3.4 Solving Exponential Equations

Property I: If $a^x = a^y$ then $x = y$.

Ex. 1 Solve:

$$4^x = 32$$
$$(2^2)^x = 2^5$$
$$2^{2x} = 2^5$$
I
$$2x = 5 \quad \Rightarrow \quad \boxed{x = 5/2}$$

Ex. 2 Solve:

$$27^x = 3^{x^2-4}$$
$$(3^3)^x = 3^{x^2-4}$$
$$3^{3x} = 3^{x^2-4}$$
I
$$3x = x^2 - 4$$
$$0 = x^2 - 3x - 4$$
$$0 = (x+1)(x-4)$$

So $\boxed{x = -1, 4}$

Property II: If $x = y$ then $\log_a(x) = \log_a(y)$

Ex. 3 Solve:

$$4^x = 3$$

II

$$\log 4^x = \log 3$$

$$x \cdot \log 4 = \log 3$$

$$x = \frac{\log 3}{\log 4} = \boxed{0.792}$$

Ex. 4 Solve:

$$e^{2x} = 7$$

II

$$\ln e^{2x} = \ln 7$$

$$2x = \ln 7$$

$$x = \frac{\ln 7}{2} = \boxed{0.973}$$

$$\boxed{\ln e = 1}$$

Ex. 5 Solve: $10^{x+1} + 3 = 8$

$10^{x+1} = 5$ Isolate the exponential.

II

$\log(10^{x+1}) = \log 5$

$(x+1)\log 10 = \log 5$

$x+1 = \log 5$

$x = \log 5 - 1 = \boxed{-0.301}$

Ex. 6 Solve: $2^x = 5^{2x+1}$

II

$\ln 2^x = \ln 5^{2x+1}$

$x \ln 2 = (2x+1)\ln 5$

$x \ln 2 = 2x \ln 5 + \ln 5$

$\underline{x} \ln 2 - 2\underline{x} \ln 5 = \ln 5$

factor: $x(\ln 2 - 2\ln 5) = \ln 5$

$x = \dfrac{\ln 5}{\ln 2 - 2\ln 5} = \boxed{-0.637}$

3.4 Solving Logarithmic Equations

Property III: If $\log_a x = y$ then $x = a^y$

Ex. 7 Solve: $2\log x + 7 = 207$

$2\log x = 200$

Isolate the log

$\log x = 100$

III $x = 10^{100}$ ← Googol

Ex. 8 Solve: $\log x + \log(x-3) = 1$

$\log[x(x-3)] = 1$

III $x(x-3) = 10^1$

$x^2 - 3x = 10$

$x^2 - 3x - 10 = 0$

$(x+2)(x-5) = 0$

Solutions: $x = \cancel{-2}, 5$ $\log(-2)$ is not a REAL #

Property IV: If $\log_a x = \log_a y$ then $x = y$.

Ex. 9 Solve:

$$2 \ln x - \ln 2 = \ln(x+4)$$

$$\ln x^2 - \ln 2 = \ln(x+4)$$

$$\ln\left(\frac{x^2}{2}\right) = \ln(x+4)$$

IV

$$\frac{x^2}{2} = x + 4$$

$$x^2 = 2x + 8$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

Solutions: $x = 4, \cancel{-2}$