

1.2 Functions

In English, the word "function" refers to what something does.

In math, a function is a rule that connects one set to another set in a particular way.



A function takes each value from an input set (Domain) and relates it to a value in an output set (Range).

Ex. ① Rule: Output the number of sides.

Input	Output
Triangle	
Pentagon	
Octagon	

Most functions that you'll encounter in calculus will be a little more abstract than that. They'll usually just relate one variable to another in the form of an equation.

Ex. ② Complete the table.

Rule: $y = x^2 - 4$	
Input	Output
x	y
-3	
0	
2	

There is an important limitation about functions that we need to know.

Definition: A function relates each element of one set (Domain) with exactly one element of another set (Range).

Ex. ③ Consider the equation: $x^2 + y^2 = 4$

Solve for y : $y^2 = 4 - x^2 \Rightarrow y =$

Rule: $y =$

Input	Output
0	
1	
$\sqrt{3}$	

Note:

For each value of x we get two values of y .

Because certain operations in math are undefined (eg. division by zero) the domain of a function may need to be restricted.

Ex. ④ Find the domain of the function.

Function	Domain
$y = x^2 - 4$	
$y = 1/t$	
$y = \sqrt{4 - x^2}$	

Function Notation

We write a function like $y = x^2 - 4$ in this form: $f(x) = x^2 - 4$

We read this as:

Note that x is just a placeholder. It shows us where the input goes and what happens to it. Any letter will do.

Ex. ⑤ Given that $f(x) = x^2 + x$, find the following:

a) $f(2) =$

b) $f(-3) =$

c) $f(a) =$

d) $f(2x) =$

e)

f) $f(x+4) =$

Note: There is nothing special about the letter f . It serves as a name.

We could have written:

Definition: The Difference Quotient for a function f is given by

the formula:
$$DQ = \frac{f(x+h) - f(x)}{h} \quad (\text{where } h \neq 0)$$

Ex. ⑥ Find the DQ for each of the following functions.

a) $f(x) = 1 - 2x$

$$DQ = \frac{f(x+h) - f(x)}{h}$$

b) $g(x) = x^2 - x$

$$DQ =$$