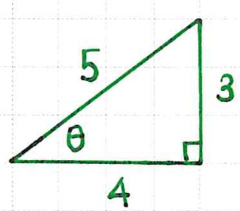
4.7 The Inverse Trigonometric Functions (Day 1)

Recall that if we know the sides of a right triangle, we can find the missing angles by using an inverse trig function.

Ex.

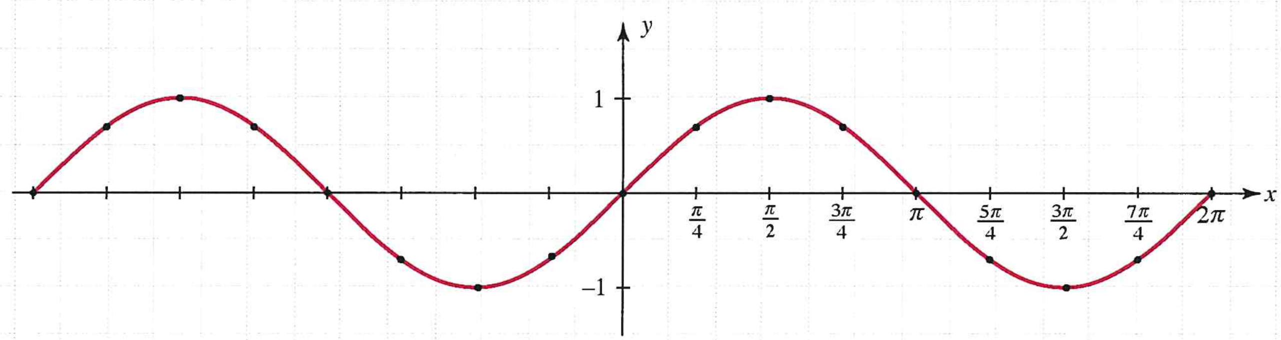


Now, $\sin\theta =$

The inverse sine function [$\sin^{-1}(x)$ or $\arcsin(x)$] takes one number as input ($3/5$) and returns one angle (0.644) as output.

The problem with finding inverse sine:

Consider the graph of $y = \sin x$ over the interval $[-2\pi, 2\pi]$.



Note that the equation: $\sin x = \frac{1}{2}$

has infinitely many solutions: $x =$

We want $x = \sin^{-1}(\frac{1}{2})$ to have

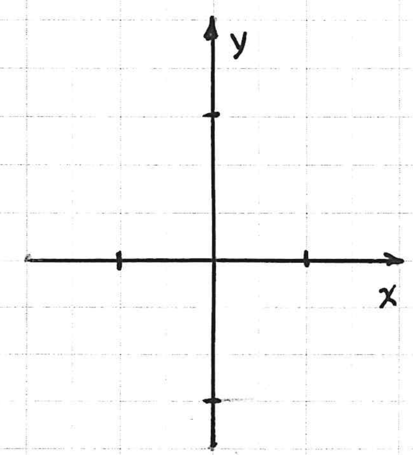
To fix this one-to-one problem, we restrict the domain of $y = \sin x$

to the interval Its range is still $[-1, 1]$.

Then we DEFINE $y = \sin^{-1}(x) = \arcsin(x)$ to be a function

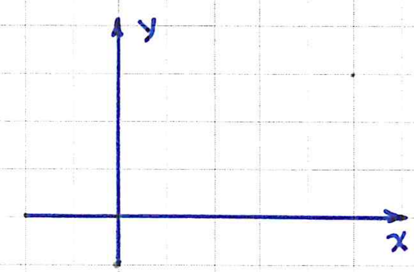
with Domain: and Range:

The graph of $y = \sin^{-1} x$:

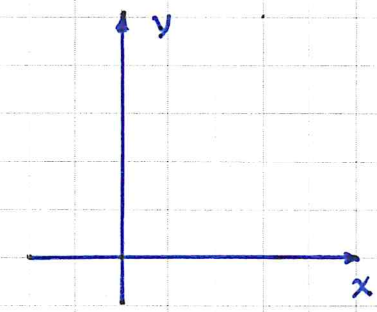


Ex. 1 Evaluate the following:

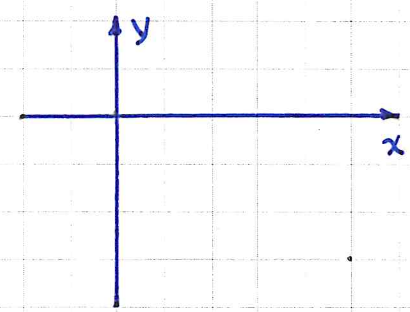
a) $\sin^{-1}(1/2)$



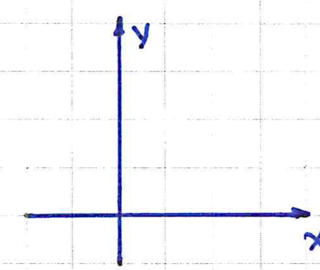
b) $\sin^{-1}(\sqrt{3}/2)$



c) $\arcsin(-1/2)$



d) $\arcsin(1)$



e) $\sin^{-1}(-2)$

Ex. 2 Use the properties of the inverse sine function to evaluate:

a) $\sin(\sin^{-1}(0.25))$

b) $\sin(\arcsin(-0.123))$

NOTE: When $-1 \le x \le 1$, then $\sin(\sin^{-1} x) =$

c) $\sin^{-1}(\sin(\pi/4))$

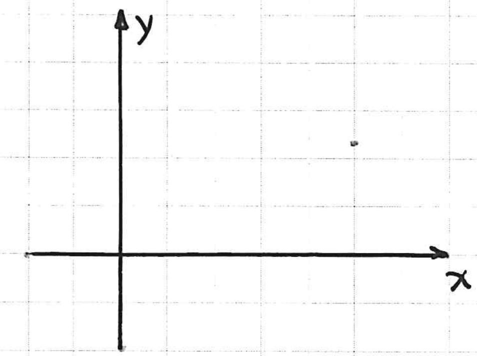
d) $\sin^{-1}(\sin(3\pi/4))$

e) $\arcsin(\sin(7\pi/6))$

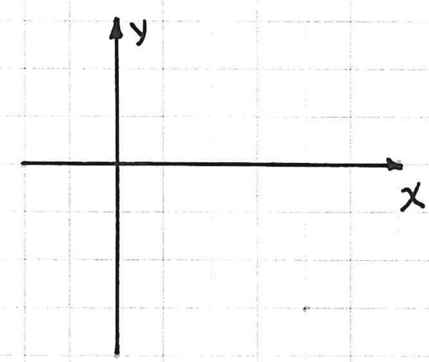
NOTE: When $-\pi/2 \le x \le \pi/2$, then $\sin^{-1}(\sin(x)) =$

Ex. 3 Find the EXACT values.

a) $\tan\left(\sin^{-1}(5/12)\right)$



b) $\cos\left(\arcsin(-3/5)\right)$



c) $\csc\left(\arcsin(3/4)\right)$