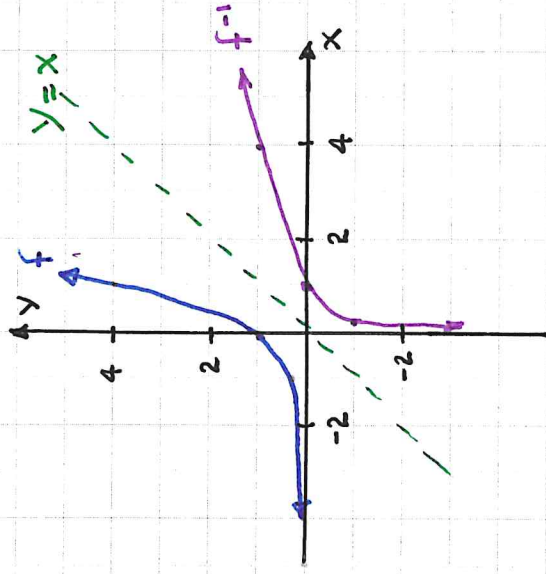


3.2 Logarithmic Functions

Graph of $f(x) = 4^x$

x	y
-1	1/4
0	1
1	4



Sketch the graph of $f^{-1}(x)$.

x	y
1/4	-1
1	0
4	1

Since $f(x)$ is a one-to-one function, it has an inverse.

Ex. ① Find a formula for $f^{-1}(x)$.

1) $y = 4^x$

2) $x = 4^y$

3) Solve for y : We need a NEW function.

We write: $y = \log_4(x)$

4) $f^{-1}(x) = \log_4(x)$

READ: "logarithm base 4 of x"

Graph of $g(x) = \left(\frac{1}{4}\right)^x$

x	y
-1	4
0	1
1	$\frac{1}{4}$

Since $g(x)$ is a one-to-one function, it has an inverse.

Ex. ② Find a formula for $g^{-1}(x)$

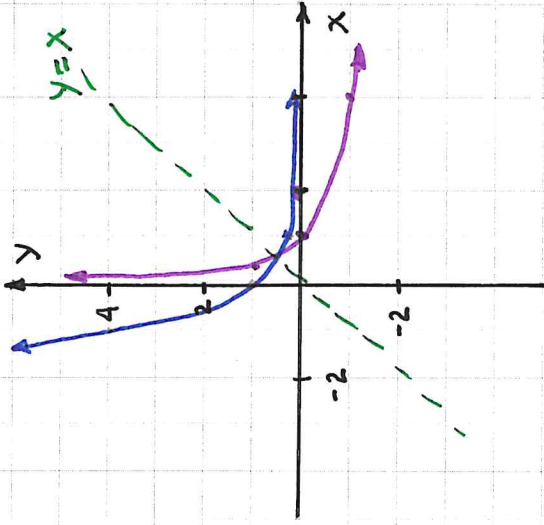
1) $y = \left(\frac{1}{4}\right)^x$

2) $x = \left(\frac{1}{4}\right)^y$

3) Solve for y :

$$y = \log_{1/4}(x)$$

4) $g^{-1}(x) = \log_{1/4}(x)$



Sketch the graph of $g^{-1}(x)$.

x	y
4	-1
1	0
$\frac{1}{4}$	1

Logarithmic Functions (§3.2)

****Note:** If $a > 0$ and $a \neq 1$ then $\log_a x = y$ is equivalent to $x = a^y$.**
 ↖ EQUIVALENT ↗

1. Write the following logarithmic equations in exponential form:

a) $\log_3 243 = 5 \rightarrow 243 = 3^5$

b) $\log_{36} 6 = \frac{1}{2} \rightarrow 6 = 36^{1/2}$

c) $-1 = \log_{10} \left(\frac{1}{10}\right) \rightarrow 10^{-1} = \frac{1}{10}$

d) $0 = \log_e 1 \rightarrow e^0 = 1$

2. Write the following exponential equations in logarithmic form:

a) $9 = 3^2 \rightarrow \log_3 9 = 2$

b) $27^{1/3} = 3 \rightarrow \log_{27} 3 = 1/3$

c) $\frac{1}{100} = 10^{-2} \rightarrow \log_{10} (1/100) = -2$

d) $e^{-1} = \frac{1}{e} \rightarrow \log_e \left(\frac{1}{e}\right) = -1$

3. Find the exact value of each logarithm without using a calculator.

a) $\log_5 25 = a \rightarrow 25 = 5^a \rightarrow a = 2$

$$\log_5 25 = 2$$

b) $\log_{49} 7 = b \rightarrow 7 = 49^b \rightarrow b = 1/2$

$$\log_{49} 7 = 1/2$$

c) $\log_3 \left(\frac{1}{9}\right) = c \rightarrow \frac{1}{9} = 3^c \rightarrow c = -2$

$$\log_3 (1/9) = -2$$

d) $\log_{10} 1000 = d \rightarrow 1000 = 10^d \rightarrow d = 3$

$$\log_{10} 1000 = 3$$

4. Use a calculator to evaluate each logarithm.

a) $\log_{10} 2 \Rightarrow \log 2 \rightarrow 0.301$

b) $\log_e \pi \Rightarrow \ln \pi \rightarrow 1.145$