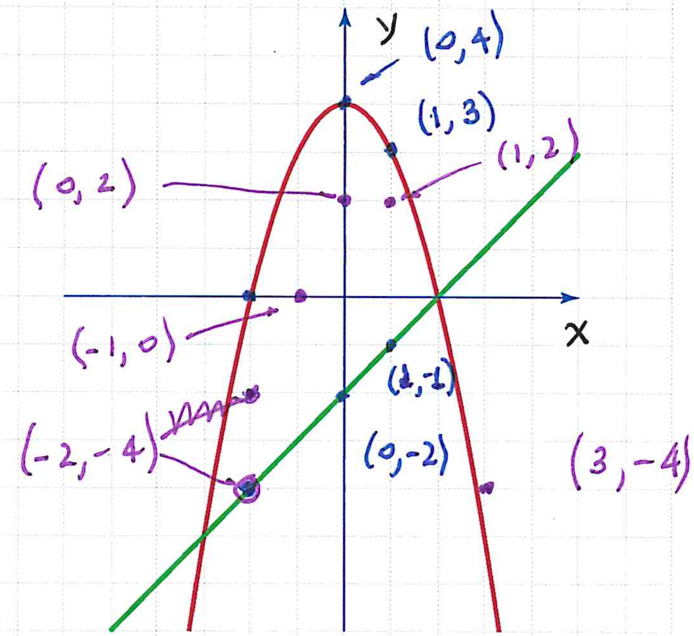
1.5 Combining Functions

Ex. ① Suppose that

$f(x) = -x^2 + 4$

and

$g(x) = x - 2$



Find the following:

a) $f(1) + g(1) = 3 + (-1) = 2$

b) $f(0) + g(0) = 4 + (-2) = 2$

c) $f(-1) + g(-1) = 3 + (-3) = 0$

d) $f(-2) + g(-2) = 0 + (-4) = -4$

Notation:

e) $(f+g)(3) = f(3) + g(3) = -5 + (1) = -4$

f) $(f+g)(x) = f(x) + g(x) = (-x^2+4) + (x-2) = \boxed{-x^2+x+2}$

g) $(f+g)(3) = -3^2 + 3 + 2 = -9+5 = -4$

h) $(f-g)(x) = f(x) - g(x) = (-x^2+4) - (x-2) = -x^2 - x + 6$

i) $(f \cdot g)(x) = f(x) \cdot g(x) = (-x^2+4)(x-2) = -x^3 + 2x^2 + 4x - 8$

$a^2 - b^2 = (a+b)(a-b)$

j) $(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{-x^2+4}{x-2} = \dfrac{4-x^2}{x-2} = \dfrac{(2+x)\cancel{(2-x)}}{\underset{-\cancel{(2-x)}}{x-2}} = -(2+x)$

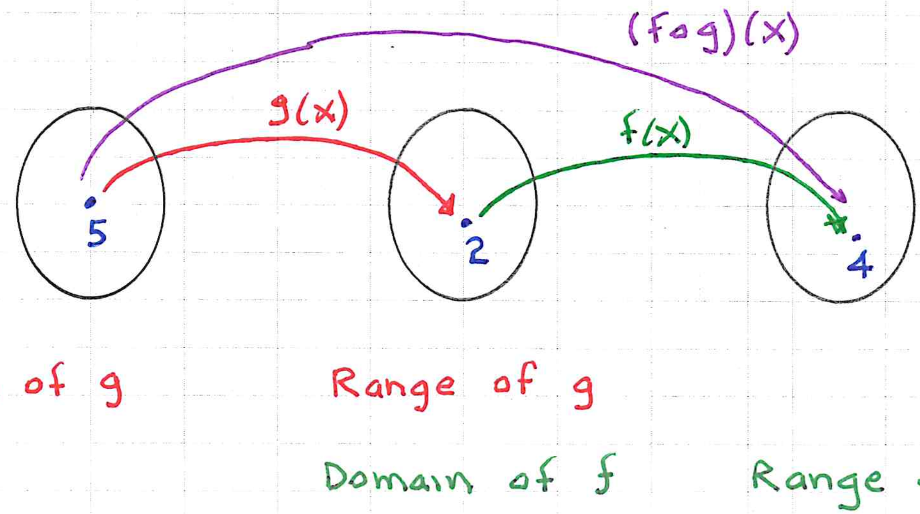
k) Find the domain of $(f/g)(x)$ is all real #'s except $x = 2$.

Definition: The composition of the function f with the function g is given by: $(f \circ g)(x) = f(g(x))$.

Ex. 2 Let $f(x) = x^2$ and $g(x) = \sqrt{x-1}$

Find: a) $(f \circ g)(5) = f(g(5)) = f(2) = 2^2 = \boxed{4}$

Mapping Diagram



b) $(f \circ g)(7) = f(g(7)) = f(\sqrt{6}) = 6$

c) $(f \circ g)(-1) = f(g(-1)) = ?$ Not defined. Since $g(-1) = \sqrt{-2}$.

d) $(f \circ g)(x) = f(g(x)) = f(\sqrt{x-1}) = (\sqrt{x-1})^2 = \boxed{x-1}$

e) $(g \circ f)(5) = g(f(5)) = g(25) = \sqrt{24} = \sqrt{4 \cdot 6} = 2\sqrt{6}$

Ex. 3 Given a function $C(x)$, find functions f and g so that $C(x) = (f \circ g)(x)$

a) $C(x) = |x+4|$

Now $f(x) = |x|$ and $g(x) = x+4$ will work.

Since $f(g(x)) = f(x+4) = |x+4| = C(x)$

b) $C(x) = \sqrt{x^2+1}$

Now $f(x) = \sqrt{x}$ and $g(x) = x^2+1$ will work

Since $f(g(x)) = f(x^2+1) = \sqrt{x^2+1} = C(x)$

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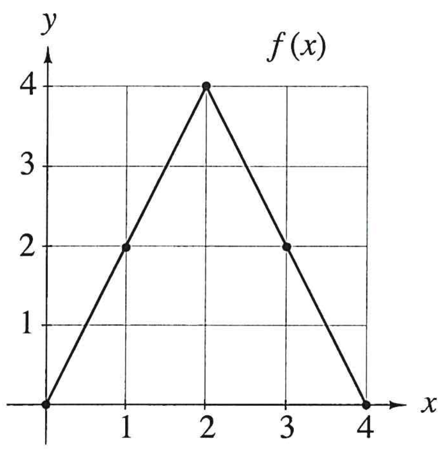
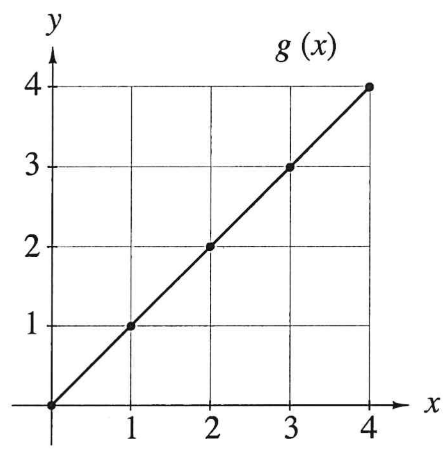
Combining Functions (§1.5)

Let f and g be functions with overlapping domains. If x lies in the domains of both functions, then the following are defined as:

1. **Sum:** $(f+g)(x) = f(x)+g(x)$
2. **Difference:** $(f-g)(x) = f(x)-g(x)$
3. **Product:** $(f \cdot g)(x) = f(x) \cdot g(x)$
4. **Quotient:** $(f/g)(x) = f(x)/g(x)$, provided $g(x) \neq 0$.
5. **Composition:** $(f \circ g)(x) = f(g(x))$

Graphical Examples:





Ex. 4 Find:

a) $(f+g)(3) = f(3) + g(3) = 2 + 3 = \boxed{5}$

b) $(f \cdot g)(2) = f(2) \cdot g(2) = 4 \cdot 2 = \boxed{8}$

c) $(f/g)(4) = \dfrac{f(4)}{g(4)} = \dfrac{0}{4} = \boxed{0}$

d) $(f \circ g)(1) = f(g(1)) = f(1) = \boxed{2}$