

# Introduction to Right Triangle Trigonometry (§4.3)

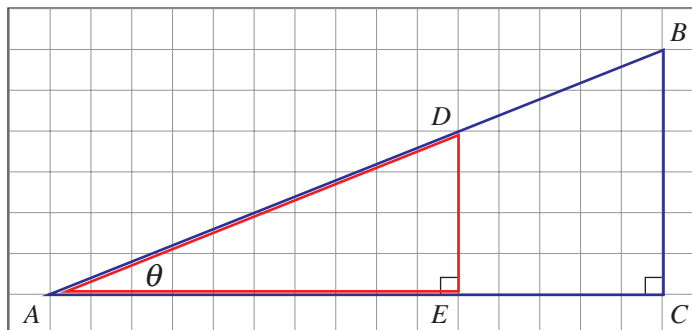
Consider the adjacent pair of overlapping triangles:

Note that  $\triangle ABC \sim \triangle ADE$ .

So:  $\frac{BC}{DE} = \frac{AC}{AE}$

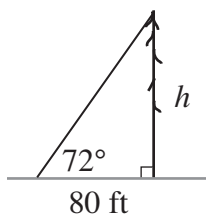
Which implies:  $\frac{BC}{AC} = \frac{DE}{AE}$

This ratio  $\left(\frac{BC}{AC}\right)$  depends only on the size of angle  $\theta$ .

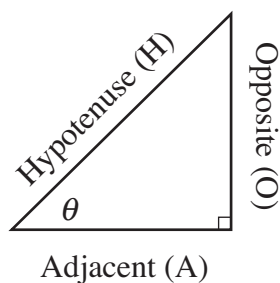


**Definition:** The *tangent* of angle  $\theta$  is given by  $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

**Example 1:** Use the tangent ratio to find the height of the tree in the figure below.



### Definition of the Trigonometric Ratios

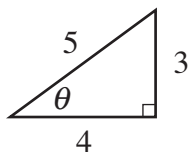


Sine of  $\theta = \sin \theta = \frac{\text{Opposite (O)}}{\text{Hypotenuse (H)}}$

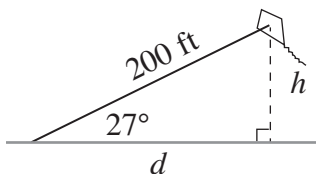
Cosine of  $\theta = \cos \theta = \frac{\text{Adjacent (A)}}{\text{Hypotenuse (H)}}$

Tangent of  $\theta = \tan \theta = \frac{\text{Opposite (O)}}{\text{Adjacent (A)}}$

**Example 2:** Find  $\sin \theta$ ,  $\cos \theta$ , and  $\tan \theta$  for the following triangle.



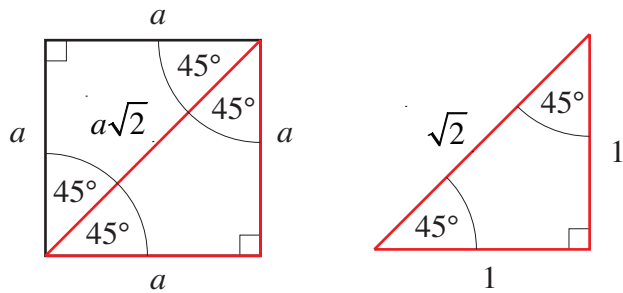
**Example 3:** Find the two missing sides in the figure below.



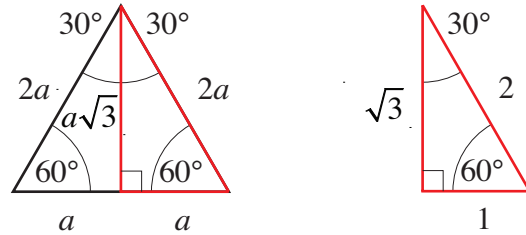
## Two Special Right Triangles

There are two special right triangles that frequently arise in trigonometry, the  $45^\circ-45^\circ-90^\circ$  triangle and the  $30^\circ-60^\circ-90^\circ$  triangle. The sides of each of these triangles have special relationships.

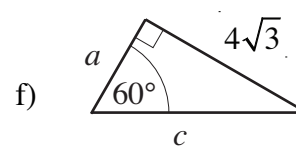
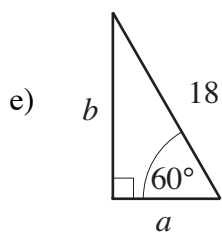
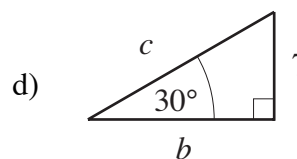
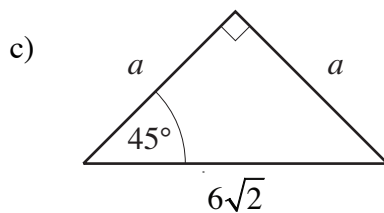
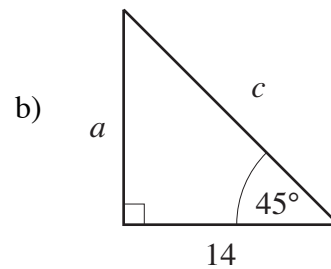
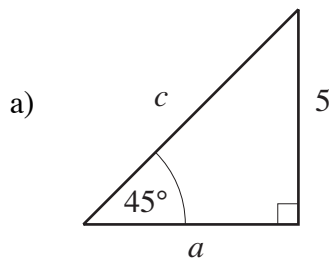
**$45^\circ-45^\circ-90^\circ$**



**$30^\circ-60^\circ-90^\circ$**



**Example 4:** For each of the following triangles, find the missing side lengths.



**Example 5:** Find the **exact values** of the following trigonometric ratios.

a)  $\sin 30^\circ$

b)  $\cos 45^\circ$

c)  $\tan 60^\circ$

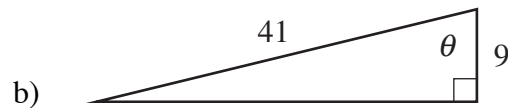
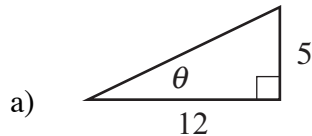
MATH 27 **Introduction to Right Triangle Trigonometry** Assn #24

**NOTE:** These problems are to be done on Engineering paper, using the standard homework format.

1. Complete the following definitions of the trigonometric ratios by using the abbreviations: Hypotenuse  $\rightarrow$  H, Opposite  $\rightarrow$  O, and Adjacent  $\rightarrow$  A.

a)  $\sin \theta = \frac{?}{?}$                       b)  $\cos \theta = \frac{?}{?}$                       c)  $\tan \theta = \frac{?}{?}$

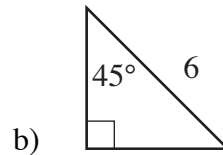
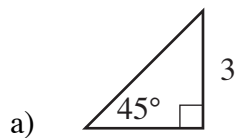
2. Use the Pythagorean theorem to find the missing side of the right triangle and then write the trigonometric ratios ( $\sin \theta$ ,  $\cos \theta$ ,  $\tan \theta$ ) for the given angle.



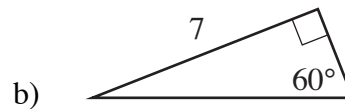
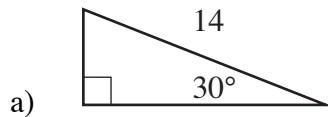
3. Given that  $\triangle ABC$  is a right triangle with right angle at  $C$ , find **exact** answers to the following.

a) Find  $\tan A$  if  $\sin A = 7/25$                       b) If  $\tan B = \sqrt{5}/2$  find  $\cos A$ .

4. For each triangle, find the **exact** values of the missing side lengths.



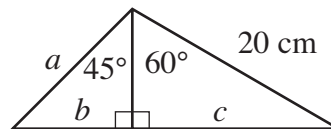
5. For each triangle, find the **exact** values of the missing side lengths.



6. Complete the following table with **exact** values.

$\theta$	$30^\circ$	$45^\circ$	$60^\circ$
$\sin \theta$			
$\cos \theta$			
$\tan \theta$			

7. Find the **exact** values of each unknown length in the adjacent figure.



8. An equilateral triangle is inscribed in a circle with radius 10 m. What is the **exact** perimeter of the triangle?

