MATH 27

Properties of Logarithms (§3.3)

Recall: If $a>0$ and $a\neq 1$ then $\log_a x = y$ is equivalent to $x = a^y$.

Properties of the Definition of the Logarithm

(i) $\log_a 1 = 0$

(ii) $\log_a a = 1$

(iii) $\log_a a^x = x$

(iv) $a^{\log_a x} = x$

Inverse Properties

$f(x) = a^x$

$g(x) = \log_a x$

$8^{\log_8 64} = 64$

$8^2 = 64$

Computational Properties of the Logarithm

Product: $\log_a(UV) = \log_a U + \log_a V$

Quotient: $\log_a\left(\frac{U}{V}\right) = \log_a U - \log_a V$

Power: $\log_a(U)^n = n\log_a U$

1. Use the properties of logarithms to expand each expression completely.

a) $\ln(P\cdot V\cdot T) = \ln P + \ln(V\cdot T) = \ln P + \ln V + \ln T$

b) $\log(m\cdot c^2) = \log m + \log c^2 = \log m + 2\log c$

c) $\log_5\left(\frac{x}{yz}\right) = \log_5 x - \log_5(yz) = \log_5 x - (\log_5 y + \log_5 z)$

$= \log_5 x - \log_5 y - \log_5 z$

d) $\log_2(x^2 - y^2) = \log_2[(x+y)\cdot(x-y)] = \log_2(x+y) + \log_2(x-y)$

2. Use the properties of logarithms to condense each expression to the logarithm of a single quantity.

a) $\frac{1}{2}\ln x + 5\ln y - 2\ln z = \ln x^{1/2} + \ln y^5 - \ln z^2$

$\frac{1}{2}\ln x + \ln y^5 + \ln z^{-2}$

$= \ln(x^{1/2}\cdot y^5) - \ln z^2$

$= \ln\left(\frac{x^{1/2}\cdot y^5}{z^2}\right)$

b) $\log(x^2-9) - \log(x+3) = \log\left(\frac{x^2-9}{x+3}\right) = \log\left(\frac{(x+3)(x-3)}{x+3}\right)$

$= \log(x-3)$

3. Find the exact value of the logarithm without using a calculator.

a) $\log_6 \sqrt[3]{6} = \log_6 6^{1/3} = \frac{1}{3}\log_6 6 = \frac{1}{3}$

b) $\log_2(-16)$ Domain of $\log_a x$: $(0, \infty)$

↳ Not a real #

c) $\log_4 2 + \log_4 32 = \log_4 (2 \cdot 32) = \log_4 64 = 3$

d) $\ln \sqrt[5]{e^3} = \ln e^{3/5} = \frac{3}{5}\ln e = \frac{3}{5}$

Change-of-Base Formula: $\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log_{10} x}{\log_{10} a} = \frac{\log_e x}{\log_e a}$

4. Use the change-of-base formula and a calculator to evaluate the following.

a) $\log_2 10 = \frac{\log(10)}{\log(2)} = \frac{1}{\log 2} = 3.322$

b) $\log_\pi e = \frac{\ln e}{\ln \pi} = \frac{1}{\ln \pi} = 0.874$

5. Graph the function $f(x) = \log_4(x)$ in the window [–10,10] x [–2,18]

$f(x) = \log_4(x) = \frac{\ln x}{\ln 4}$