2.6 Rational Functions (Part 1)
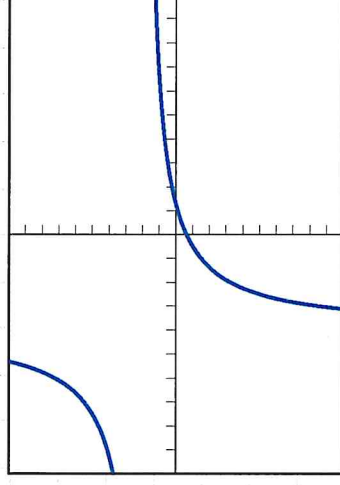
Definition: A rational function is the quotient of two polynomials.

$$f(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \ldots + b_1 x + b_0}$$

Ex. ① Consider the graph of $g(x) = \frac{4x-5}{2x+8}$

Note that the function is undefined
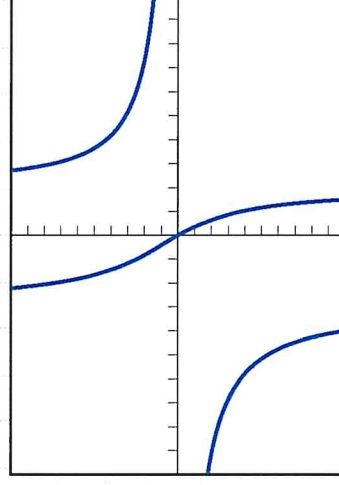
when $x = -4$.

This results in a

HA:

VA:

Ex. ② Consider the graph of $h(x) = \frac{15x}{x^2 + x - 6}$
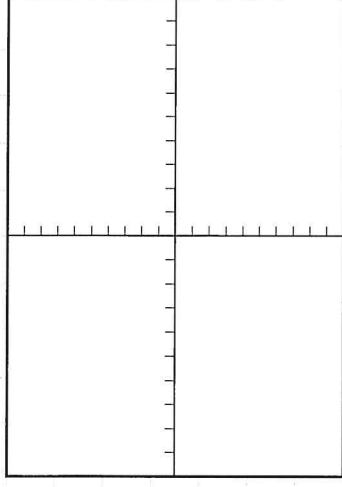

$h(x)$ is undefined when

HA:

VA:

Ex. ③ Use a calculator to sketch the

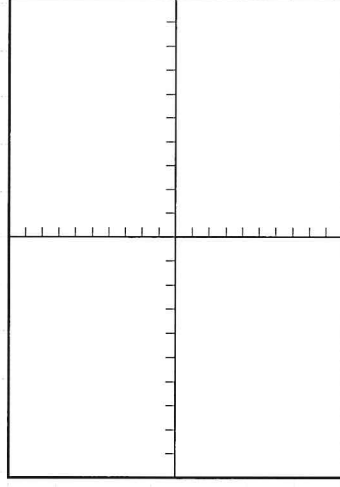graph of $m(x) = \dfrac{3x^2}{x^2 - 16}$

$m(x)$ is undefined when

VA: HA:

Ex. ④ Use a calculator to sketch the

graph of $n(x) = \dfrac{15x}{x^2 + 1}$

$n(x)$ is undefined when

VA: HA:

Conclusion: When simplified completely, the rational function

$f(x) = \dfrac{N(x)}{D(x)}$ has a VA at the

of $D(x)$.

Definition: The line $y = b$ is a horizontal asymptote (HA) of f

if $f(x) \to b$ as $x \to \infty$ or $x \to -\infty$.

Conclusion: The graph of f to $f(x) = \dfrac{N(x)}{D(x)} = \dfrac{a_n x^n + \ldots + a_0}{b_m x^m + \ldots + b_0}$
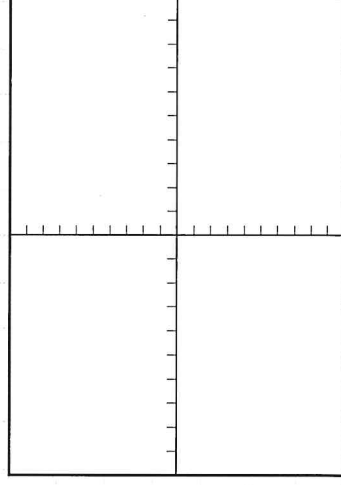
has at most one horizontal asymptote determined by comparing

the degrees of $N(x)$ and $D(x)$.

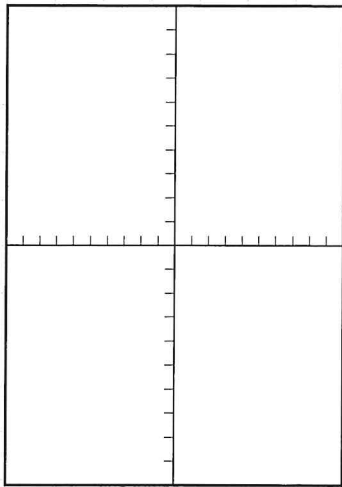a) If the graph of f has the line $y = 0$ as its HA.

b) If the graph of f has the line $y = a_n/b_m$ as its HA.

c) If the graph of f does not have a HA.

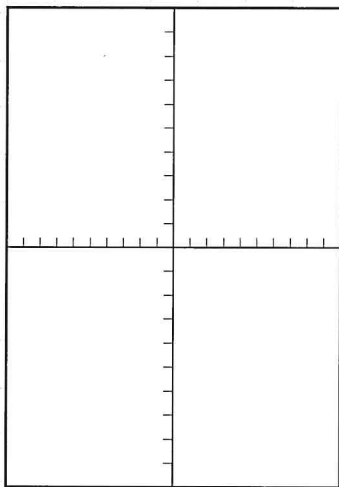Ex. ⑤ Sketch: $r(x) = \dfrac{28x - 44 - 2x^2}{x^2 - 6x + 9}$

Ex. ⑥ Sketch: $g(x) = \dfrac{3x^2 - 24x + 12}{x^2 + 4}$

HA: VA:

Ex. ⑦ Sketch: $r(x) = \dfrac{x^2 - 1}{x^2 - 2x - 3}$

HA: VA: