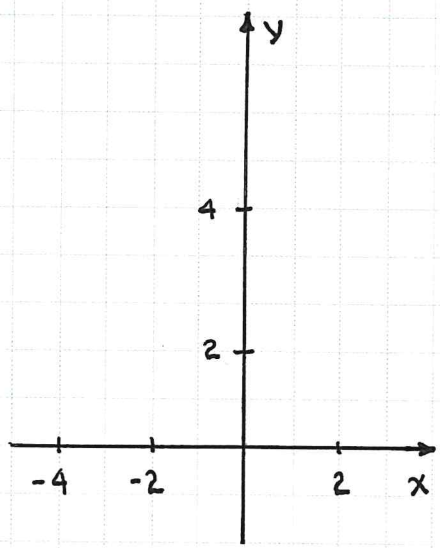
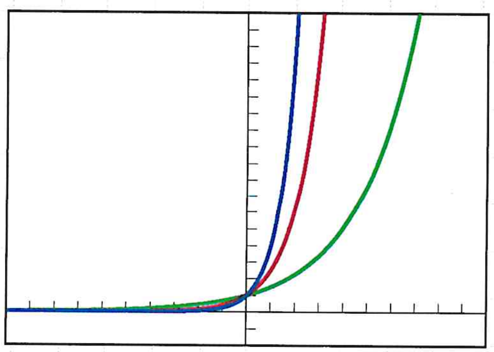
3.1 Exponential Functions

Definition: A function of the form $f(x) = a^x$ (where $a > 0$ and $a \neq 1$) is called an exponential function.

Ex. 1 Sketch: $f(x) = 2^x$

x	y
3	
2	
1	
0	
-1	
-2	





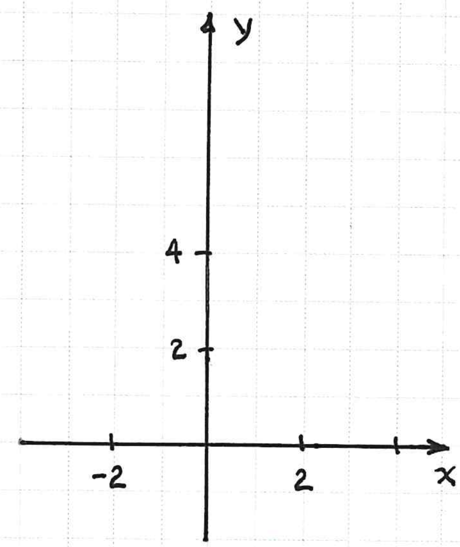
$y =$

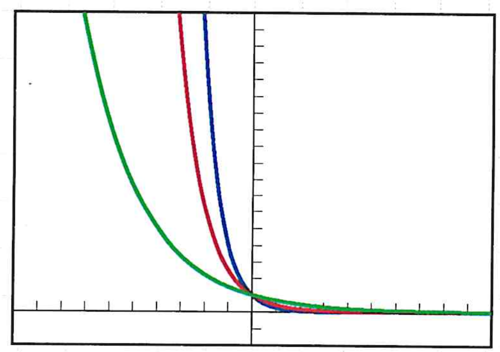
$y =$

$y =$

Ex. 2 Sketch: $g(x) = (\frac{1}{2})^x$

x	y
2	
1	
0	
-1	
-2	
-3	





$y =$

$y =$

$y =$

Notes: 1) $p(x) = x^2$ is not an exponential function.

2) $g(x) = (-2)^x$ is not an exponential function.

3) For $a > 1$ we have exponential

4) For $0 < a < 1$ we have exponential

5) For $f(x) = a^x$ the x-axis is the

6) Of all the choices we have for a, there is one that is often convenient. It is the irrational number

It's called the natural base.

Ex. 3 Under ideal conditions, a colony of bacteria will grow exponentially for a limited time. Suppose that the population of a certain type of bacteria is $P(t) = 1000e^{0.293t}$ (t is in hours).

a) Find $P(0)$

b) Find $P(10)$

Exponential Functions (§3.1)

Compound Interest

When funds are deposited into an account paying compound interest, the following formula is used to calculate the amount in the account after a certain number of years:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Where A = amount in the account (future value), P = principal (present value), r = annual interest rate, n = number of compoundings per year, and t = number of years.

4. In the year 1626, \$24 is deposited into an account paying $3\frac{1}{2}\%$. Find the amount in the account now
   a) if it earns simple interest ($A = P + Prt$).

   b) if it earns interest compounded monthly.

   c) if it earns interest compounded daily.

Continuous Compound Interest

If interest is compounded continuously, the formula becomes $A = Pe^{rt}$.

5. Find the amount in the account now for Example 4 if it earns continuous compound interest

Carbon Dating

6. Radioactive carbon 14 is used in the process known as **carbon dating**. The half-life of carbon 14 is approximately 5730 years. The function $Q(t) = 100e^{-0.0001216t}$ gives the quantity present, in grams, of a sample after t years.

   a) Find $Q(0)$

   b) Find $Q(2000)$

   c) Find $Q(5730)$