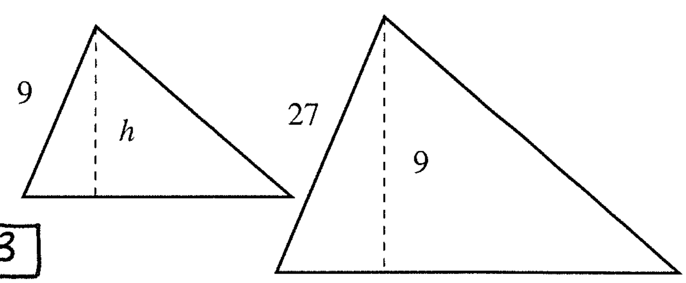
Solve: $100 = 25 + 325\,e^{-0.1005t}$ for t.

$$\frac{75}{325} = e^{-0.1005t}$$

$$\ln\left(\frac{75}{325}\right) = -0.1005t$$

$$t = \frac{\ln(75/325)}{-0.1005t} \approx \boxed{15 \text{ minutes}}$$

8. The adjacent figure shows two similar triangles (not to scale) and their altitudes. Some segment lengths are also given.



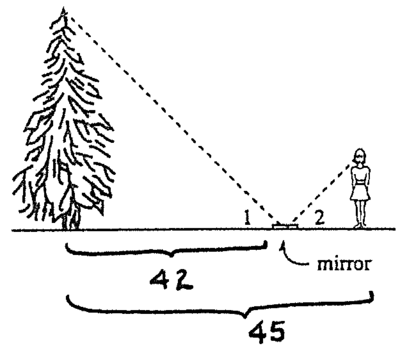
a) Determine the length of the altitude h.

$$\frac{h}{9} = \frac{9}{27} \Rightarrow h = \frac{81}{27} = \boxed{3}$$

b) Determine the ratio of the areas of the two triangles.

$$\frac{\text{Area Sm } \Delta}{\text{Area Lg } \Delta} = \left(\frac{9}{27}\right)^2 = \left(\frac{1}{3}\right)^2 = \boxed{\frac{1}{9}}$$

9. A woman uses a mirror to find the height of a tree by placing the mirror horizontally on the ground and walking backward on the line formed by the tree and mirror until she can see the top of the tree in the mirror. The laws of physics tell us that $\angle 1 = \angle 2$. Use similar triangles to solve the following:



a) What is the height of the tree if her eyes are 5 ft. from the ground, her feet are 3 ft. from the mirror, and the mirror is 42 ft. from the base of the tree?

$$\frac{h}{42} = \frac{5}{3} \Rightarrow h = \frac{42(5)}{3} = \boxed{70 \text{ ft.}}$$

b) A man whose eyes are 10 ft from the ground replaces the woman. (This means that his feet are the same distance from the tree as hers.) How far from the base of the tree would the mirror have to be placed so that he could see the top of the tree in the mirror?

Let x = the distance from the tree to the mirror

Then $\frac{70}{x} = \frac{10}{45-x} \Rightarrow 70(45-x) = 10x$

$$\Rightarrow 70(45) = 80x \Rightarrow x = \frac{3150}{80} = \boxed{39\tfrac{3}{8} \text{ ft.}}$$

EXTRA CREDIT

a) Use the Change-of-Base Formula to show that: $(\log_a b)\cdot(\log_b a) = 1$

$$(\log_a b)\cdot(\log_b a) = \frac{\ln b}{\ln a}\cdot\frac{\ln a}{\ln b} = 1. \quad \checkmark$$

b) Use the Change-of-Base Formula, algebra, and other rules of logarithms to show that: $\frac{\log_2 x}{\log_6 x} = 1 + \log_2 3$

$$\frac{\log_2 x}{\log_6 x} = \frac{\ln x/\ln 2}{\ln x/\ln 6} = \frac{\ln x}{\ln 2}\cdot\frac{\ln 6}{\ln x} = \frac{\ln 6}{\ln 2}$$

$$= \frac{\ln(2\cdot 3)}{\ln 2} = \frac{\ln 2 + \ln 3}{\ln 2} = \frac{\ln 2}{\ln 2} + \frac{\ln 3}{\ln 2}$$

$$= 1 + \frac{\ln 3}{\ln 2} = 1 + \log_2 3 \quad \checkmark$$