

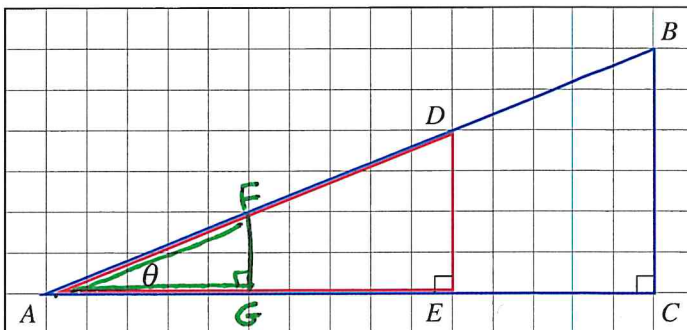
Introduction to Right Triangle Trigonometry (§4.3)

Consider the adjacent pair of overlapping triangles:

Note that $\triangle ABC \sim \triangle ADE$.

So: $\frac{BC}{DE} = \frac{AC}{AE} \quad \left(\frac{6}{4} = \frac{15}{10} \right)$

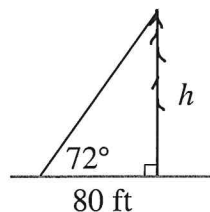
Which implies: $\frac{BC}{AC} = \frac{DE}{AE} \quad \left(\frac{6}{15} = \frac{4}{10} \right)$



This ratio $\left(\frac{V}{H} \right)$ depends only on the size of angle θ . $\frac{BC}{AC} = \frac{DE}{AE} = \frac{FG}{AG} = \frac{2}{5}$

Definition: The *tangent* of angle θ is given by $\tan \theta = \frac{\text{Opposite side}}{\text{Adjacent side}} \quad \left(\frac{\text{Across}}{\text{Next to}} \right)$

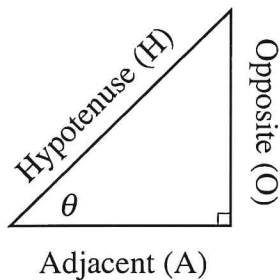
Example 1: Use the tangent ratio to find the height of the tree in the figure below.



Note: $\tan 72^\circ = \frac{h}{80}$

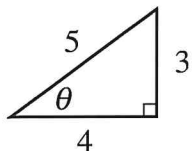
So $h = 80 \tan 72^\circ = \boxed{246 \text{ ft.}}$

Definition of the Trigonometric Ratios



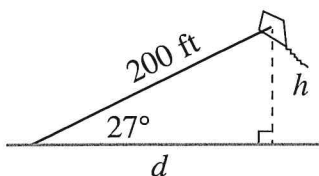
Sine of $\theta = \sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{O}{H} \quad \text{SOH}$
 Cosine of $\theta = \cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{A}{H} \quad \text{CAH}$
 Tangent of $\theta = \tan \theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{O}{A} \quad \text{TOA}$

Example 2: Find $\sin \theta$, $\cos \theta$, and $\tan \theta$ for the following triangle.



$\sin \theta = \frac{3}{5} \quad \cos \theta = \frac{4}{5} \quad \tan \theta = \frac{3}{4}$

Example 3: Find the two missing sides in the figure below.



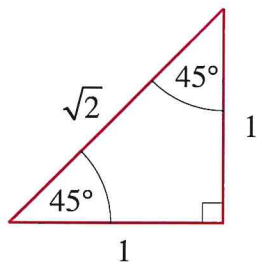
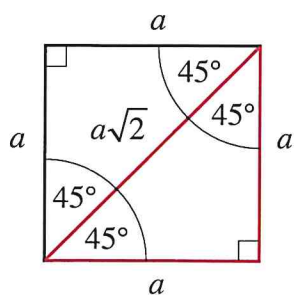
For h : $\sin 27^\circ = \frac{h}{200} \Rightarrow h = 200 \sin 27^\circ = 91 \text{ ft}$

For d : $\cos 27^\circ = \frac{d}{200} \Rightarrow d = 200 \cos 27^\circ = 178 \text{ ft}$

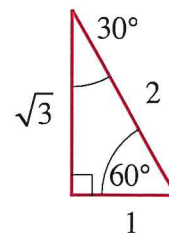
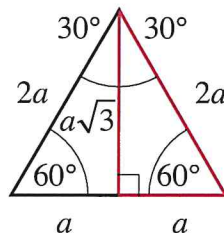
Two Special Right Triangles

There are two special right triangles that frequently arise in trigonometry, the $45^\circ-45^\circ-90^\circ$ triangle and the $30^\circ-60^\circ-90^\circ$ triangle. The sides of each of these triangles have special relationships.

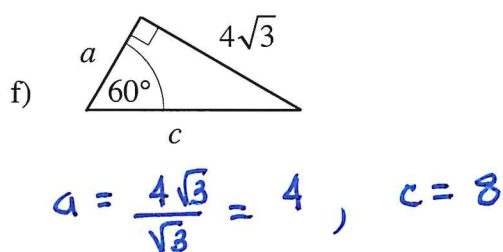
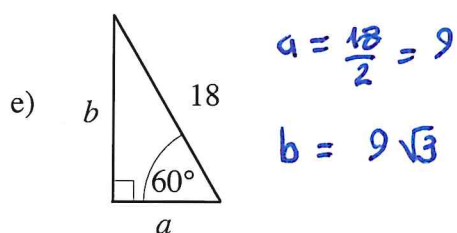
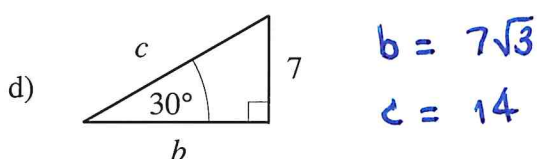
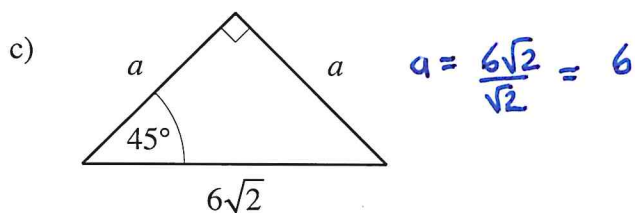
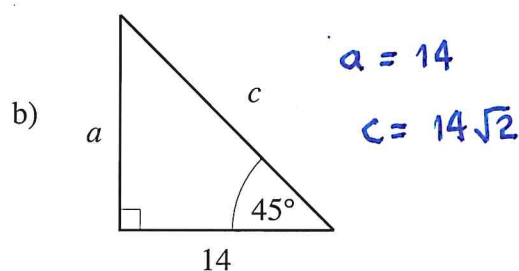
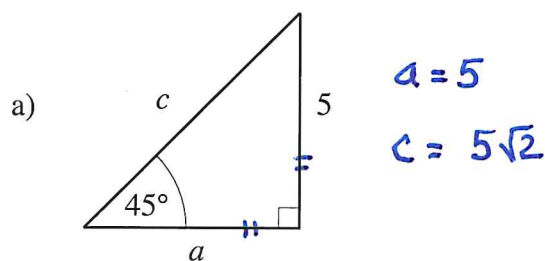
$45^\circ-45^\circ-90^\circ$



$30^\circ-60^\circ-90^\circ$



Example 4: For each of the following triangles, find the missing side lengths.



Example 5: Find the exact values of the following trigonometric ratios.

a) $\sin 30^\circ = \frac{O}{H} = \frac{1}{2}$

b) $\cos 45^\circ = \frac{A}{H} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

c) $\tan 60^\circ = \frac{O}{A} = \frac{\sqrt{3}}{1} = \sqrt{3}$