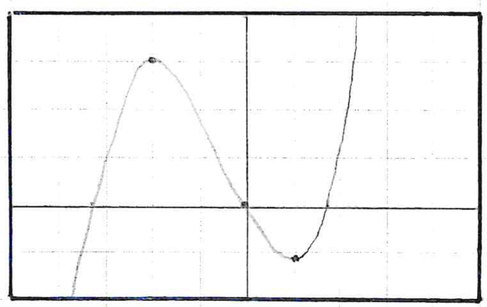
1.3 Graphs of Functions

Recall that if $y = f(x)$ then for every x in the domain of f there can be only one value of y. This leads to a simple graphical test for functions.

The Vertical Line Test

A set of points in the coordinate plane is the graph of y as a function of x if and only if no vertical line intersects the graph in more than one point.

Ex. ① Sketch the graph of $f(x) = x^3 + 3x^2 - 24x$ in the window $[-10, 10] \times [-50, 100]$.



Notes: 1) The graph appears to have a "peak" (relative maximum) at $x =$

Use [2nd] [CALC] [maximum]

2) The graph appears to have a "valley" (relative minimum) at $x =$

Use [2nd] [CALC] [minimum]

Note: For the following material, we will assume that the graph of a function is traced out from left to right. The direction in which x increases.

For the graph of the function in Ex. ①, it can be seen that the graph is rising when x is less than -4. We say that the function is ________ on the interval: $(-\infty, -4)$

The graph is ________ when x is between -4 and 2. We say that the function is ________ on the interval: $(-4, 2)$

If a function neither rises nor falls, we say that it is ________.

Note that a function's behavior will change at a relative maximum or minimum.

Definition: A piece-wise function is a function that is defined by two or more equations over a specified domain.

Ex. ② Consider the function:

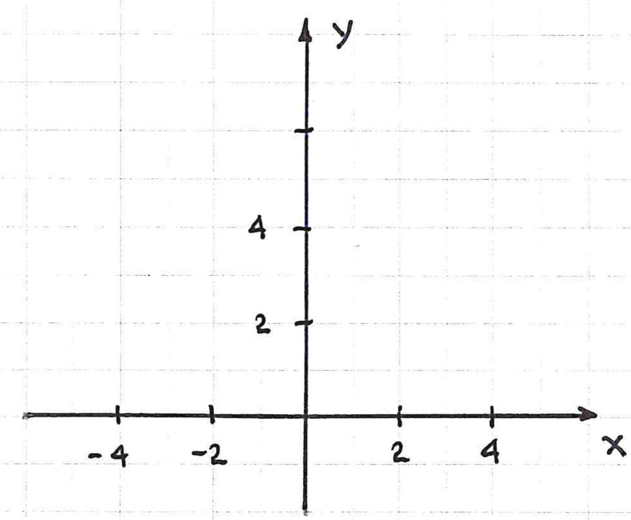
$$g(x) = \begin{cases} 4 - x/2, & \text{when } x \le 0 \\ x^2, & \text{when } x > 0 \end{cases}$$

a) Find $g(2)$ =

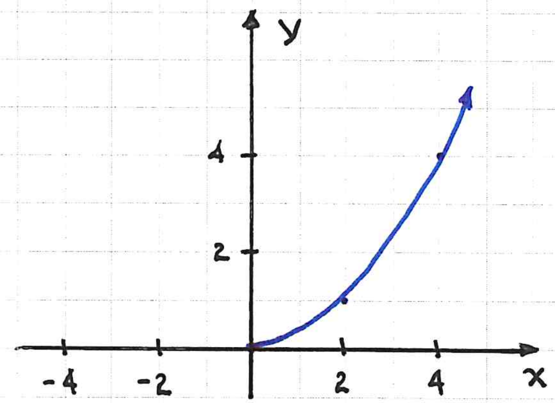
b) Find $g(-4)$ =

c) Find $g(0)$ =

d) Sketch the graph of $y = g(x)$ by hand.



Consider the partial graph of the function $y = f(x)$.



Reflect the graph in the y-axis.

A function that is symmetric in the y-axis is called an   function.

Note that

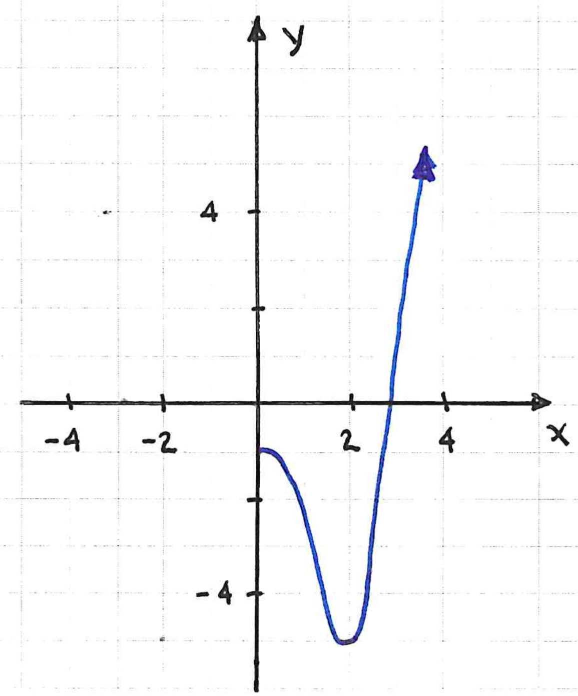
In fact, for any

Ex. ③ a) Show that the function

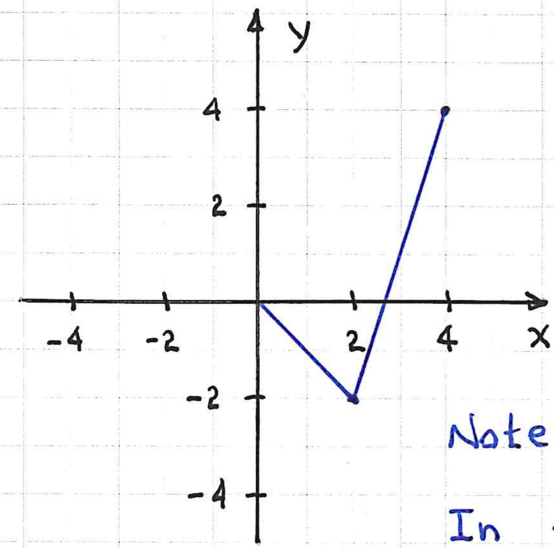
$g(x) = \frac{1}{4}x^4 - 2x^2 - 1$

is an even function.

b) Complete the graph of $y = g(x)$.



Consider the partial graph of the function $y = f(x)$.



Reflect the graph in both axes.

A function with this property

is called an ______ function.

Note that

In fact, for any

Ex. ④ a) Show that the function $g(x) = \frac{1}{4}(x^3 - 16x)$ is an odd function.

b) Sketch the graph of

$y = g(x)$ in the

window $[-6, 6] \times [-8, 8]$.