4.4 Trigonometric Functions of Any Angle (Day 1)

Definition: Let θ be an angle in standard position and let $P(x,y)$ be any point on the terminal side of θ with $r = d(\overline{OP}) = \sqrt{x^2+y^2}$.

Then

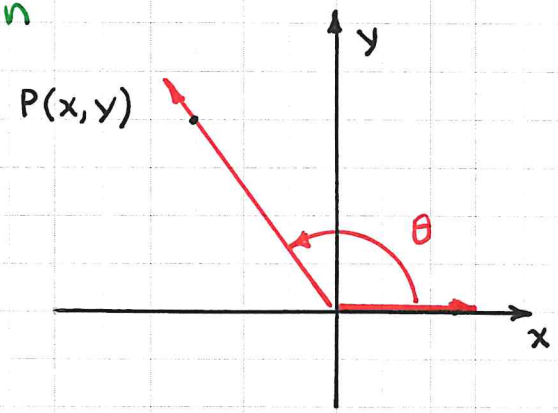


Figure 1

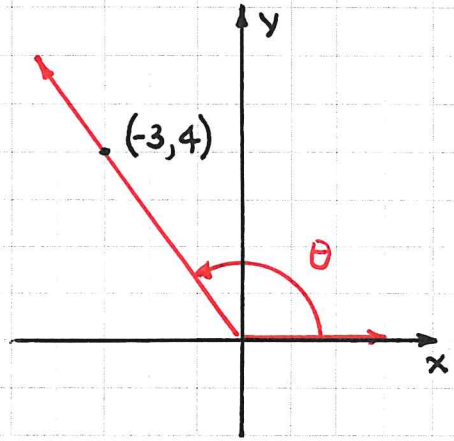
$\sin\theta =$ ____ $\quad \csc\theta =$ ____

$\cos\theta =$ ____ $\quad \sec\theta =$ ____

$\tan\theta =$ ____ $\quad \cot\theta =$ ____

Ex ① Find the values of the trigonometric functions of the given angles.

a)



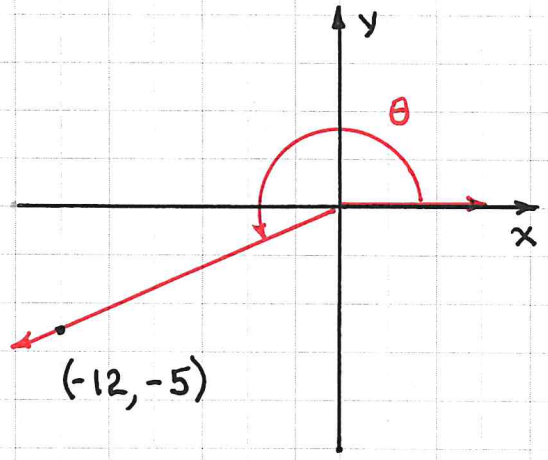
Note: $r =$

$\sin\theta =$ ____ $\quad \csc\theta =$ ____

$\cos\theta =$ ____ $\quad \sec\theta =$ ____

$\tan\theta =$ ____ $\quad \cot\theta =$ ____

b)



Note: $r =$

$\sin\theta =$ ____ $\quad \csc\theta =$ ____

$\cos\theta =$ ____ $\quad \sec\theta =$ ____

$\tan\theta =$ ____ $\quad \cot\theta =$ ____

Observations:

① $r = \sqrt{x^2 + y^2}$

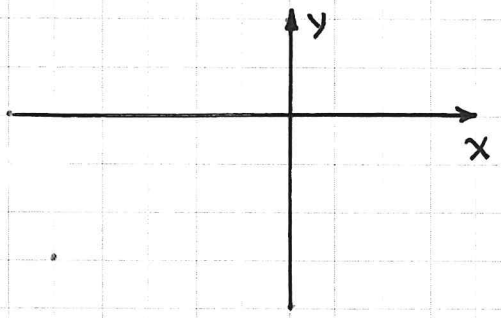
② Trigonometric functions can be

③ $\sin\theta$ and $\cos\theta$ are always

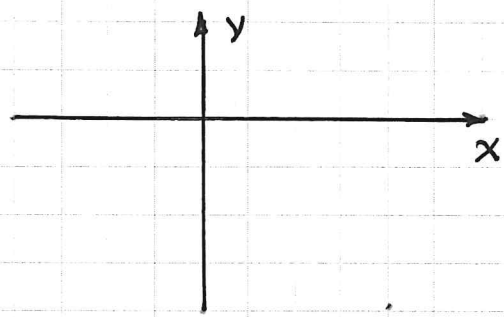
Definition: Let θ be an angle in standard position. Then its reference angle is the positive angle θ' formed by the terminal side of θ and the x-axis.

Ex. ② Find the reference angle θ'.

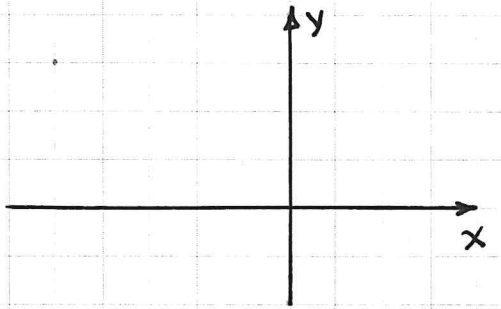
a) $\theta = 210°$



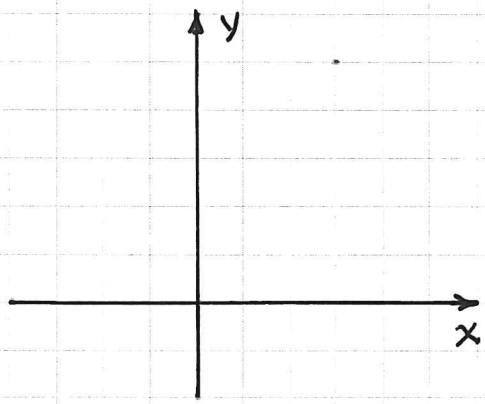
b) $\theta = -45°$



c) $\theta = 5\pi/6$

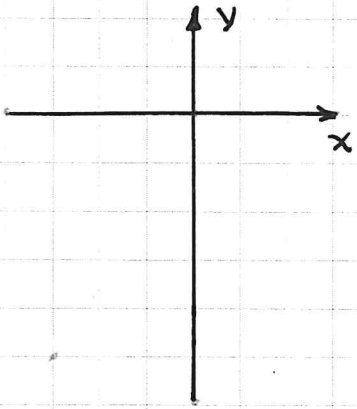


d) $\theta = 7\pi/3$

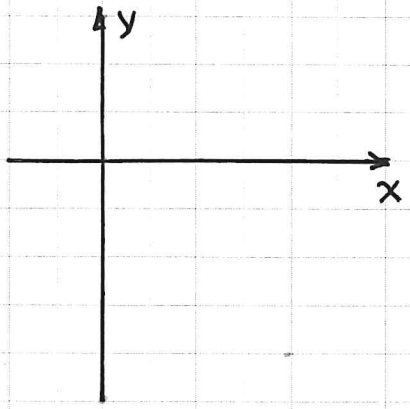


Ex. ③ Find the EXACT values.

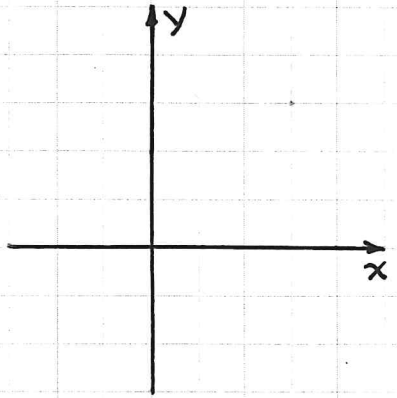
a) cos 240°



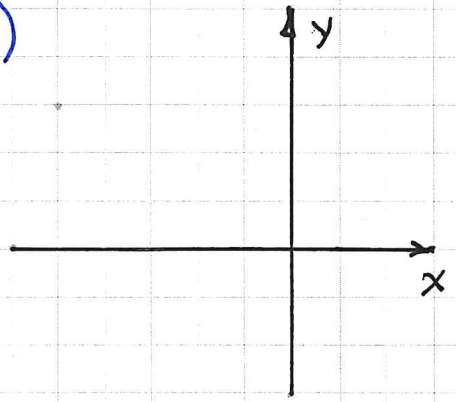
b) sin (-45°)



c) tan (405°)



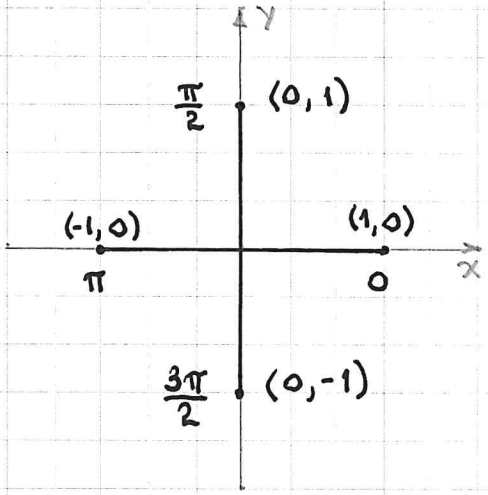
d) tan ($5\pi/6$)



A quadrant angle is an angle whose terminal side lies on an axis.

These include: $0°$ (0), $90°$ $(\pi/2)$, $180°$ (π), $270°$ $(3\pi/2)$

To evaluate the trigonometric function of these angles, use the definition given at the beginning of this lecture and the following diagram.



Ex. ④ Give the EXACT values of the following, where possible.

a) $\sin 0 =$

b) $\cos \pi =$

c) $\tan \frac{\pi}{2} =$

d) $\csc \frac{3\pi}{2} =$