Exponential and Logarithmic Models (§3.5)

1. If money is placed in an account paying interest compounded continuously, then the amount in the account at time t can be calculated by the formula $A = Pe^{rt}$, where P represents the principal and r the interest rate.

   $2.5\% = .025$

   How long will it take \$1000 to double at $2\frac{1}{2}\%$ compounded continuously?

Solve: $2000 = 1000\, e^{.025t}$ for t.

$$2 = e^{.025t}$$

$$\ln 2 = \ln e^{.025t} = .025t$$

$$t = \frac{\ln 2}{.025} \approx 27.7 \text{ years}$$

2. The population of a region at time t (in years) is modeled by the formula $P = P_0 e^{rt}$, where P_0 is the initial population of the region and r the growth constant.

   a) The population of Sonoma County in 2000 was 459 thousand and in 2010 it was 484 thousand. Determine r to four decimal places.

Solve: $484\,\cancel{000} = 459\,\cancel{000}\, e^{r \cdot 10}$

$$\frac{484}{459} = e^{10 \cdot r}$$

$$\ln\left(\frac{484}{459}\right) = \ln e^{10 \cdot r} = 10r$$

Thus, $r = \dfrac{\ln(484/459)}{10} \approx \boxed{0.0053}$ (0.53%)

   b) If the growth rate remains the same, then when will the population be 1 million?

Solve: $1\,000 = 459\, e^{0.0053t}$ for t.

$$\ln\left(\frac{1000}{459}\right) = \ln e^{0.0053t} = 0.0053t$$

So $t = \ln(1000/459)/0.0053 \approx \boxed{147 \text{ years}}$

3. The population of a colony of bacteria is modeled by $P = \dfrac{240{,}000}{1+23e^{-0.1398t}}$, where t is measured (in days) from the beginning of the experiment. When will the population be 100,000?

Solve: $10\cancel{0{,}000} = \dfrac{24\cancel{0{,}000}}{1 + 23e^{-0.1398t}}$ for t.

$$10(1 + 23e^{-0.1398t}) = 24$$

$$1 + 23e^{-0.1398t} = 2.4$$

$$23e^{-0.1398t} = 1.4$$

$$e^{-0.1398t} = \frac{1.4}{23}$$

$$\ln e^{-0.1398t} = \ln(1.4/23)$$

$$-0.1398t = \ln(1.4/23)$$

$$t = \ln(1.4/23)/(-0.1398) \approx 20 \text{ days}$$

4. It was discovered at the beginning of the 20th century that radioactive materials decay in a way that can be modeled by the formula $A = A_0e^{rt}$, where A_0 is the initial amount of the material present.

Strontium–90 is a radioactive material with a half-life of 28 years. It is one of the waste products from nuclear fission reactors.

a) Determine r to four decimal places.

In 28 years we will have $\frac{1}{2}A_0$

Solve: $\frac{1}{2}\cancel{A_0} = \cancel{A_0}e^{r\cdot 28}$ for r.

$\Rightarrow \ln(1/2) = \ln e^{28r} = 28r$

So $r = \ln(1/2)/28 \approx \boxed{-0.025}$

b) How long will it be before 1% of the original sample remains?

Note: $1\% = .01$

Solve: $.01A_0 = A_0e^{-0.025t}$ for t

$$.01 = e^{-0.025t}$$

$$\ln(.01) = \ln e^{-0.025t} = -0.025t$$

So $t = \ln(.01)/(-0.025) \approx 184$ years