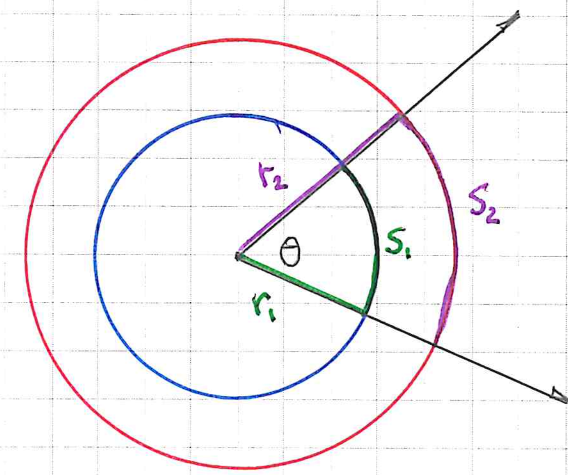
4.1 Angle Measure (II) - Radians

DEFINITION: The radian measure of an angle whose vertex is at the center of a circle is found by forming the ratio of the length of the intercepted arc to the radius of the circle.



Note: $\theta = \frac{S_1}{r_1} = \frac{S_2}{r_2}$

The radian measure of θ is given by: $\boxed{\theta = \frac{s}{r}}$

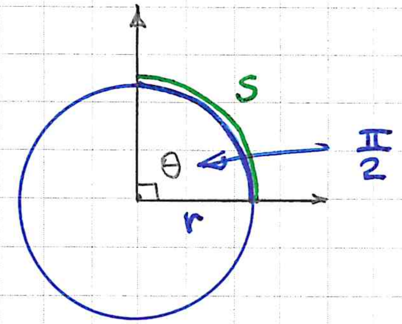
Notes: 1) r and s are assumed to be in the same units.

2) Radians are pure numbers. - No units.

Ex. $\theta = \frac{s}{r} = \frac{36\,\cancel{cm}}{12\,\cancel{cm}} = 3$ (Sometimes we write: 3^r)

3) Radian measure does not depend on the size of the circle.

Ex. ① Find the radian measure of 90°.



Recall: $C = 2\pi r$. So $s = \frac{1}{4}(2\pi r)$

Thus $\theta = \frac{s}{r} = \frac{\frac{1}{4}(2\pi \cancel{r})}{\cancel{r}} = \frac{1}{4}(2\pi) = \boxed{\frac{\pi}{2}}$

Ex. ② Complete the chart:

θ°	0°	90°	180°	45°	360°	60°	30°
θ^r	0	$\frac{\pi}{2}$	π	$\frac{\pi}{4}$	2π	$\frac{\pi}{3}$	$\frac{\pi}{6}$

FORMULA: The following proportion can be used to convert between radians and degrees.

$$\frac{\theta^\circ}{180^\circ} = \frac{\theta^r}{\pi}$$

Ex. 3 Find the radian measure of:

a) $150^\circ \Rightarrow \dfrac{150^\circ}{180^\circ} = \dfrac{\theta}{\pi} \Rightarrow \theta = \dfrac{15\cancel{0}^\circ \pi}{18\cancel{0}^\circ} = \boxed{\dfrac{5\pi}{6}}$

b) $\left(\frac{1}{2}\right)^\circ \Rightarrow \dfrac{\left(\frac{1}{2}\right)^\circ}{180^\circ} = \dfrac{\theta}{\pi} \Rightarrow \theta = \dfrac{\left(\frac{1}{2}\right)^{\cancel{\circ}} \pi}{180^{\cancel{\circ}}} \cdot \dfrac{2}{2} = \boxed{\dfrac{\pi}{360^\circ}}$

Ex. 4 Find the degree measure of:

a) $\dfrac{2\pi}{3} \Rightarrow \dfrac{\theta^\circ}{180^\circ} = \dfrac{2\pi/3}{\pi} \Rightarrow \theta^\circ = \dfrac{180^\circ (2\cancel{\pi}/3)}{\cancel{\pi}} = 60^\circ \cdot 2 = \boxed{120^\circ}$

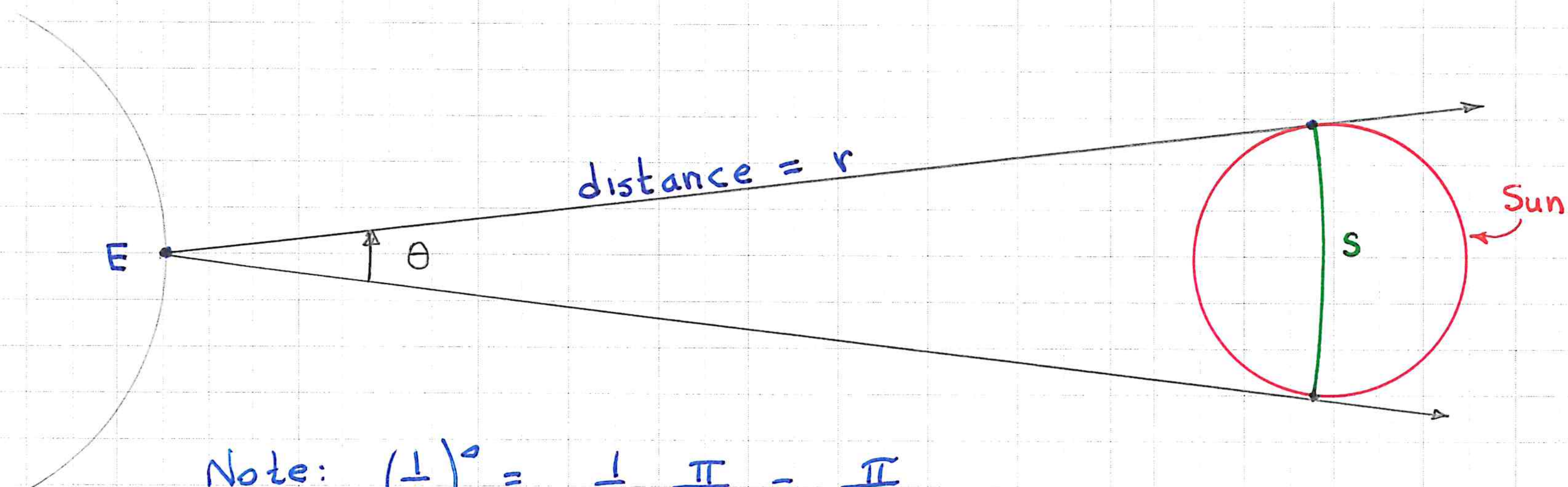
b) $\dfrac{11\pi}{6} \Rightarrow \dfrac{\theta^\circ}{180^\circ} = \dfrac{11\cancel{\pi}/6}{\cancel{\pi}} \Rightarrow \theta^\circ = 180^\circ \left(11/6\right) = 330^\circ$

c) $1 \Rightarrow \dfrac{\theta^\circ}{180^\circ} = \dfrac{1}{\pi} \Rightarrow \theta^\circ = \dfrac{180^\circ}{\pi} = 57.296^\circ$

FORMULA: Since $\theta = \frac{s}{r}$, it follows that $\boxed{s = r\,\theta}$

(When θ is measured in radians.)

Ex. 5 Use $s = r\theta$ to approximate the distance to the sun if it is known that $s = 811{,}000$ miles and $\theta = (1/2)^\circ$



Note: $\left(\frac{1}{2}\right)^\circ = \frac{1}{2}\,\frac{\pi}{180} = \frac{\pi}{360}$

$s = r\theta \Rightarrow 811{,}000 = r\left(\frac{\pi}{360}\right) \Rightarrow r = \frac{811{,}000 \cdot 360}{\pi}$

$= 92{,}933{,}754$ miles