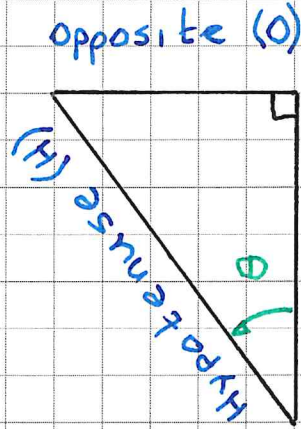


4.3 Right Triangle Trigonometry

Definitions of the Trigonometric Ratios



$$\text{Sine of } \theta = \frac{O}{H}$$

$$\text{Cosine of } \theta = \frac{A}{H}$$

$$\text{Tangent of } \theta = \frac{O}{A}$$

The Reciprocal Ratios

$$\frac{1}{\sin \theta} = \frac{H}{O}$$

$$\frac{1}{\cos \theta} = \frac{H}{A}$$

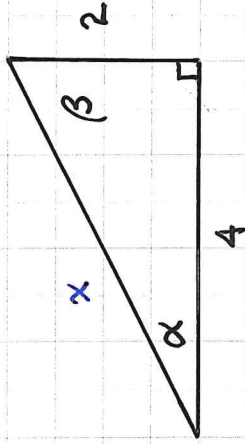
$$\frac{1}{\tan \theta} = \frac{A}{O}$$

$$\text{Cosecant of } \theta =$$

$$\text{Secant of } \theta =$$

$$\text{Cotangent of } \theta =$$

Ex. ① Given:



$$\begin{aligned}x^2 &= 4^2 + 2^2 \\ &= 16 + 4 \\ &= 20\end{aligned}$$

$$x = \sqrt{20} = 2\sqrt{5}$$

a) Find all 6 trigonometric ratios for α .

$$\sin \alpha = \frac{1 \cdot 2}{2\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos \alpha = \frac{2 \cdot 4}{2\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\tan \alpha = \frac{2}{4} = \frac{1}{2}$$

$$\cot \alpha = 2$$

$$\csc \alpha = \sqrt{5}$$

$$\sec \alpha = \frac{\sqrt{5}}{2}$$

b) Do the same for β .

$$\sin \beta = \frac{2 \cdot 4}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\cos \beta = \frac{1 \cdot 2}{2\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\tan \beta = \frac{4}{2} = 2$$

$$\csc \beta = \frac{\sqrt{5}}{2}$$

$$\sec \beta = \sqrt{5}$$

$$\cot \beta = \frac{1}{2}$$

$$\frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\frac{\sqrt{5}}{2} = \frac{5}{2\sqrt{5}}$$

Ex. ② Use a calculator to find the following values.

$$a) \cot 12.3^\circ = \frac{1}{\tan 12.3^\circ} = 4.586$$

$$b) \csc 32.1^\circ = \frac{1}{\sin(32.1^\circ)} = 1.882$$

$$c) \sec 27^\circ 27' 27'' = \frac{1}{\cos(27 + 27/60 + 27/3600)} \\ = 1.127$$

$$d) \cot 1.23 = \frac{1}{\tan(1.23)} = 0.355$$

Ex. ③ Sketch the right triangle corresponding to θ and write the values of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

a) $\cot \theta = 12/5$

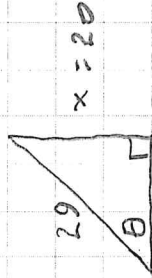


$$\sin \theta = 5/13$$

$$\cos \theta = 12/13$$

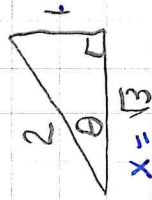
$$\tan \theta = 5/12$$

b) $\sec \theta = 29/21$



$$x^2 = \sqrt{29^2 - 21^2} = 20$$

c) $\csc \theta = 2$



$$x = \sqrt{2^2 - 1^2} = \sqrt{3}$$

$$\sin \theta = 20/29$$

$$\cos \theta = 21/29$$

$$\tan \theta = 20/21$$

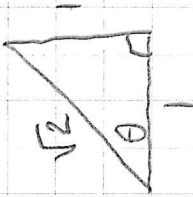
$$\sin \theta = 1/2$$

$$\cos \theta = \sqrt{3}/2$$

$$\tan \theta = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

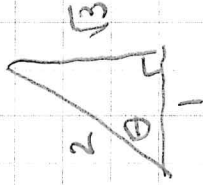
Ex. ④ Find the value of θ in degrees and radians.

a) $\tan \theta = 1$



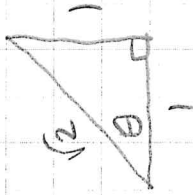
$$\theta = 45^\circ, \theta^r = \pi/4$$

b) $\cos \theta = \frac{1}{2}$



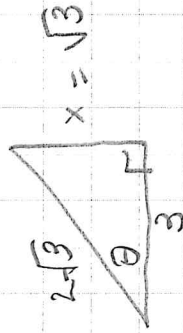
$$\theta = 60^\circ, \theta^r = \frac{\pi}{3}$$

c) $\csc \theta = \frac{\sqrt{2}}{1}$



$$\theta = 45^\circ, \theta^r = \pi/4$$

d) $\sec \theta = \frac{2\sqrt{3}}{3}$



$$\theta = 30^\circ, \theta^r = \frac{\pi}{6}$$

$$x^2 = (2\sqrt{3})^2 - 3^2 = 12 - 9 = 3$$