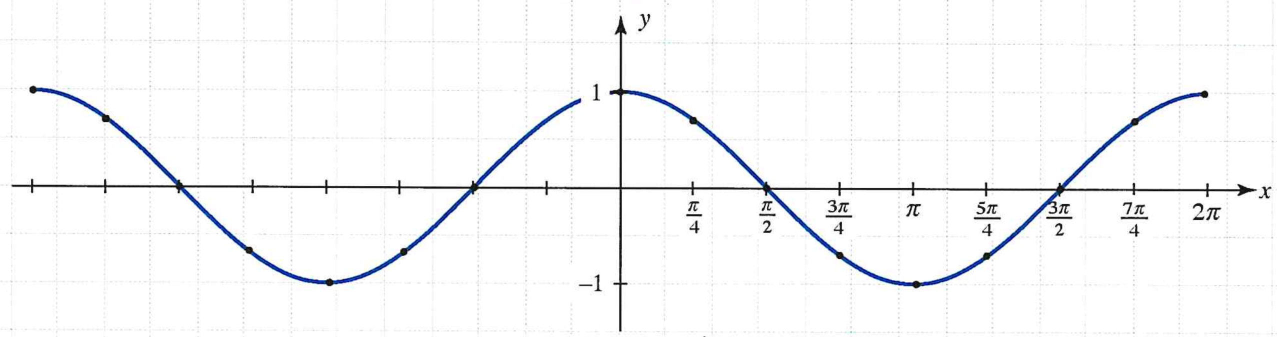
$\boxed{\text{4.7 The Inverse Trigonometric Functions}}$ (Part 2)

Consider the graph of $y = \cos x$ over the interval $[-2\pi, 2\pi]$.



Note that the equation: $\cos x = -\frac{1}{2} \Rightarrow x = \cos^{-1}(-\frac{1}{2})$ has infinitely many solutions:

We want $x = \cos^{-1}(-\frac{1}{2})$ to have

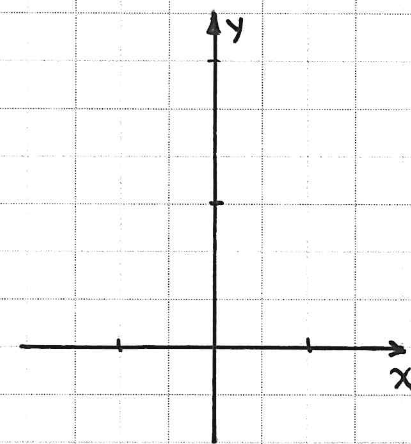
but the graph of $y = \cos x$ is not

To fix this problem, we restrict the domain of $y = \cos x$ to the interval Its range is still $[-1, 1]$.

Then we DEFINE $y = \cos^{-1} x = \arccos x$ to be a function

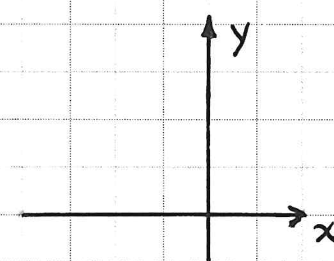
with Domain: and Range:

The graph of $y = \cos^{-1} x$:

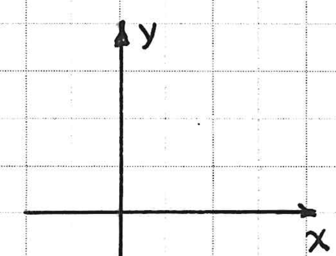


Ex. ① Evaluate the following:

a) $\cos^{-1}(-1/2)$



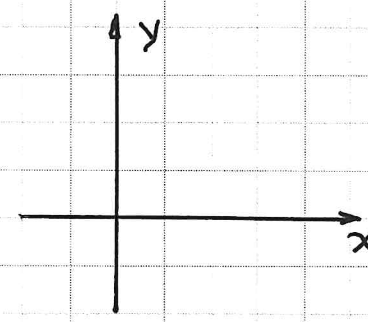
b) $\arccos(\sqrt{3}/2)$



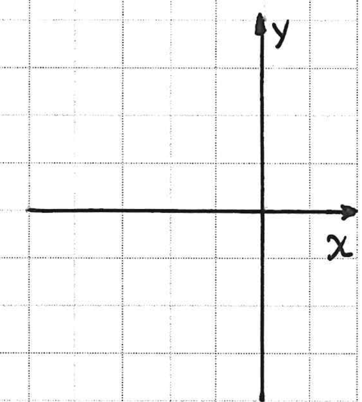
c) $\cos^{-1}(\pi)$

Ex. 2 Find the EXACT values:

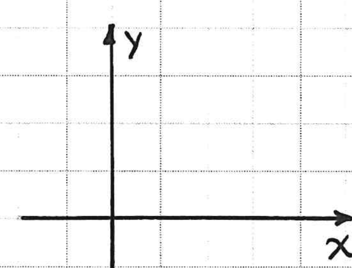
a) $\cos\left(\cos^{-1}\left(-\sqrt{2}/2\right)\right.$



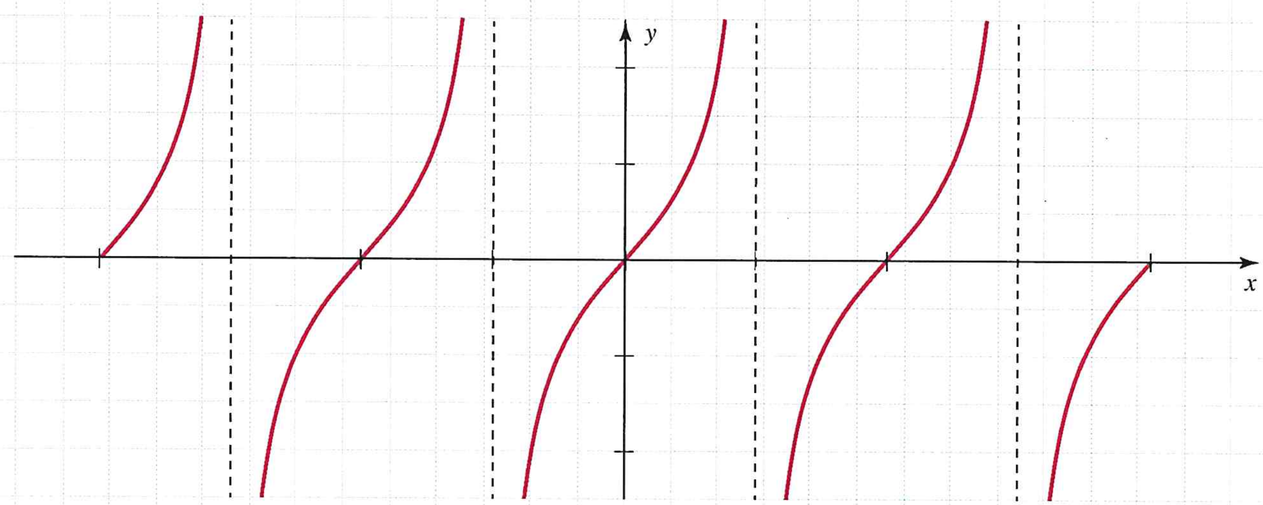
b) $\cos^{-1}\left(\cos\left(7\pi/6\right)\right)$



c) $\cot\left(\arccos\left(3/5\right)\right)$



Consider the graph of $y = \tan x$ over the interval $[-2\pi, 2\pi]$.

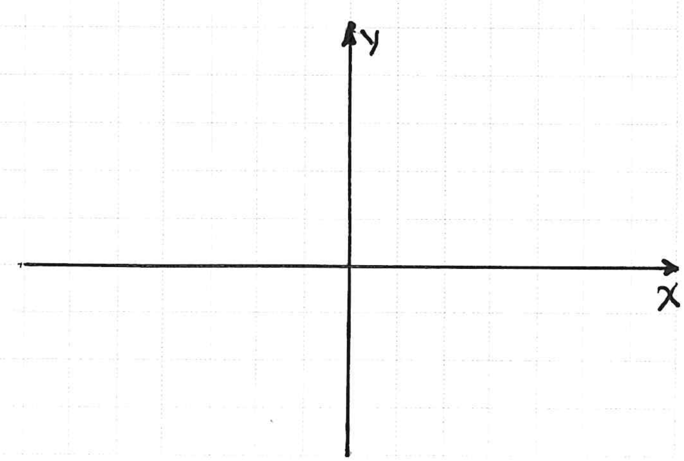


Note that the equation $\tan x = 1 \Rightarrow x = \tan^{-1}(1)$ has infinitely many solutions: $x =$

We want $x = \tan^{-1}(1)$ to have ONE solution.

We restrict the domain of $y = \tan x$ to

It's range is still $(-\infty, \infty)$.

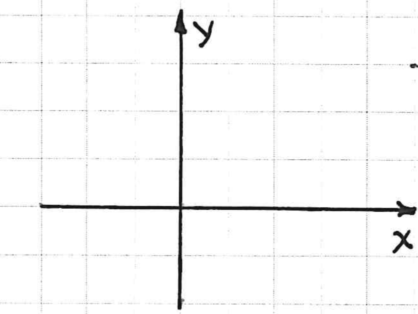


Then we DEFINE $y = \tan^{-1} x = \arctan x$

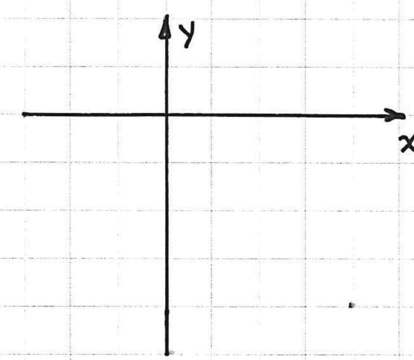
with Domain: and Range:

Ex. 3 Find the EXACT values.

a) $\tan^{-1}(\sqrt{3}/3)$



b) $\tan^{-1}(-1)$



c) $\sin(\arctan(-\sqrt{3}))$