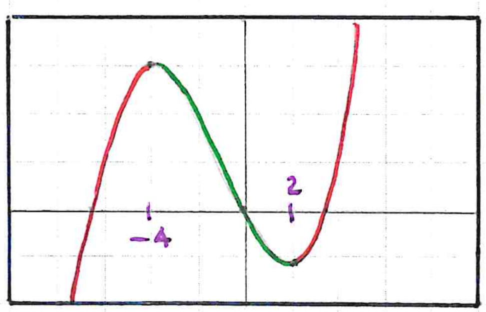
1.3 Graphs of Functions

Recall that if $y = f(x)$ then for every x in the domain of f there can be only one value of y. This leads to a simple graphical test for functions.

The Vertical Line Test

A set of points in the coordinate plane is the graph of y as a function of x if and only if no vertical line intersects the graph in more than one point.

Ex. 1 Sketch the graph of $f(x) = x^3 + 3x^2 - 24x$ in the window $[-10, 10] \times [-50, 100]$.



Rel. Max = 80 ; Rel. Min = -28

Notes: 1) The graph appears to have a "peak" (relative maximum) at $x = -4$

Use $\boxed{2^{nd}}$ $\boxed{\text{CALC}}$ $\boxed{\text{maximum}}$

2) The graph appears to have a "valley" (relative minimum) at $x = 2$

Use $\boxed{2^{nd}}$ $\boxed{\text{CALC}}$ $\boxed{\text{minimum}}$

Note: For the following material, we will assume that the graph of a function is traced out from left to right. The direction in which x increases.

For the graph of the function in Ex. 1, it can be seen that the graph is rising when x is less than -4. We say that the function is increasing on the interval: $(-\infty, -4)$ and $(2, \infty)$

The graph is falling when x is between -4 and 2. We say that the function is decreasing on the interval: $(-4, 2)$

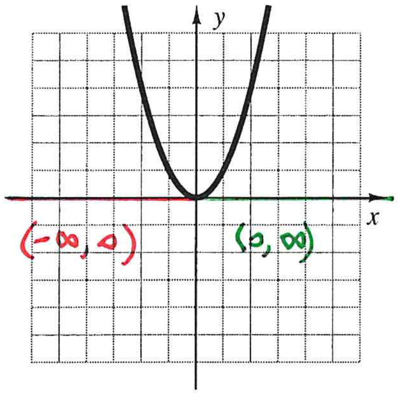
If a function neither rises nor falls, we say that it is constant.

Note that a function's behavior will change at a relative maximum or minimum.

Functions and the Vertical Line Test

Use the vertical line test to determine which of the following graphs are graphs of functions.

Increasing (——)　　　Decreasing (——)



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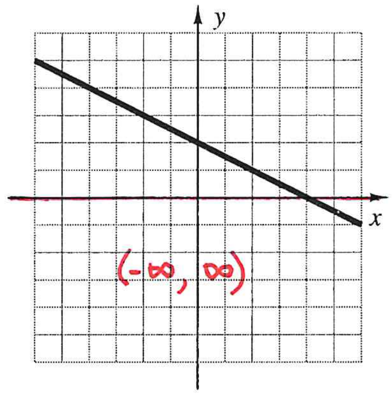

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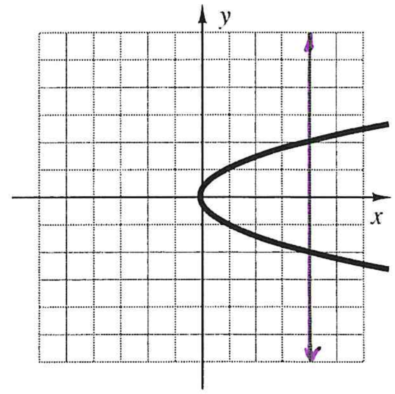

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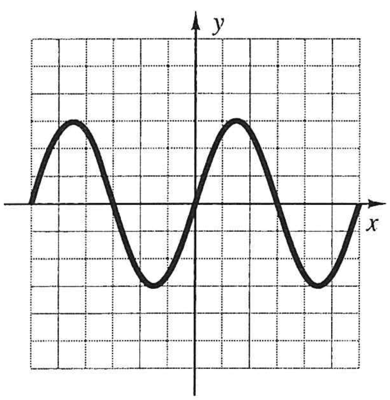

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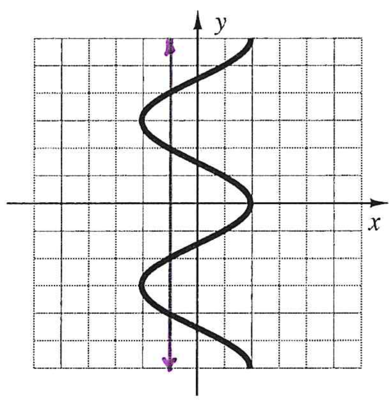

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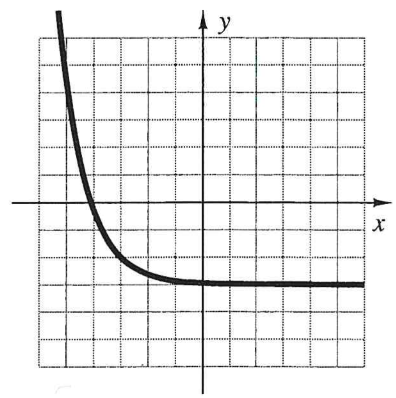

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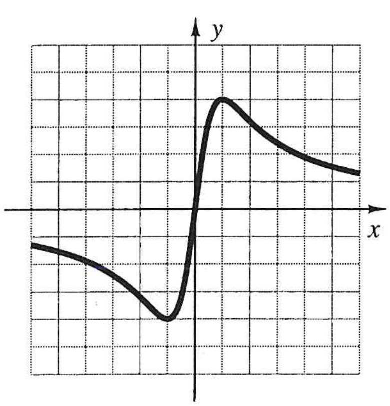

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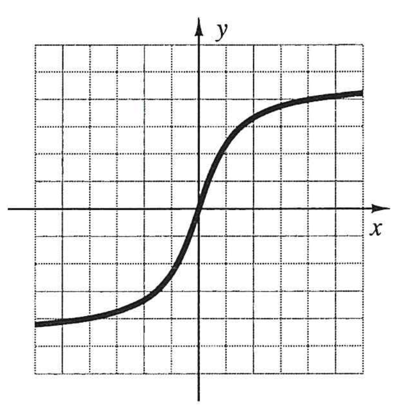

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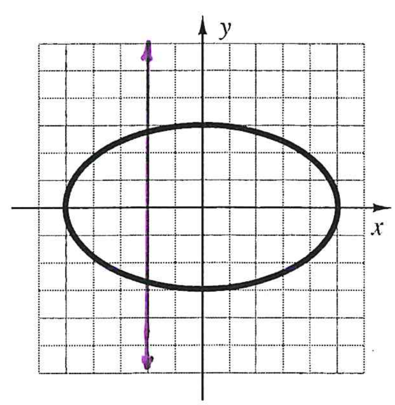


NO

Definition: A piece-wise function is a function that is defined by two or more equations over a specified domain.

Ex. ② Consider the function: $g(x) = \begin{cases} 4 - x/2, & \text{when } x \le 0 \\ x^2, & \text{when } x > 0 \end{cases}$

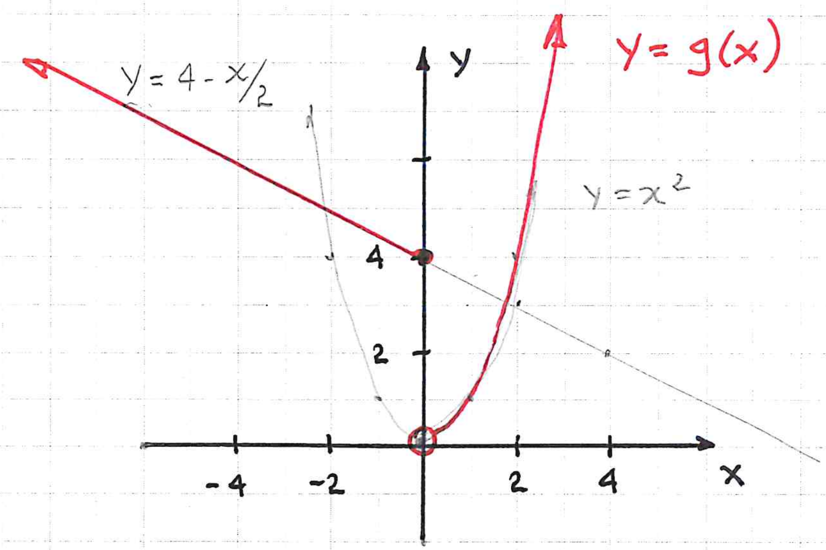
"split Domain"

a) Find $g(2) = 2^2 = 4$

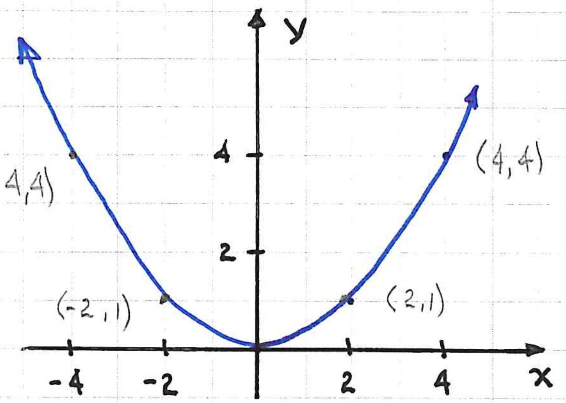
b) Find $g(-4) = 4 - (-4)/2 = 6$

c) Find $g(0) = 4 - 0/2 = 4$

d) Sketch the graph of $y = g(x)$ by hand.



Consider the partial graph of the function $y = f(x)$.



Reflect the graph in the y-axis.

A function that is symmetric in the y-axis is called an EVEN function.

Note that $f(-4) = f(4) = 4$ and $f(-2) = f(2) = 1$

In fact, for any x, $f(-x) = f(x)$

Ex. 3 a) Show that the function

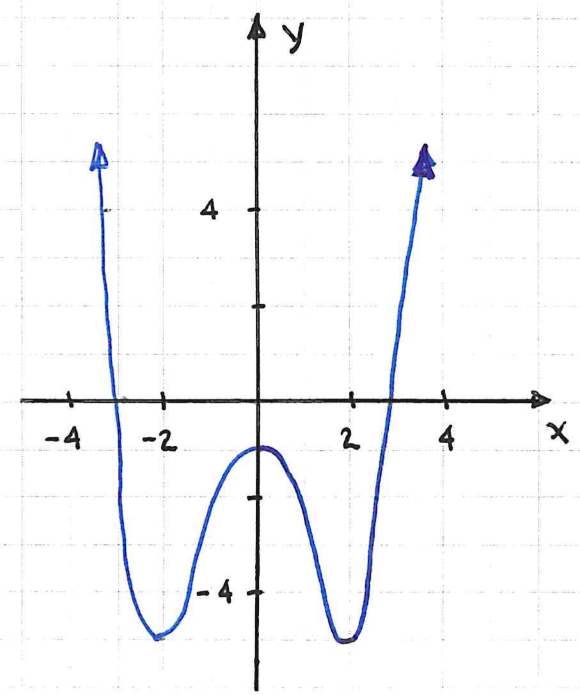
$$g(x) = \tfrac{1}{4}x^4 - 2x^2 - 1$$

is an even function.

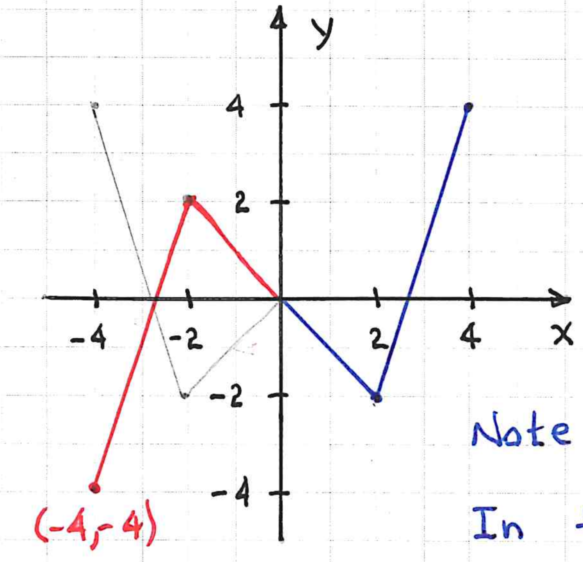
$$g(-x) = \tfrac{1}{4}(-x)^4 - 2(-x)^2 - 1$$

$$= \tfrac{1}{4}x^4 - 2x^2 - 1 = g(x) \checkmark$$

b) Complete the graph of $y = g(x)$.



Consider the partial graph of the function $y = f(x)$.



Reflect the graph in both axes.

A function with this property is called an ODD function.

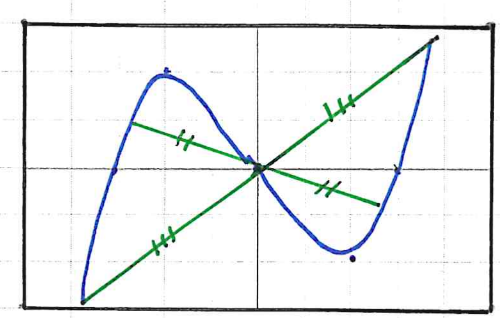
Note that $f(-4) = -4 = -f(4)$ and $f(-2) = 2 = -f(2)$

In fact, for any x, $f(-x) = -f(x)$

Ex. 4 a) Show that the function $g(x) = \frac{1}{4}(x^3 - 16x)$ is an odd function.

Now $g(-x) = \frac{1}{4}((-x)^3 - 16(-x)) = \frac{1}{4}(-x^3 + 16x) = -\frac{1}{4}(x^3 - 16x) = -g(x)$

b) Sketch the graph of $y = g(x)$ in the window $[-6, 6] \times [-8, 8]$.



Symmetric in the Origin