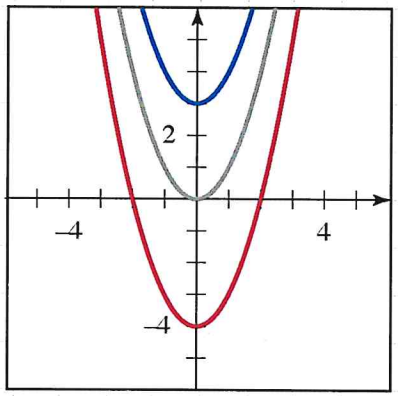
1.4 Transforming the Graphs of Functions

(I) Vertical and Horizontal Shifts

The graph of a function can be shifted vertically or horizontally by adding a constant to the original function or to x.

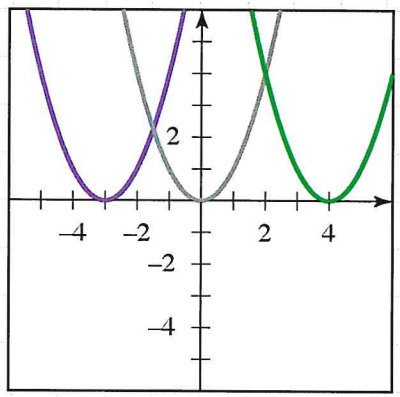
Vertical Shift

$y = f(x) = x^2$



Horizontal Shift

$y = f(x) = x^2$



V. Shift c units up:

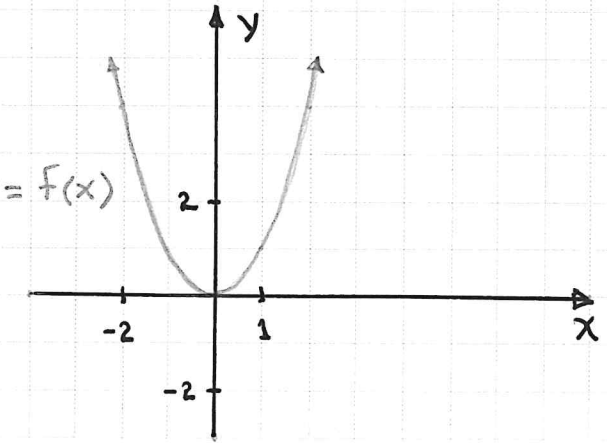
V. Shift c units down:

H. Shift c units left:

H. Shift c units right:

Ex. ① Given that $f(x) = x^2$, sketch the graph of $h(x) = f(x-3) - 2$

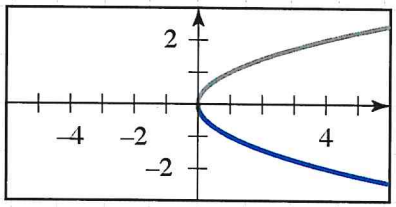
Steps:



(II) Reflections in the x and y axes.

$y = f(x) = \sqrt{x}$

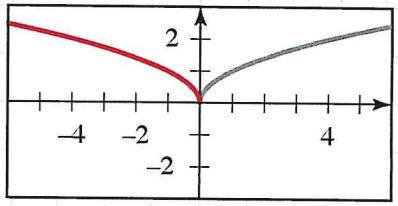
$m(x) = -\sqrt{x}$



Reflection in the x-axis:

$y = f(x) = \sqrt{x}$

$h(x) = \sqrt{(-x)}$

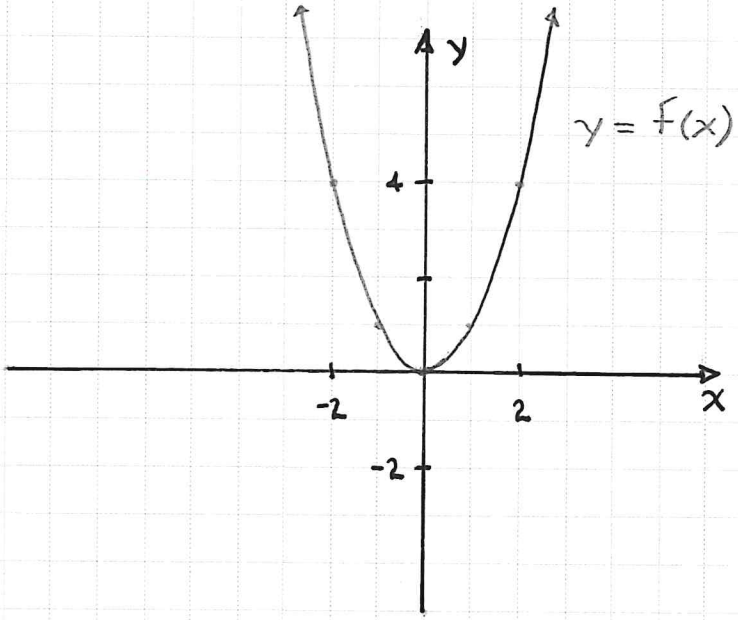


Reflection in the y-axis:

Ex. ② Given that $f(x) = x^2$, sketch the graph of

$h(x) = -f(x+5) + 3$

Steps:



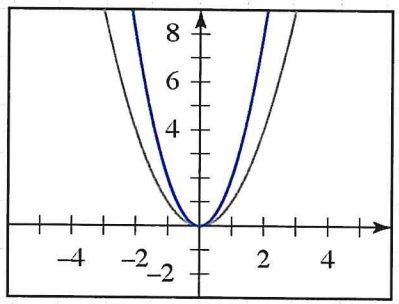
(III) Vertical Scaling

When a function is multiplied by a positive constant, the original graph is either stretched or compressed.

Stretch ($c>1$)

$y = f(x) = x^2$

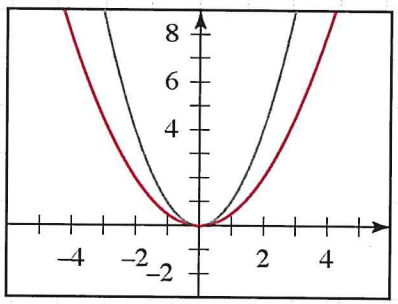
$m(x) = 2x^2$



Shrink ($0<c<1$)

$y = f(x) = x^2$

$n(x) = \frac{1}{2}x^2$



V. Stretch when $c>1$:

V. Shrink when $0<c<1$:

Ex. ③ Given that $f(x) = |x|$, sketch the graph of

$h(x) = \frac{-1}{2} f(x-1) + 4$

Steps: