MATH 27 **Properties of Logarithms (§3.3)**

Recall: If $a > 0$ and $a \neq 1$ then $\log_a x = y$ is equivalent to $x = a^y$.

Properties of the Definition of the Logarithm

(i) $\log_a 1 = 0$

(ii) $\log_a a = 1$

(iii) $\log_a a^x = x$

(iv) $a^{\log_a x} = x$

Computational Properties of the Logarithm

Product: $\log_a (UV) = \log_a U + \log_a V$

Quotient: $\log_a \left(\frac{U}{V}\right) = \log_a U - \log_a V$

Power: $\log_a (U)^n = n \log_a U$

1. Use the properties of logarithms to expand each expression completely.

   a) $\ln(P \cdot V \cdot T)$

   b) $\log(m \cdot c^2)$

   c) $\log_5 \left(\frac{x}{yz}\right)$

   d) $\log_2 (x^2 - y^2)$

2. Use the properties of logarithms to condense each expression to the logarithm of a single quantity.

   a) $\frac{1}{2} \ln x + 5 \ln y - 2 \ln z$

   b) $\log(x^2 - 9) - \log(x + 3)$

3. Find the exact value of the logarithm without using a calculator.

a) $\log_6 \sqrt[3]{6}$

b) $\log_2(-16)$

c) $\log_4 2 + \log_4 32$

d) $\ln \sqrt[5]{e^3}$

Change-of-Base Formula: $\log_a x = \dfrac{\log_b x}{\log_b a}$

4. Use the change-of-base formula and a calculator to evaluate the following.

a) $\log_2 10$

b) $\log_\pi e$

5. Graph the function $f(x) = \log_4(x)$ in the window [–10,10] x [–2,18]