5.1 The Fundamental Trigonometric Identities

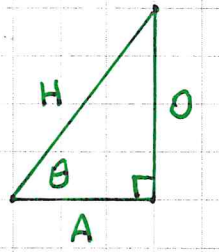
Algebraic Equation: $x^2 + 5x + 6 = 0$

Algebraic Identity: $a(b+c) = ab + ac$

Trigonometric Equation: $\sin x = 1$

Trigonometric Identity: $(\sin x)^2 + (\cos x)^2 = 1$

Recall the Ratio Definitions:



$\sin\theta =$ $\qquad$ $\csc\theta =$

$\cos\theta =$ $\qquad$ $\sec\theta =$

$\tan\theta =$ $\qquad$ $\cot\theta =$

The Reciprocal Identities

$\csc x = \_\_\_\_$ $\qquad$ $\sec x = \_\_\_\_$ $\qquad$ $\cot x = \_\_\_\_$

$\sin x = \_\_\_\_$ $\qquad$ $\cos x = \_\_\_\_$ $\qquad$ $\tan x = \_\_\_\_$

The Quotient Identities: ① $\tan x = \frac{\sin x}{\cos x}$ ② $\cot x = \frac{\cos x}{\sin x}$

Proof of ①: $\frac{\sin x}{\cos x} =$

A Pythagorean Identity: $(\sin x)^2 + (\cos x)^2 = 1$

Proof: $(\sin x)^2 + (\cos x)^2 =$

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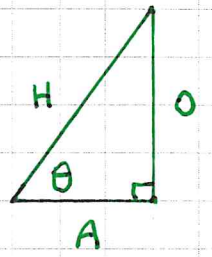

It is customary to write this identity in the form:

Another Pythagorean Identity: $\tan^2 x + 1 = \sec^2 x$

Proof: [Hint: Start with $\sin^2 x + \cos^2 x = 1$, then divide both sides by $\cos^2 x$.]

$$\sin^2 x + \cos^2 x = 1$$

The Final Pythagorean Identity:

Note: All of these identities are found on Pg. 350 of the textbook.

The Fundamental Identities can be used to simplify trigonometric expressions.

Ex. ① Simplify the following expressions.

a) $\cos x \cdot \tan x$

b) $\dfrac{\csc x}{\cot x}$

c) $\sin^2 x - \sin^2 x \cdot \cos^2 x$

Note that $\sin^2 x + \cos^2 x = 1$ can be rewritten as

d) $\cos x \, (\sec x - \cos x)$

e) $\sec x - \tan x \cdot \sin x$

f) $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{1 + \sin x}$

g) $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta}$