

**Definitions:**

Recall that when the graph of a function is moved either to the right or to the left, we say that it has been translated horizontally. The amount of the translation is called the **horizontal shift**.

For the trigonometric functions, this is known as the **Phase Shift**.

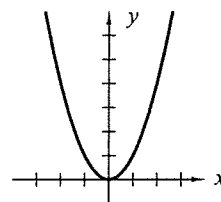


Figure 1:  $y = x^2$

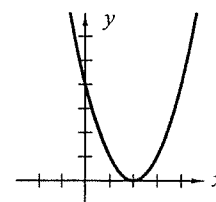
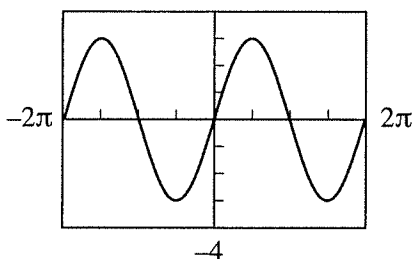


Figure 2:  $y = (x-2)^2$

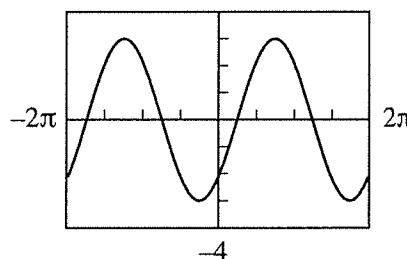
**Examples:** Consider the graphs of the following functions. Determine the phase shift for each one.

1.  $y = 3 \sin x$   $c =$   
4



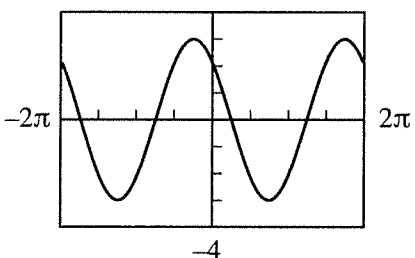
PS =

2.  $y = 3 \sin(x - \pi/4)$   $c =$   
4



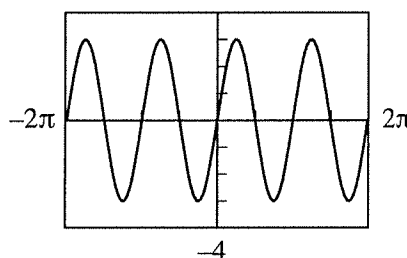
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3.  $y = 3 \sin(x + 3\pi/4)$   $c =$   
4



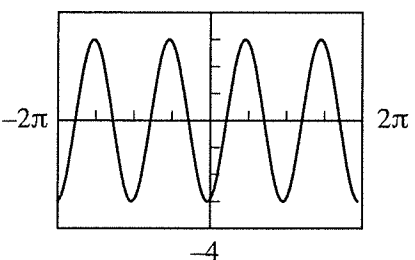
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4.  $y = 3 \sin 2x$   $c =$   
4



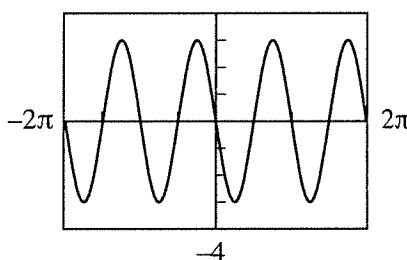
PS =

5.  $y = 3 \sin(2x - \pi/2)$   $c =$   
4



PS =

6.  $y = 3 \sin(2x + \pi)$   $c =$   
4



PS =

**Observation:** For the graphs of  $y = a \sin(bx - c)$  and  $y = a \cos(bx - c)$  the phase shift equals:

Procedure for sketching the graphs of  $y = a \sin(bx - c)$  and  $y = a \cos(bx - c)$ .

**Step 1** Find the amplitude:  $|a|$  and the period  $2\pi/b$ . Draw the ceiling and the floor.

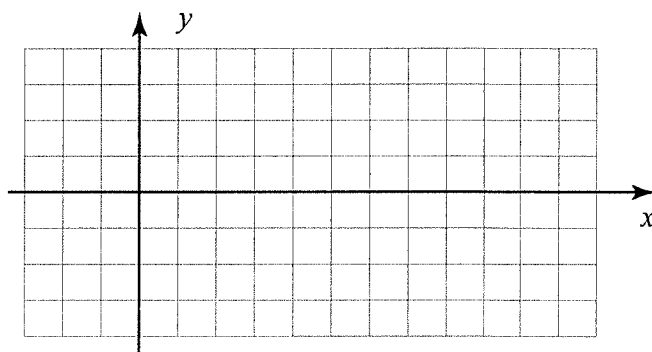
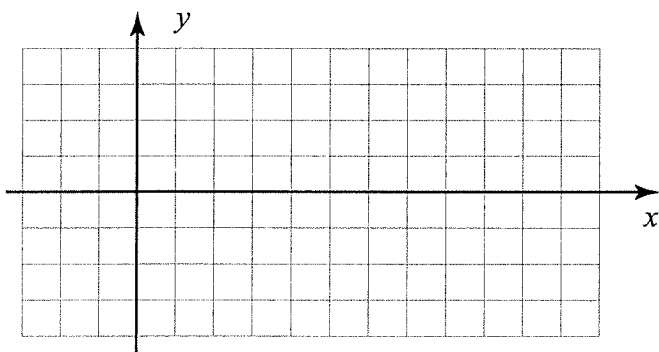
**Step 2** Solve  $bx - c = 0$ . Now,  $x = c/b$  is the left wall (Phase Shift). Draw the left wall. Then  $x = c/b + 2\pi/b$  is the right wall.

**Step 3** Divide the period into quarter points and sketch one cycle in the frame.

**Exercise:** Sketch the graphs of one period of the following functions on the coordinate system provided.

1.  $y = 2 \sin(x + \pi/4)$

2.  $y = 3 \cos(x - \pi/2)$



3.  $y = -3 \sin(\pi x + \pi/2)$

4.  $y = 2 \cos(\pi x/2 - \pi/4) + 1$

