4.4 Trigonometric Functions of Any Angle (Day 1)

Definition: Let θ be an angle in standard position and let $P(x,y)$ be any point on the terminal side of θ with $r = d(\overline{OP}) = \sqrt{x^2+y^2}$.

Then

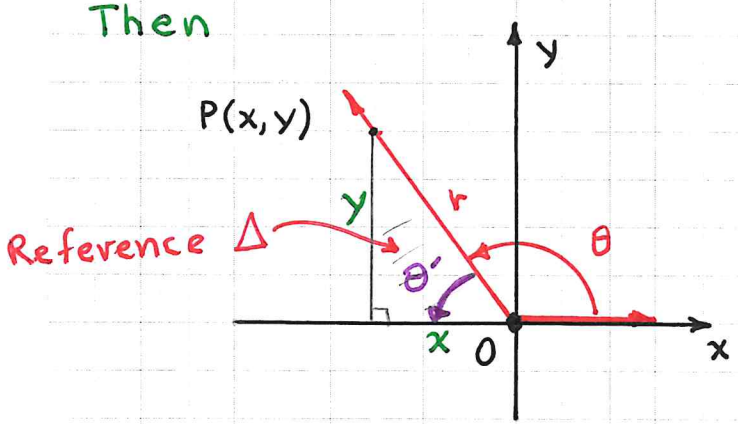


Figure 1

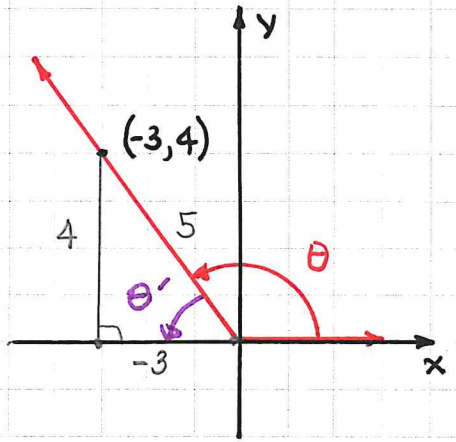
$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Ex ① Find the values of the trigonometric functions of the given angles.

a)



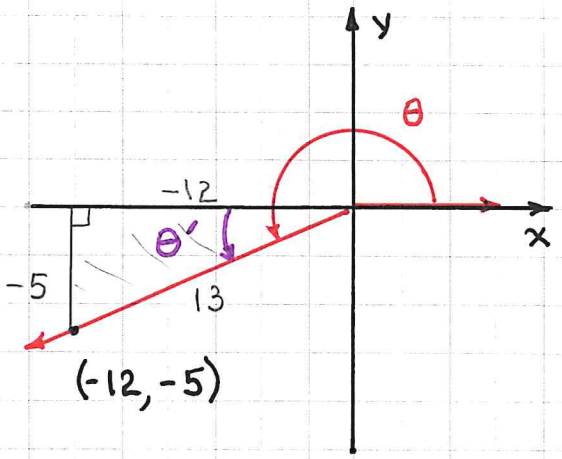
Note: $r = \sqrt{(-3)^2 + 4^2} = 5$

$$\sin\theta = \frac{4}{5} \qquad \csc\theta = \frac{5}{4}$$

$$\cos\theta = \frac{-3}{5} \qquad \sec\theta = \frac{-5}{3}$$

$$\tan\theta = \frac{4}{-3} = \frac{-4}{3} \qquad \cot\theta = \frac{-3}{4}$$

b)



Note: $r = \sqrt{(-12)^2 + (-5)^2} = 13$

$\sin\theta = \frac{-5}{13}$ $\quad \csc\theta = \frac{-13}{5}$

$\cos\theta = \frac{-12}{13}$ $\quad \sec\theta = \frac{-13}{12}$

$\tan\theta = \frac{5}{12}$ $\quad \cot\theta = \frac{12}{5}$

Observations:

① $r = \sqrt{x^2 + y^2}$ is always positive (never zero)

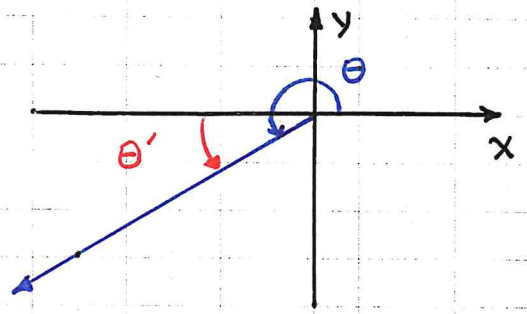
② Trigonometric functions can be positive or negative or zero or undefined.

③ $\sin\theta$ and $\cos\theta$ are always defined.

Definition: Let θ be an angle in standard position. Then its reference angle is the positive angle θ' formed by the terminal side of θ and the x-axis.

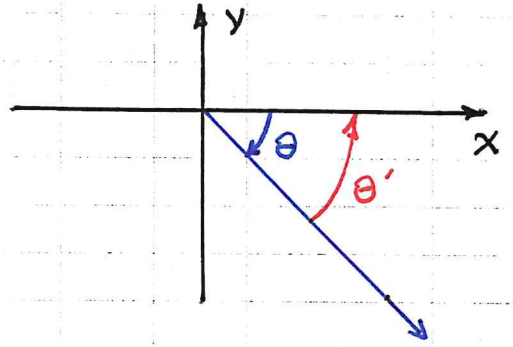
Ex. ② Find the reference angle θ'.

a) $\theta = 210°$



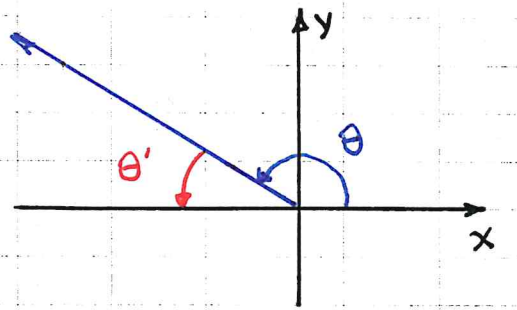
$\theta' = 210° - 180° = 30°$

b) $\theta = -45°$



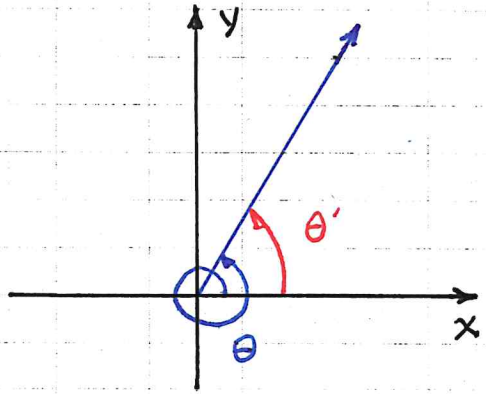
$\theta' = 45°$

c) $\theta = 5\pi/6$ $(5 \times 30 = 150°)$



$\theta' = 180° - 150° = 30° = \boxed{\frac{\pi}{6}}$

d) $\theta = 7\pi/3$ $(7 \times 60 = 420°)$

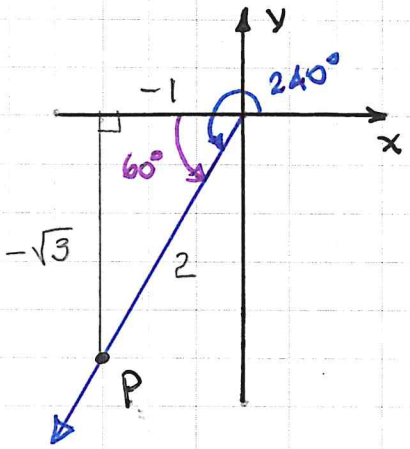


$\theta' = 420° - 360° = 60° = \boxed{\frac{\pi}{3}}$

Ex. ③ Find the EXACT values.

a) $\cos 240°$

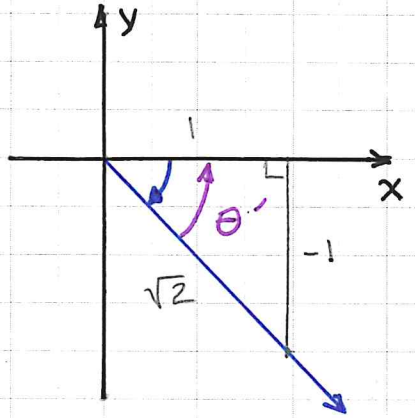
$\theta' = 60°$



$\cos 240° = \frac{-1}{2}$

b) $\sin(-45°)$

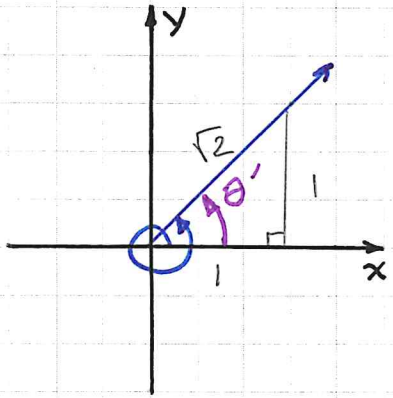
$\theta' = 45°$



$\sin(-45°) = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

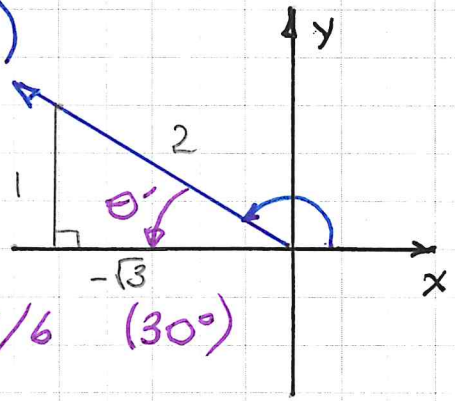
c) $\tan(405°)$

$\theta' = 45°$



$\tan(405°) = 1$

d) $\tan(5\pi/6)$



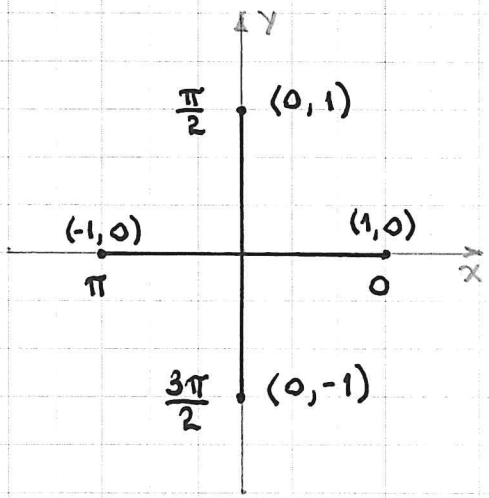
$\theta' = \pi/6 \quad (30°)$

$\tan\left(\frac{5\pi}{6}\right) = \frac{1}{-\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

A quadrant angle is an angle whose terminal side lies on an axis.

These include: $0°\ (0)$, $90°\ (\pi/2)$, $180°\ (\pi)$, $270°\ (3\pi/2)$

To evaluate the trigonometric function of these angles, use the definition given at the beginning of this lecture and the following diagram.



Note: $r = 1$

Ex. ④ Give the EXACT values of the following, where possible.

a) $\sin 0 = \frac{y}{r} = \frac{0}{1} = 0$

b) $\cos \pi = \frac{x}{r} = \frac{-1}{1} = -1$

c) $\tan \frac{\pi}{2} = \frac{y}{x} = \frac{1}{0}$ Undefined

d) $\csc \frac{3\pi}{2} = \frac{r}{y} = \frac{1}{-1} = -1$