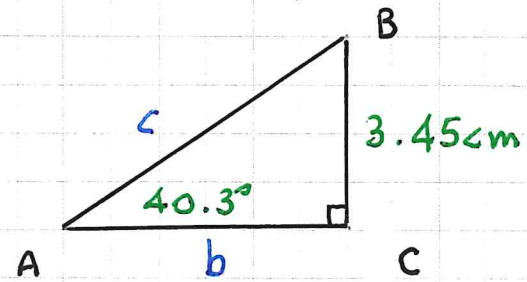
4.8 Right Triangle Applications

Ex. ① Solve the triangle. (Find the missing parts.)



$A = 40.3°$ $\quad a = 3.45$ cm

$B = 49.7°$ $\quad b = 4.07$ cm

$C = 90°$ $\quad c = 5.33$ cm

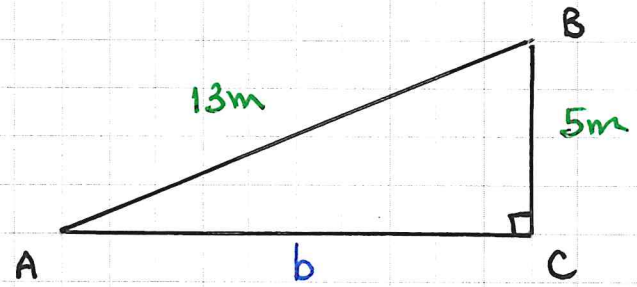
1) Find $B = 90° - 40.3° = 49.7°$

2) Find b. $\tan 40.3° = \frac{3.45}{b} \Rightarrow b \tan 40.3° = 3.45$

$$\Rightarrow b = \frac{3.45}{\tan 40.3°} = 4.07$$

3) Find c. $\sin 40.3° = \frac{3.45}{c} \Rightarrow c = \frac{3.45}{\sin 40.3°} = 5.33$

Ex. ② Solve the triangle.



$A = 22.6°$ $a = 5\text{ m}$

$B = 67.4°$ $b = 12\text{ m}$

$C = 90°$ $c = 13\text{ m}$

1) Find b. (The PT)

$$a^2 + b^2 = c^2 \Rightarrow 5^2 + b^2 = 13^2 \Rightarrow b^2 = \sqrt{13^2 - 5^2} \Rightarrow b = 12$$

2) Find A: $\sin A = \frac{5}{13} \Rightarrow A = \sin^{-1}\left(\frac{5}{13}\right) = 22.6°$

3) Find B: $\cos B = \frac{5}{13} \Rightarrow B = \cos^{-1}\left(\frac{5}{13}\right) = 67.4°$

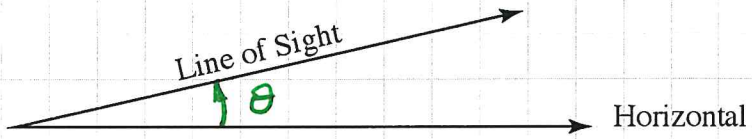
MATH 27

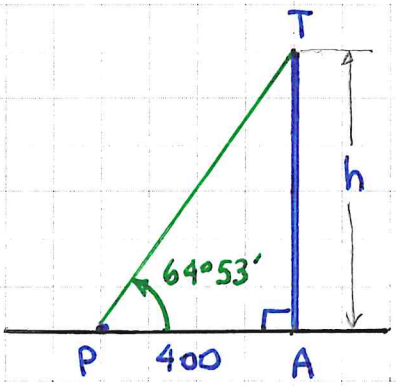
Right Triangle Applications (§4.8)

Ex. 3. From a point 400 ft. from the base of the TransAmerica pyramid in San Francisco, the angle of elevation to the top of the pyramid is found to be 64°53'. Find the height of the pyramid to the nearest foot.

The Angle of Elevation:

UP





Now, $\tan 64°53' = \frac{h}{400}$

$\Rightarrow h = 400 \tan 64°53'$

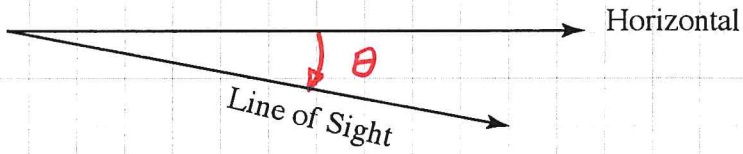
$= 400 \tan\left(64° + \frac{53}{60}\right)$

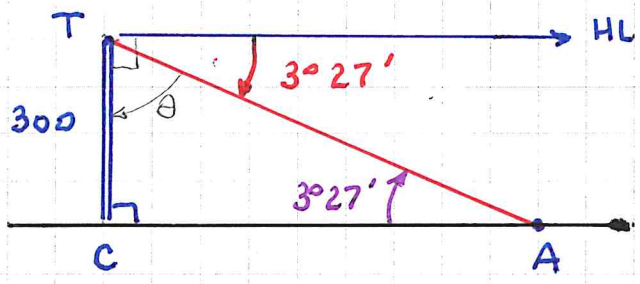
$= \boxed{853 \text{ ft.}}$

Ex. 4. An observer in a castle tower 300 feet above level ground notices an approaching army and finds that the angle of depression to the army is 3°27'. Find the distance from the army to the observer to the nearest foot.

The Angle of Depression:

DOWN





Find AT

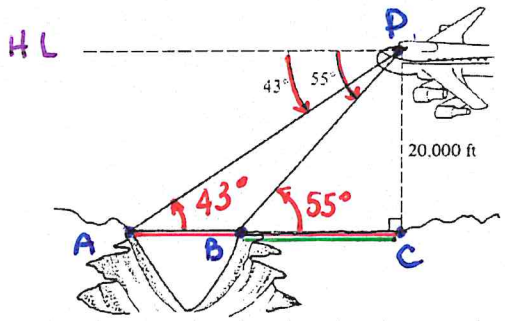
So, $\sin 3°27' = \frac{300}{AT} \Rightarrow AT = \frac{300}{\sin 3°27'}$

$\Rightarrow AT = \frac{300}{\sin(3 + 27/60)} = \boxed{4985 \text{ ft.}}$

MATH 27

Right Triangle Applications (§4.8)

Ex. 5. An airplane is flying toward a canyon. From the plane, the angle of depression to the near side of the canyon is 55° and the angle of depression to the far side is 43°. If the altitude of the plan is 20,000 feet, then what is the distance across the canyon to the nearest foot?



Goal: Find AB

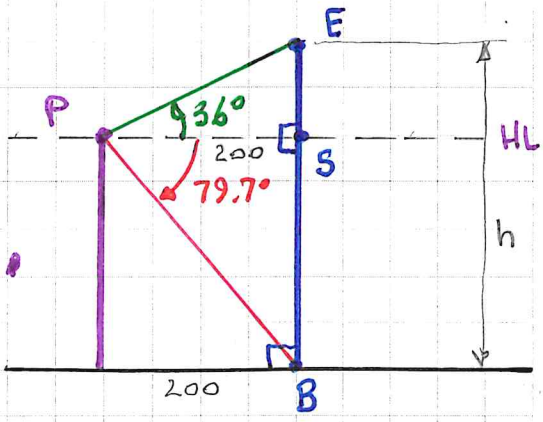
Note: $AB = AC - BC$

Now, $\tan 43° = \frac{20K}{AC} \Rightarrow AC = \frac{20K}{\tan 43°} = 21{,}447$

and $\tan 55° = \frac{20K}{BC} \Rightarrow BC = \frac{20K}{\tan 55°} = 14{,}004$

So $AB = AC - BC =$ **7443 ft**

Ex. 6. From the roof of a building 200 feet from a line through the center of the Empire State Building, the angle of elevation to the top of the ESB is 36° while the angle of depression to its base is 79.7°. Find the height of the ESB to the nearest foot.



Goal: Find EB

Note: $EB = ES + SB$

Now, $\tan 36° = \frac{ES}{200} \Rightarrow ES = 200 \tan 36°$

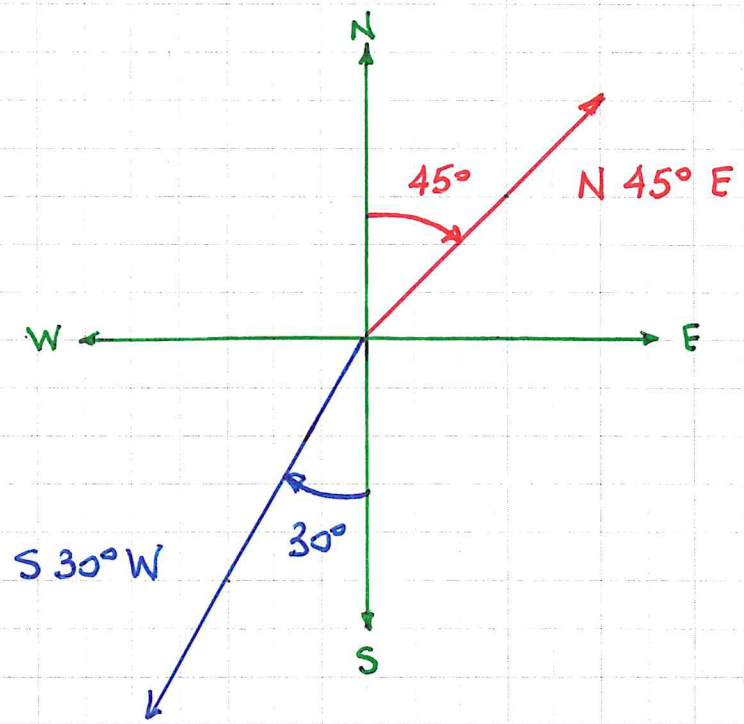
and $\tan 79.7° = \frac{SB}{200} \Rightarrow SB = 200 \tan 79.7°$

So $EB =$ **1246 ft**

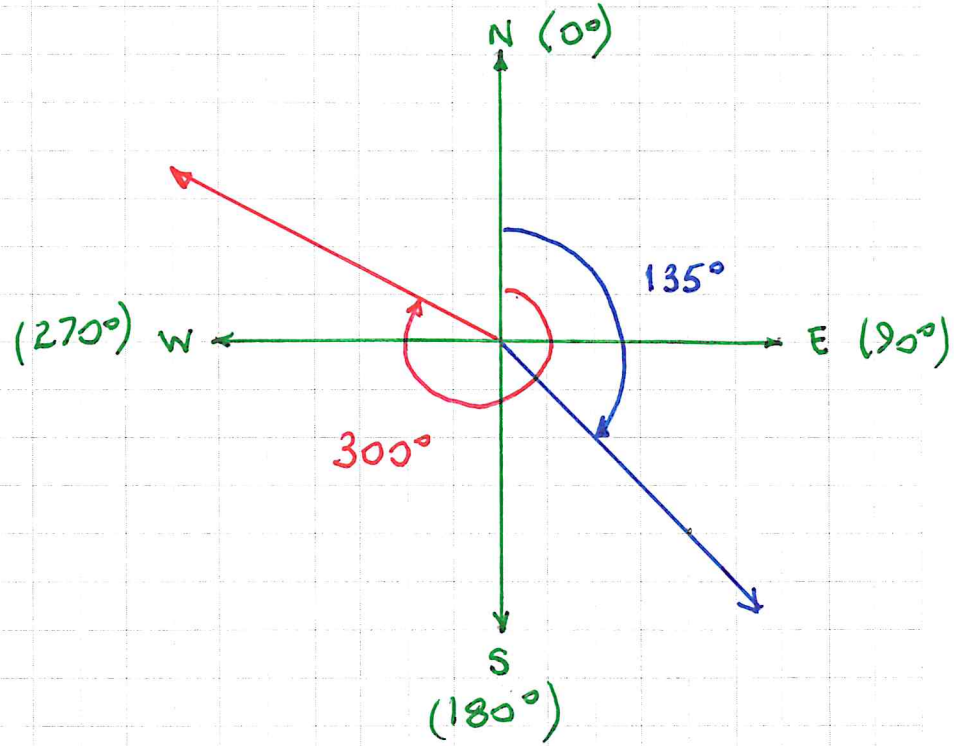
Specifying Direction

There are two common methods used for specifying direction.

I) Bearing



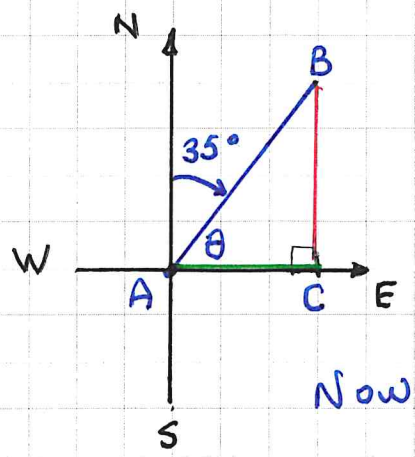
II) Heading



MATH 27

Right Triangle Applications (§4.8)

Ex. 7. A plane leaves an airport flying at 365 miles per hour on a heading of 35°. To the nearest mile, how far north and east will the plane be from the airport after two hours?



Note: $\theta = 90° - 35° = 55°$

and after two hours the plane has flown $365 \times 2 = 730 = AB$

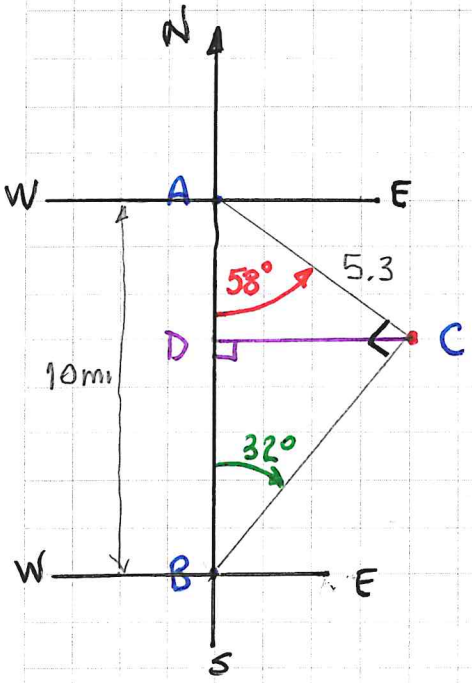
Now, $\cos 55° = \frac{AC}{730} \Rightarrow AC = 730 \cos 55° = \boxed{419 \text{ mi}}$

and $\sin 55° = \frac{BC}{730} \Rightarrow BC = 730 \sin 55° = \boxed{598 \text{ mi}}$

Ex. 8. Two coastal observers located at points A and B ten miles apart spot a submarine offshore. If A is located directly north of B and the bearings to the submarine from A and B are S58°E and N32°E respectively, then find the following distances to the nearest tenth of a mile.

a) The distance from the submarine to point A. AC

b) The distance from the submarine to the shore.



Note: $C = 180° - (58° + 32°) = 90°$

a) $\sin 32° = \frac{AC}{10} \Rightarrow AC = 10 \sin 32°$

$\Rightarrow AC = \boxed{5.3 \text{ mi}}$

b) $\sin 58° = \frac{CD}{5.3} \Rightarrow CD = 5.3 \sin 58°$

$\Rightarrow CD = \boxed{4.5 \text{ mi}}$