

1.4 Transforming the Graphs of Functions

(I) Vertical and Horizontal Shifts

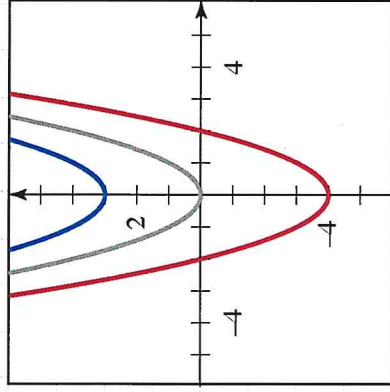
The graph of a function can be shifted vertically or horizontally by adding a constant to the original function or to x .

Vertical Shift

$$y = f(x) = x^2$$

$$m(x) = x^2 + 3$$

$$p(x) = x^2 - 4$$



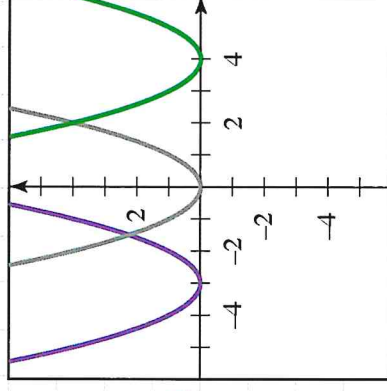
Horizontal Shift

$$y = f(x) = x^2$$

$$n(x) = (x+3)^2$$

$$c > 0$$

$$r(x) = (x-4)^2$$



V. shift c units up: $h(x) = f(x) + c$

H. shift c units left: $h(x) = f(x+c)$

V. shift c units down: $h(x) = f(x) - c$

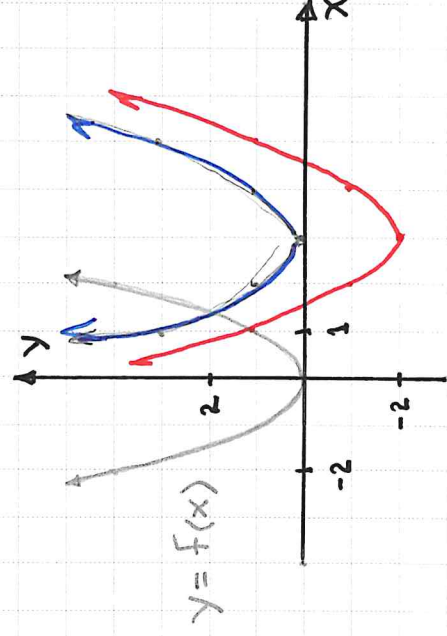
H. shift c units right: $h(x) = f(x-c)$

Ex. ① Given that $f(x) = x^2$, sketch the

graph of $h(x) = f(x-3) - 2$

Steps: (1) HS 3 →

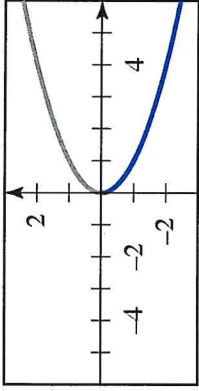
(2) VS 2 ↓ Red



(II) Reflections in the x and y axes.

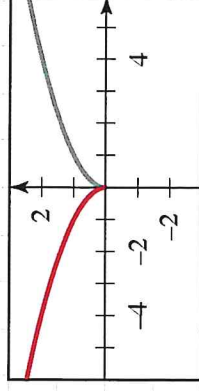
$$y = f(x) = \sqrt{x}$$

$$m(x) = -\sqrt{x}$$



$$y = f(x) = \sqrt{x}$$

$$h(x) = \sqrt{-x}$$



Reflection in the x-axis: $h(x) = -f(x)$ Reflection in the y-axis: $h(x) = f(-x)$

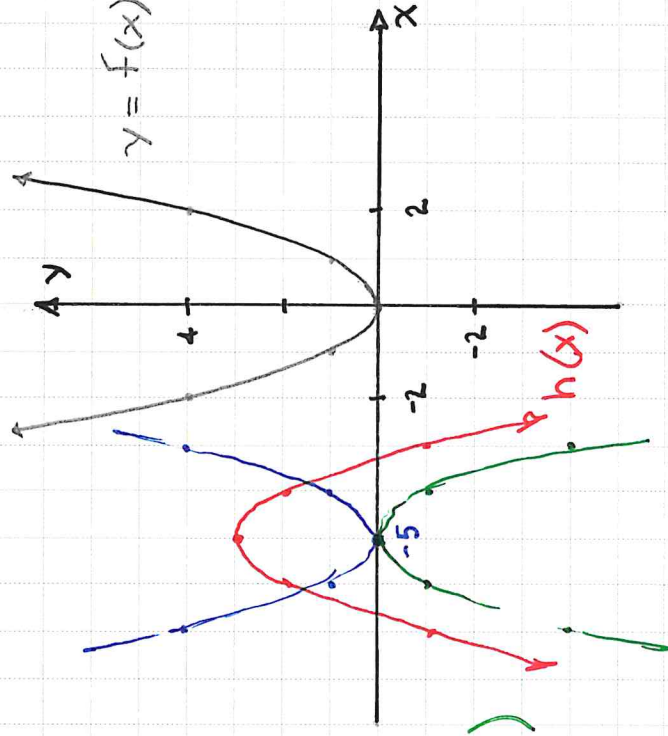
Ex. ② Given that $f(x) = x^2$,
sketch the graph of

$$h(x) = -f(x+5) + 3$$

Steps: ① H5 ←

② Rx (Reflect in x)

③ V3 ↑



* Note: When reflections are involved ORDER is important.

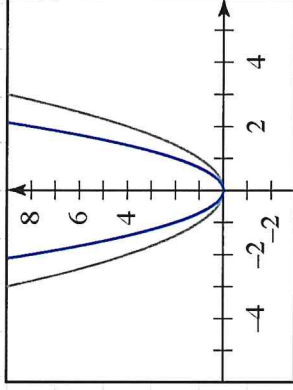
(III) Vertical Scaling

When a function is multiplied by a positive constant, the original graph is either stretched or compressed.

Stretch ($c > 1$)

$$y = f(x) = x^2$$

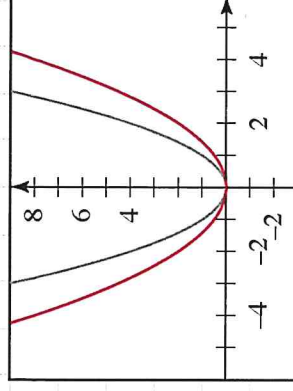
$$m(x) = 2x^2$$



Shrink ($0 < c < 1$)

$$y = f(x) = x^2$$

$$n(x) = \frac{1}{2}x^2$$



V. stretch when $c > 1$: $h(x) = c f(x)$

V. shrink when $0 < c < 1$: $h(x) = c f(x)$

Ex. ③ Given that $f(x) = |x|$,

sketch the graph of

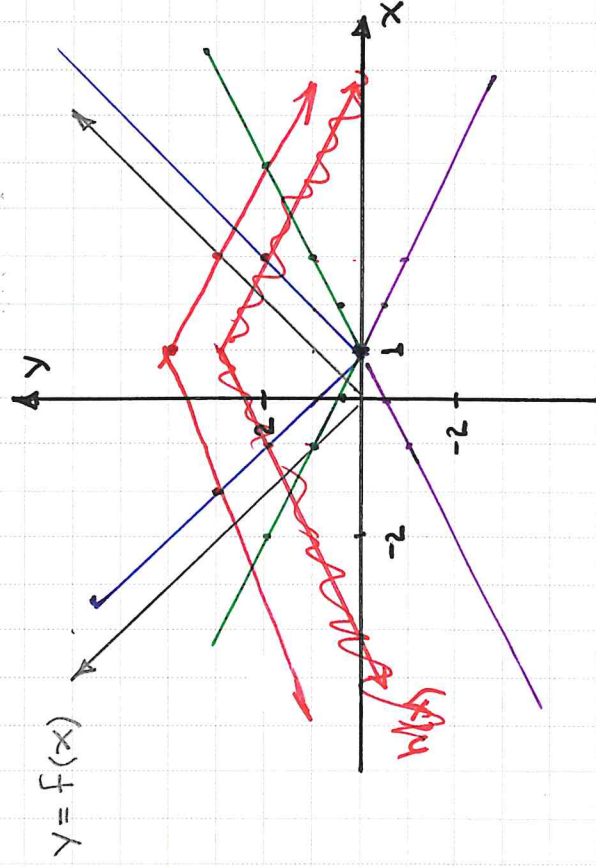
$$h(x) = -\frac{1}{2} f(x-1) + 4$$

Steps: ① HS 1 \rightarrow

② V shrink $c = \frac{1}{2}$

③ R x

④ VS 4 \uparrow

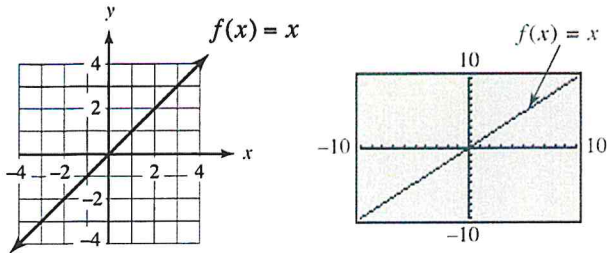


$$h(x) = -\frac{1}{2}|x-1| + 4$$

Library of Functions

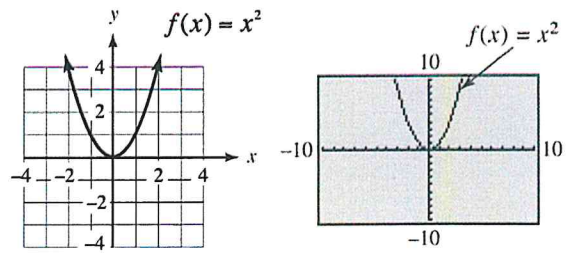
Identity Function $f(x) = x$

Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$



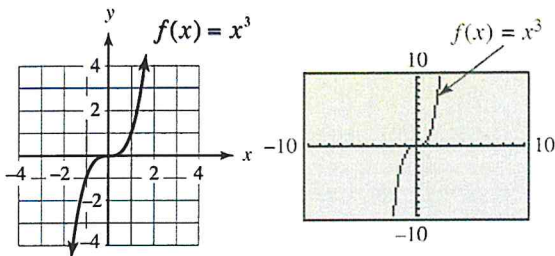
Squaring Function $f(x) = x^2$

Domain: $(-\infty, \infty)$ Range: $[0, \infty)$



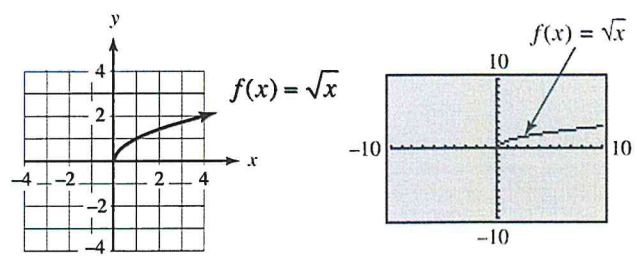
Cubing Function $f(x) = x^3$

Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$



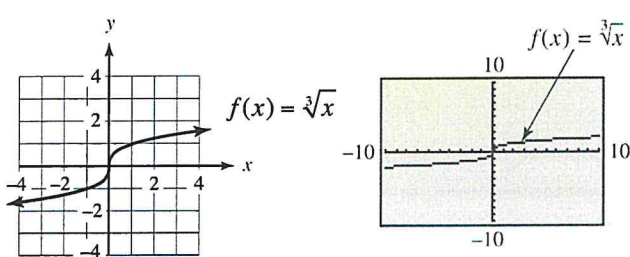
Square Root Function $f(x) = \sqrt{x}$

Domain: $[0, \infty)$ Range: $[0, \infty)$



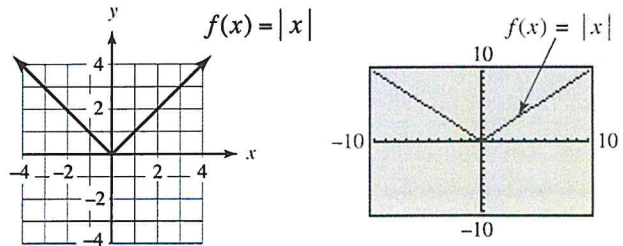
Cube Root Function $f(x) = \sqrt[3]{x}$

Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$



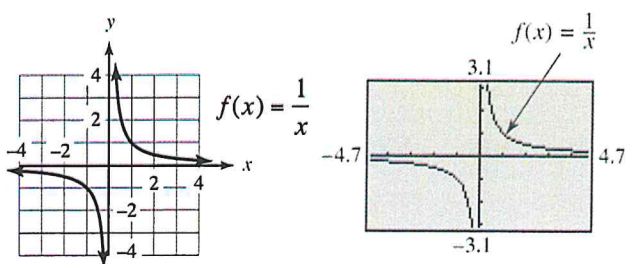
Absolute Value Function $f(x) = |x|$

Domain: $(-\infty, \infty)$ Range: $[0, \infty)$



Reciprocal Function $f(x) = \frac{1}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0) \cup (0, \infty)$



Rational Function $f(x) = \frac{1}{x^2}$

Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(0, \infty)$

