

1.6 Inverse Functions

Ex. ① Suppose that $f(x) = \sqrt[3]{x+4}$ and $g(x) = x^3 - 4$

Find the following:

a) $(f \circ g)(-1) =$

b) $(f \circ g)(4) =$

c) $(f \circ g)(x) =$

d) $(g \circ f)(x) =$

Definition: Two functions f and g are inverses of each other if $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$.

Note: Parts (c) and (d) of Ex. ① show that

$$f(x) = \sqrt[3]{x+4} \quad \text{and} \quad g(x) = x^3 - 4$$

are

Notation: We write $f^{-1}(x) = g(x)$ and $g^{-1}(x) = f(x)$.

Caution:

Important Questions

IQ #1: Do all functions have inverses?

IQ #2: Which functions DO have inverses?

Theorem: A function f has an inverse f^{-1} if and only if f is a

* The graph must pass the

IQ #3: How are the graphs of

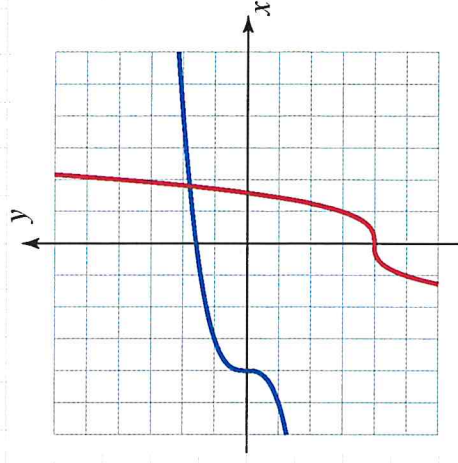
$$y = f(x) \text{ and } y = f^{-1}(x)$$

related?

Consider: $f(x) = \sqrt[3]{x+4}$

and $g(x) = f^{-1}(x) = x^3 - 4$

The graphs are



IQ #4: Given a one-to-one function f , how do we find f^{-1} ?

1)

2)

3)

4)

Ex. ② Given: $f(x) = \sqrt[3]{x+4}$. Find $f^{-1}(x)$.

Solution: 1)

Ex. ③ Given: $f(x) = x/(x+4)$. Find $f^{-1}(x)$.

Solution: 1)