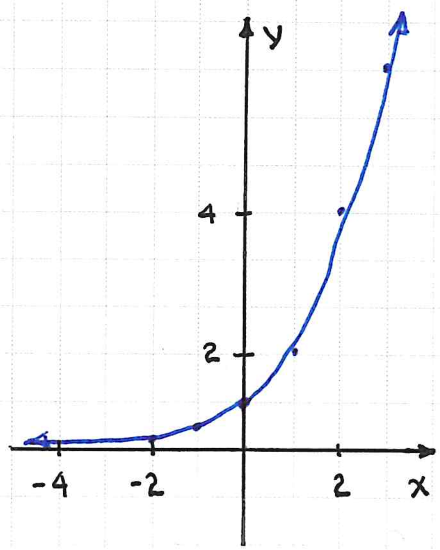
3.1 Exponential Functions

Definition: A function of the form $f(x) = a^x$ (where $a > 0$ and $a \neq 1$) is called an exponential function.

Ex. ① Sketch: $f(x) = 2^x$ $(a > 1)$

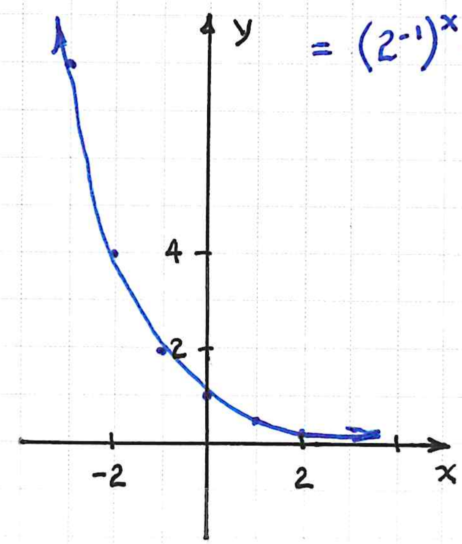
x	y
3	8
2	4
1	2
0	1
-1	1/2
-2	1/4

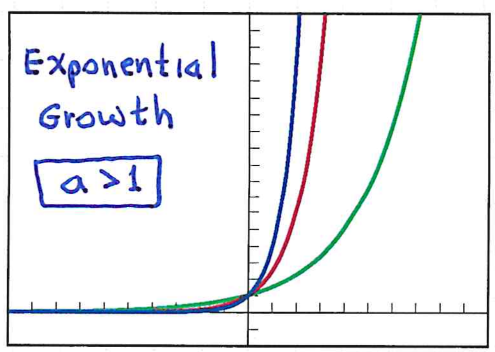


Ex. ② Sketch: $g(x) = (1/2)^x$ $(0 < a < 1)$

$= (2^{-1})^x = 2^{-x}$

x	y
2	1/4
1	1/2
0	1
-1	2
-2	4
-3	8

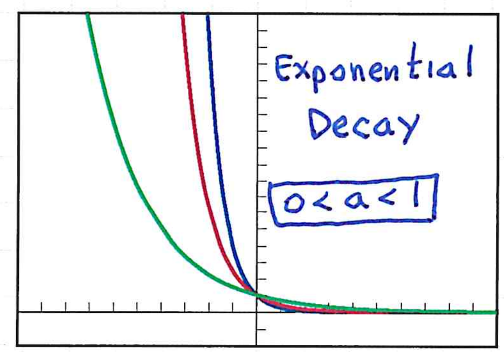




$y = 4^x$

$y = (5/2)^x$

$y = (3/2)^x$



$y = (1/4)^x = 4^{-x}$

$y = (2/5)^x = (5/2)^{-x}$

$y = (2/3)^x = (3/2)^{-x}$

Notes: 1) $p(x) = x^2$ is not an exponential function. (It's a polynomial.)

2) $g(x) = (-2)^x$ is not an exponential function. (Base < 0)

3) For $a > 1$ we have exponential GROWTH.

4) For $0 < a < 1$ we have exponential DECAY.

5) For $f(x) = a^x$ the x-axis is the HORIZONTAL ASYMPTOTE

6) Of all the choices we have for a, there is one that is often convenient. It is the irrational number

$$e = 2.718281828345\ldots$$

It's called the natural base.

THE exponential function

$$\boxed{f(x) = e^x}$$

Ex. 3 Under ideal conditions, a colony of bacteria will grow exponentially for a limited time. Suppose that the population of a certain type of bacteria is $P(t) = 1000e^{0.293t}$ (t is in hours).

a) Find $P(0) = 1000e^{0.293(0)} = 1000e^0 = 1000$.

b) Find $P(10) = 1000e^{0.293(10)} \doteq 1000e^{2.\cancel{293}\,93} = \boxed{18,728}$

MATH 27 **Exponential Functions (§3.1)**

Compound Interest

When funds are deposited into an account paying compound interest, the following formula is used to calculate the amount in the account after a certain number of years:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Where A = amount in the account (future value), P = principal (present value), r = annual interest rate, n = number of compoundings per year, and t = number of years.

P $t = 2022-1626 = 396$ $r = 3.5\% = .035$

4. In the year 1626, \$24 is deposited into an account paying $3\frac{1}{2}\%$. Find the amount in the account now
   a) if it earns simple interest ($A = P + Prt$).

   $A = 24 + 24(.035 \cdot 396) = \boxed{\$356.64}$

   b) if it earns interest compounded monthly.

   $A = 24\left(1 + \frac{.035}{12}\right)^{12 \cdot 396} = \boxed{\$24{,}590{,}740.97}$

   c) if it earns interest compounded daily.

   $A = 24\left(1 + \frac{.035}{365}\right)^{\cancel{12}\, 365 \cdot 396} = \boxed{\$25{,}075{,}187.22}$

Continuous Compound Interest

If interest is compounded continuously, the formula becomes $A = Pe^{rt}$.

5. Find the amount in the account now for Example 4 if it earns continuous compound interest

   $A = 24\, e^{.035 \cdot 396} = \boxed{\$25{,}091{,}854.52}$

Carbon Dating

6. Radioactive carbon 14 is used in the process known as **carbon dating**. The half-life of carbon 14 is approximately 5730 years. The function $Q(t) = 100e^{-0.0001216t}$ gives the quantity present, in grams, of a sample after t years.

   a) Find $Q(0)$ $= 100\,e^{-0.000126 \cdot 0} = 100\,e^{0} = 100\text{g}$

   b) Find $Q(2000)$ $= 100\,e^{-0.000126 \cdot 2000} = 78.4\text{g}$

   c) Find $Q(5730)$ $= 100\,e^{-0.000126 \cdot 5730} = 49.8\text{g}$