MATH 27
Exam #1
Spring 2018

Name: Key

Roll #: _____ Score: _______/100

Work each problem in the space provided on the exam page. Full or partial credit will be given only if all work has been shown and the answers are clearly indicated. Each numbered problem is worth 10 points.

1. Answer **T** if always true, **F** if otherwise.

F a) If $x<0$ and $y>0$ then the point (x, y) lies in the first quadrant.

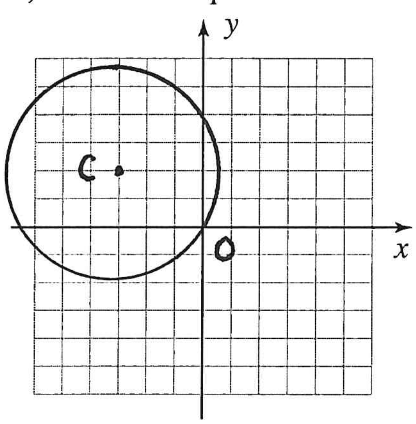
F b) The domain of the function $h(x)=2x\sqrt{3-x}$ is $[-3,+\infty)$.

F c) The solutions to the inequality $y \le 0$ are the values of x such that the graph of $y=f(x)$ lies on or above the x-axis.

T d) If $f(x)$ is a one-to-one function, then $-f(x)$ is also one-to-one.

F e) The graph of $g^{-1}(x)$ is a reflection of the graph of $g(x)$ in the y-axis.

2. a) Draw the circle centered at the point (–3, 2) and passing through the origin.
   b) Write the equation of the circle from part (a) in standard form.



Radius (r) $= \sqrt{(-3-0)^2 + (2-0)^2}$

$= \sqrt{9+4} = \sqrt{13}$

EQ: $(x-h)^2 + (y-k)^2 = r^2$

$$\boxed{(x+3)^2 + (y-2)^2 = 13}$$

3. Solve the equation $x^4 - 16x^2 + 28 = 0$ in two ways.

a) **Algebraic Solution**
[Hint: It factors.]

$(x^2-14)(x^2-2) = 0$

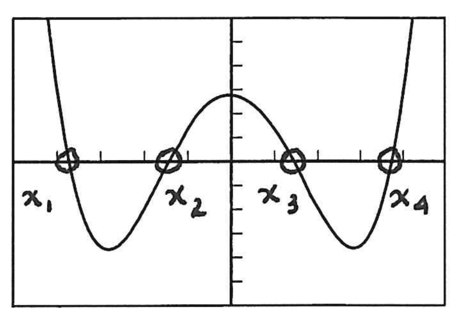
$x^2 = 14$ OR $x^2 = 2$

$$\boxed{x = \pm\sqrt{14} \quad \text{OR} \quad x = \pm\sqrt{2}}$$

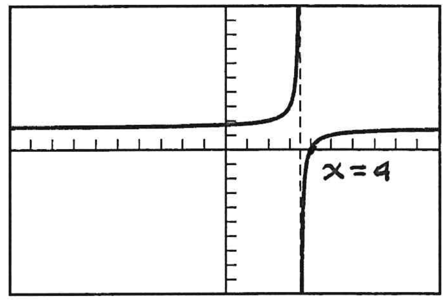
$x_1 \approx -3.742$ $x_2 \approx -1.414$

$x_3 \approx 1.414$ $x_4 \approx 3.742$

b) **Graphical Solution**
Viewing Window: $[-5,5] \times [-60,60]_{10}$.
Label the solutions on the graph.
Approximate each solution to the nearest 0.001.



4. Solve the inequality: $\dfrac{x-5}{2x-7}+1\geq 0$ **graphically**.



a) Write the equation of the function to be entered into your calculator. Use calculator notation.

Y1 = (x-5)/(2x-7) + 1 (y1 ≥ 0)

b) Sketch the graph in the standard viewing window: $[-10,10]\times[-10,10]$.

c) Write the solution using either inequality or interval notation.

Solution: $(-\infty, 3.5) \cup [4, \infty)$ OR $x < 3.5$ or $x \geq 4$

5. For the function $f(x)=3x^2-x$ evaluate the expression $\dfrac{f(x+h)-f(x)}{h}$. Simplify completely.

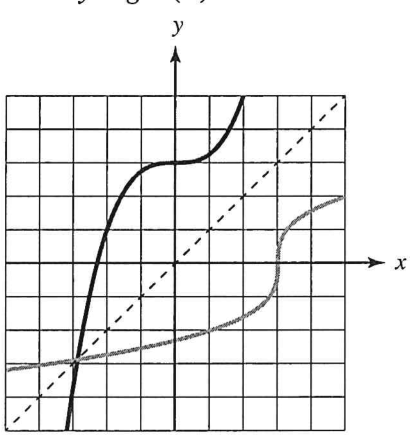
$$\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)^2-(x+h)-(3x^2-x)}{h}$$

$$= \frac{3(x^2+2xh+h^2)-x-h-3x^2+x}{h}$$

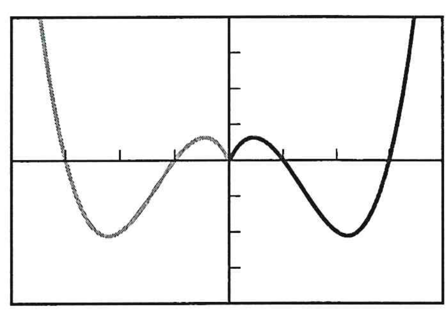
$$= \frac{6xh+3h^2-h}{h}$$

$$= \frac{h(6x+3h-1)}{h} = \boxed{6x+3h-1}$$

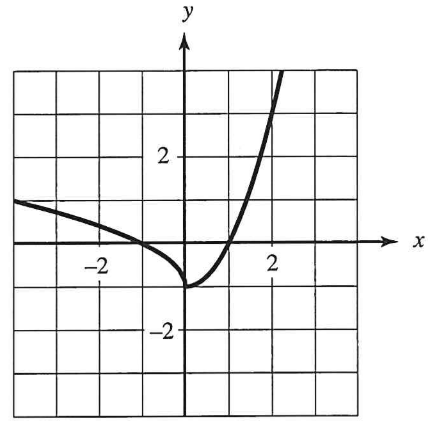
6. a) The graph of a one-to-one function $y=g(x)$ is shown. Sketch the graph of the inverse function $y=g^{-1}(x)$.



b) The graph shown below is of the function $h(x)$ for $x\geq 0$. Complete the graph if h is an **even** function.

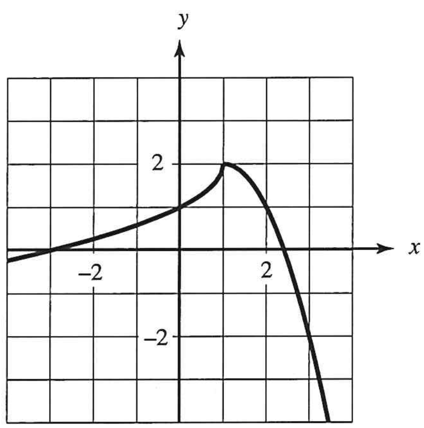


7. Let $f(x)$ be defined by the adjacent graph. Describe, in words, the effect of the transformation and sketch the graph of the resulting function.



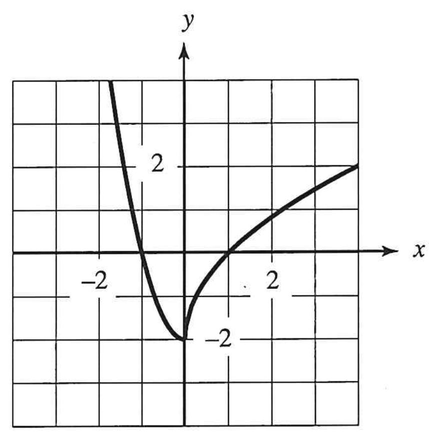
a) $g(x) = -f(x-1)+1$

1) HT 1 right
2) Reflect in x
3) VT 1 up

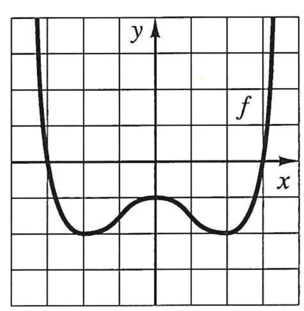


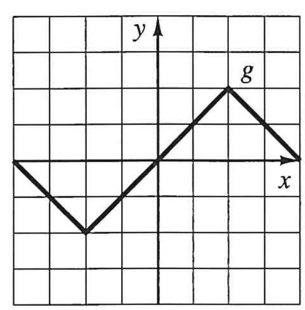
b) $h(x) = 2f(-x)$

1) Reflect in y
2) Vstretch by 2



8. Refer to the graphs of the functions f and g to compute the required quantities.





-4 a) $(f-g)(2) = f(2) - g(2)$

4 b) $(fg)(-2) = f(-2) \cdot g(-2)$

0 c) $\left(\frac{f}{g}\right)(-3) = \frac{f(-3)}{g(-3)}$

-1 d) $(f \circ g)(0) = f[g(0)]$

1 e) $(g \circ g)(3) = g[g(3)]$

9. a) Let $f(x)=\dfrac{3x-4}{x}$. Find $f^{-1}(x)$ algebraically.

(1) $y = \dfrac{3x-4}{x}$

(2) $x = \dfrac{3y-4}{y}$

(3) $xy = 3y - 4$

$xy - 3y = -4$

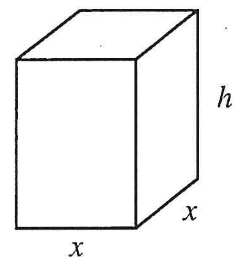
$y(x-3) = -4$

$y = \dfrac{-4}{x-3}$

or $y = \dfrac{4}{3-x}$

(4) $\boxed{f^{-1}(x) = \dfrac{4}{3-x}}$

10. Follow the instructions below to find the dimensions of a box with a square base that has a volume of 867 cubic inches and the smallest possible surface area.



Note: The surface area is the sum total of the areas of all of the sides of the box.

a) Write an equation for the surface area S of the box in terms of x and h. [Be sure to include all four sides, the top, and the bottom of the box.]

$S = 2x^2 + 4xh$ ①

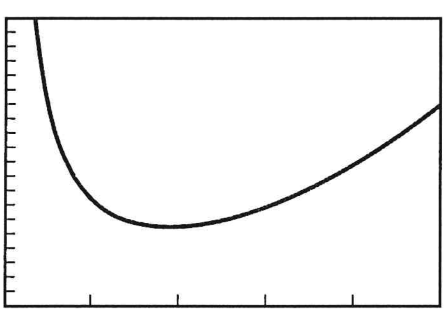
b) Write an equation, using x and h, that uses the fact that the volume of the box is 867.

$V = x^2 h = 867 \Rightarrow h = \dfrac{867}{x^2}$ ②

c) Show that the surface area of the box is given by: $S(x)=2x^2+4(867/x)$

Sub. h from ② into ①: $S(x) = 2x^2 + 4x\left(\dfrac{867}{x^2}\right)$

d) Graph the function from part (c) in the viewing window: $[0,25]_5 \times [0,2000]_{100}$. Sketch the graph in the space provided. Find the value of x that produces the smallest possible value of S.



Write the following values to the nearest 0.01

x-value: 9.54 in.

Minimum value of S: 545.55 in²