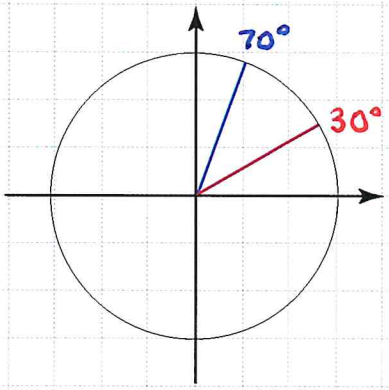
5.5 Double and Half Angle Identities (Part 2)

The half angle identities:

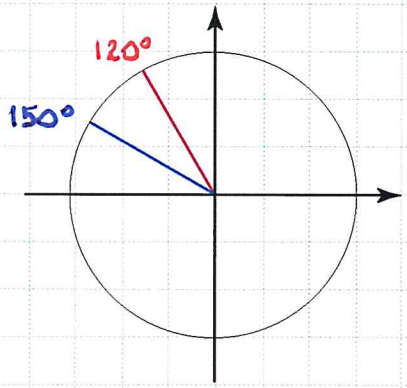
$$\sin\left(\frac{x}{2}\right)=\pm\sqrt{\frac{1-\cos x}{2}} \qquad \cos\left(\frac{x}{2}\right)=\pm\sqrt{\frac{1+\cos x}{2}} \qquad \tan\left(\frac{x}{2}\right)=\frac{1-\cos x}{\sin x}=\frac{\sin x}{1+\cos x}$$

Finding half an angle:



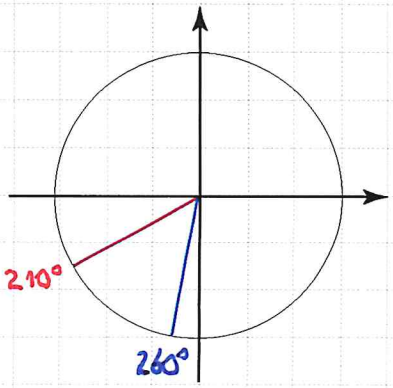
In QI, $0<\theta<\pi/2$

and



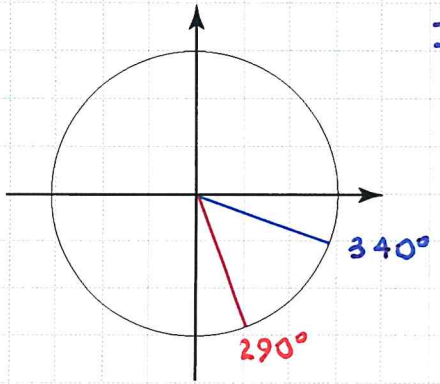
In QII, $\pi/2<\theta<\pi$

and



In QIII, $\pi<\theta<3\pi/2$

and



In QIV, $3\pi/2<\theta<2\pi$

To show that $\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$, recall that $\cos 2\theta = 2\cos^2\theta - 1$.

Now, substitute $\frac{x}{2}$ for θ.

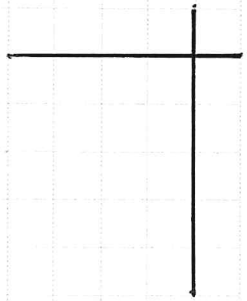
Note that $\cos\left(\frac{x}{2}\right)$ cannot be both + and −.

* The + or − sign is determined by the quadrant of the angle $\frac{x}{2}$. *

Ex. ① Given: $\cos\alpha = -3/5$, $\pi < \alpha < 3\pi/2$

Find the exact values of: a) $\sin\left(\frac{\alpha}{2}\right)$ b) $\cos\left(\frac{\alpha}{2}\right)$ c) $\tan\left(\frac{\alpha}{2}\right)$

Not that $\alpha/2$ is in



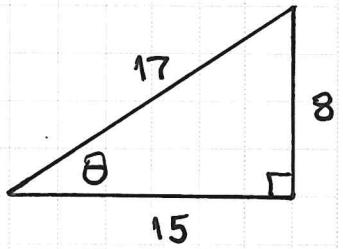
a) $\sin\left(\frac{\alpha}{2}\right) =$

b) $\cos\left(\frac{\alpha}{2}\right) =$

c) $\tan\left(\frac{\alpha}{2}\right) =$

Ex. ② Use the figure to find the exact values of:

a) $\sin\left(\frac{\theta}{2}\right)$ b) $\csc\left(\frac{\theta}{2}\right)$ c) $2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$



a) $\sin\left(\frac{\theta}{2}\right) =$

b) $\csc\left(\frac{\theta}{2}\right) =$

c) Note: $2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) =$

Ex. ③ Find the exact value of $\cos 15°$ in two ways.

a) Note that $15° = 30°/2$.

b) Note that $15° = 45° - 30°$.