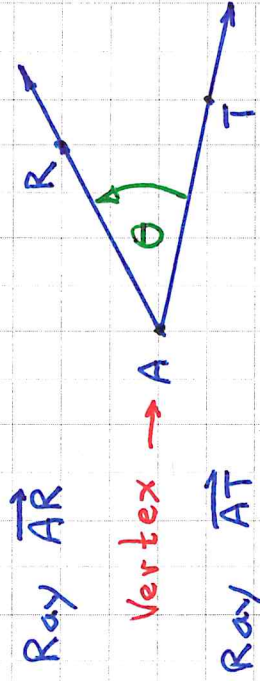


4.1 Angle Measure (I) - Degrees

DEFINITION: An angle is the union of two rays with a common endpoint.



Names: $\angle TAR$ ($\angle RAT$)

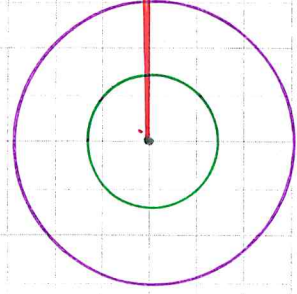
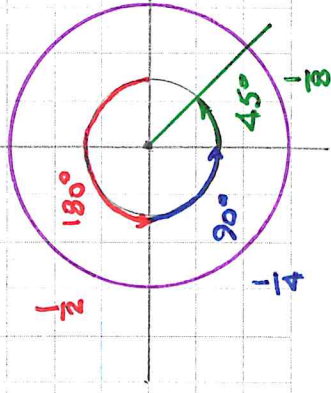
$\angle A$ $\angle \theta$

To measure an angle is to determine the amount of ROTATION between one side and the other. (Commonly measured in DEGREES.)

Ex. Measure of $\angle RAT = 40^\circ$

QUESTION: What is a degree?

ANSWER: One degree is $\frac{1}{360}$ of a circle.



Note: The measure of the angle does not depend on the size of the circle.

Finer divisions:

$\frac{1}{60}$ of a degree is called one minute:

$$1^\circ = 60' \quad (60 \text{ minutes})$$

$\frac{1}{60}$ of a minute is called one second:

$$1' = 60'' \quad (60 \text{ seconds})$$

$$\text{Thus } 1^\circ = 60 \times 60 = 3600''$$

Two formats:

a) Degrees - Minutes - Seconds (DMS) Ex. $7^\circ 15' 30''$

b) Decimal Degrees (DD) Ex. 12.345°

Ex. ① a) Convert $7^\circ 15' 30''$ to DD. Ans: $7 + \frac{15}{60} + \frac{30}{3600} = 7.258^\circ$

b) Convert 12.345° to DMS.

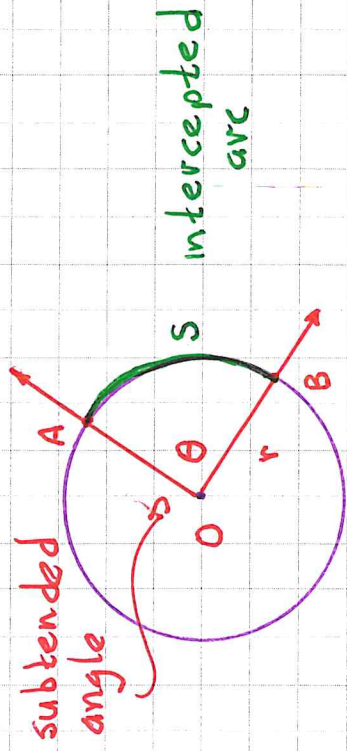
$$\text{Ans: } 12^\circ + .345 \times 60 = 12^\circ \underline{20.7}'$$

$$= 12^\circ 20' + .7 \times 60$$

$$= 12^\circ 20' 42''$$

APPLICATION: Finding Arc Length

PROBLEM: Given r and θ , find the length of \widehat{AB} (s).



SOLUTION: Recall that circumference (C) = $2\pi r$

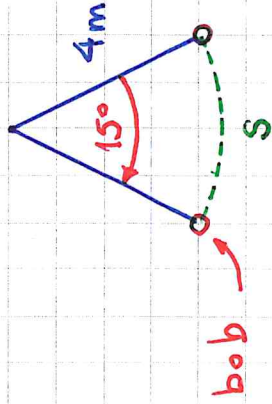
$$\text{So that } \frac{\text{Arc Length}}{\text{Circumference}} = \frac{s}{C} = \frac{s}{2\pi r} = \frac{\theta}{360^\circ}$$

$$\text{Thus } s = \frac{2\pi r \theta}{360^\circ} = \frac{\pi r \theta}{180^\circ}$$

$$s = \frac{\pi r \theta}{180^\circ} \quad (\theta \text{ in degrees})$$

Ex. ② A pendulum of length 4m swings through an arc of 15° .

How far does the "bob" travel in one swing?



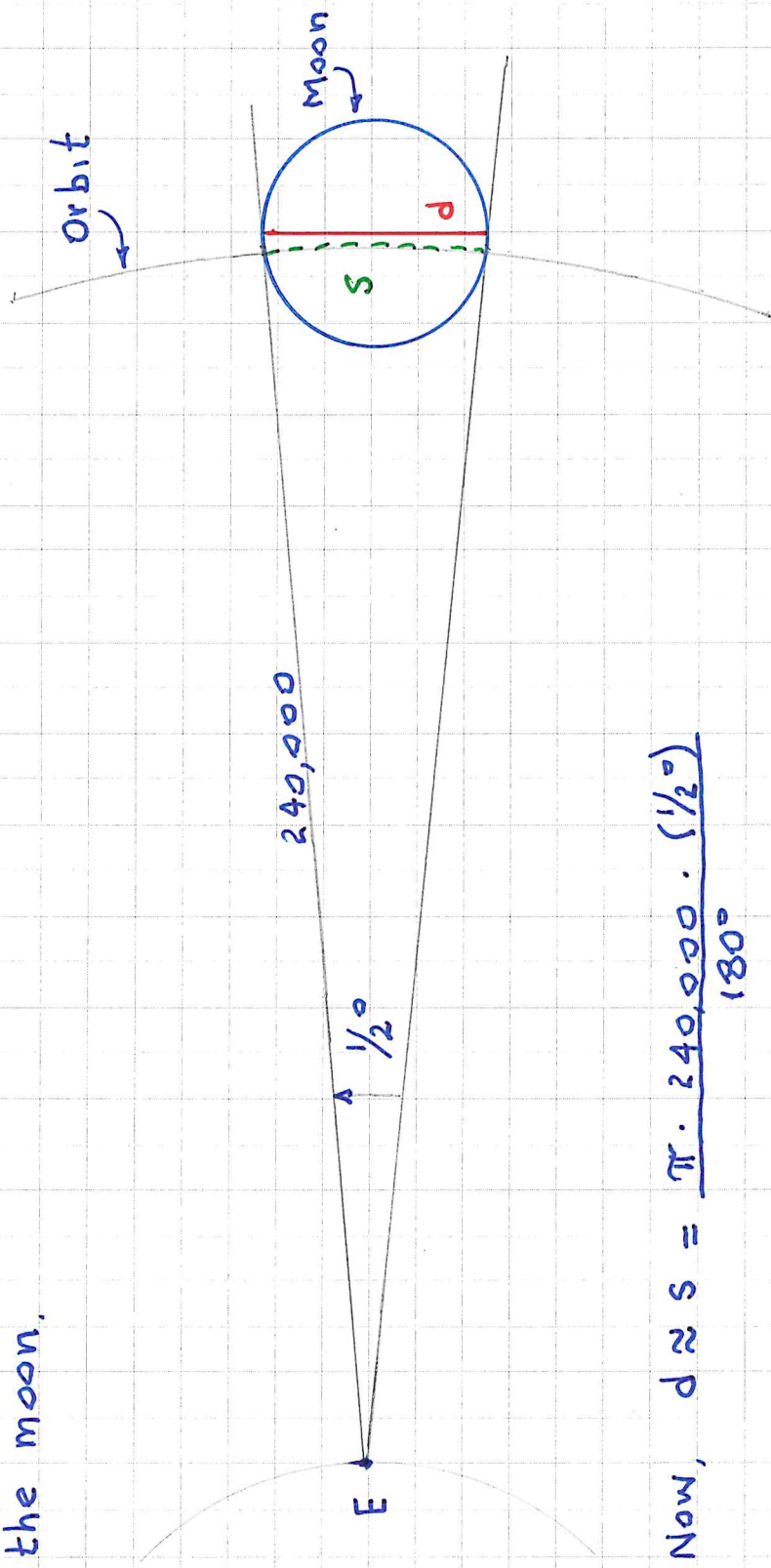
$$\text{Now, } s = \frac{\pi (4) 15^\circ}{180^\circ} = \frac{\pi \text{ m}}{3}$$

Got this far on day 1
Fa 22
(50 minutes)

The EXACT value

Approximate Ans = 1.047 m

Ex. ③ Given that the radius of the moon's orbit is 240,000 miles and that the moon subtends an angle of $\frac{1}{2}^\circ$, use the arc length formula to approximate the diameter of the moon.

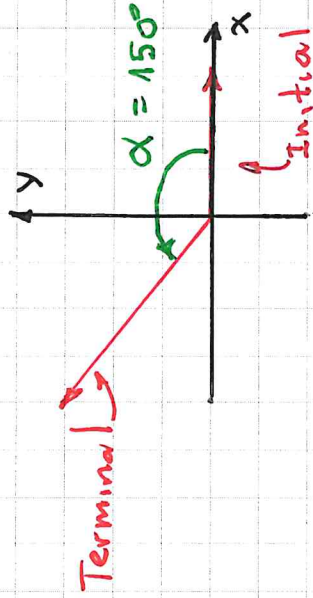


$$\text{Now, } d \approx s = \frac{\pi \cdot 240,000 \cdot (\frac{1}{2}^\circ)}{180^\circ}$$

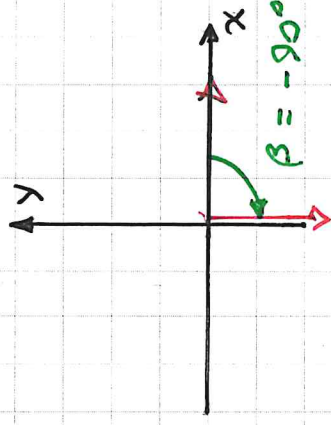
$$\approx 2,094 \text{ miles}$$

Ex. ④ Sketch the following angles in Standard Position.

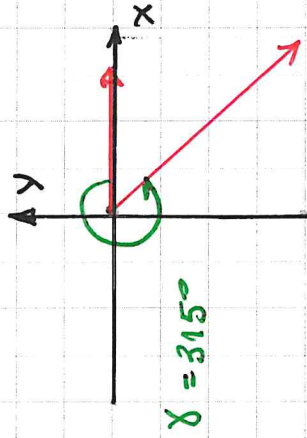
a) $\alpha = 150^\circ$



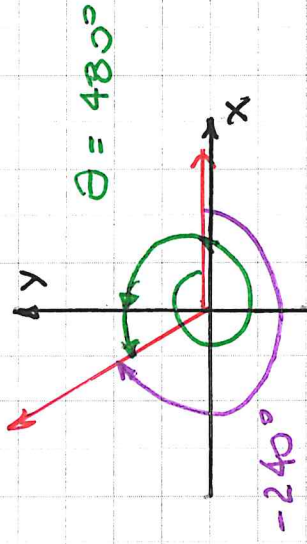
b) $\beta = -90^\circ$



c) $\gamma = 315^\circ$



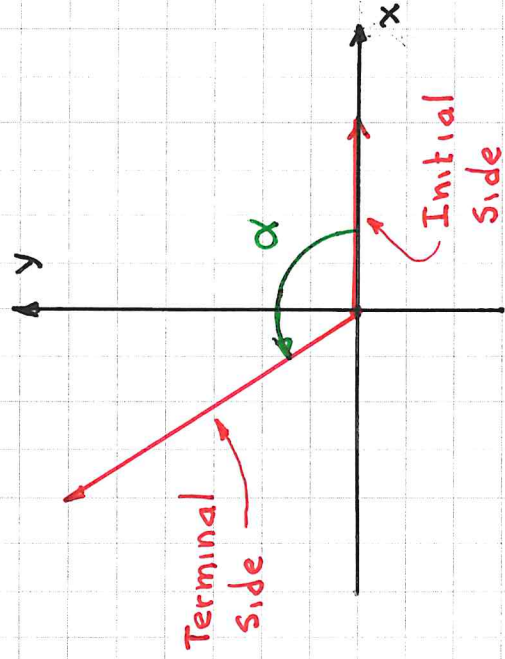
d) $\theta = 480^\circ$



Note: Two angles in Standard Position with the same terminal side are called **COTERMINAL**.

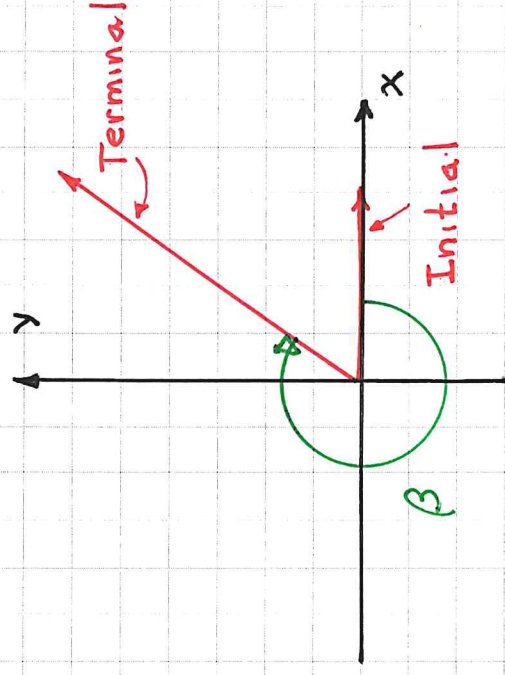
Ex: 480° and -240° are **COTERMINAL**

DEFINITION: An angle is in STANDARD POSITION when its vertex is at the origin and its initial side lies along the positive x-axis.



Counterclockwise Rotation

$\angle \alpha$ has a positive measure



Clockwise Rotation

$\angle \beta$ has a negative measure