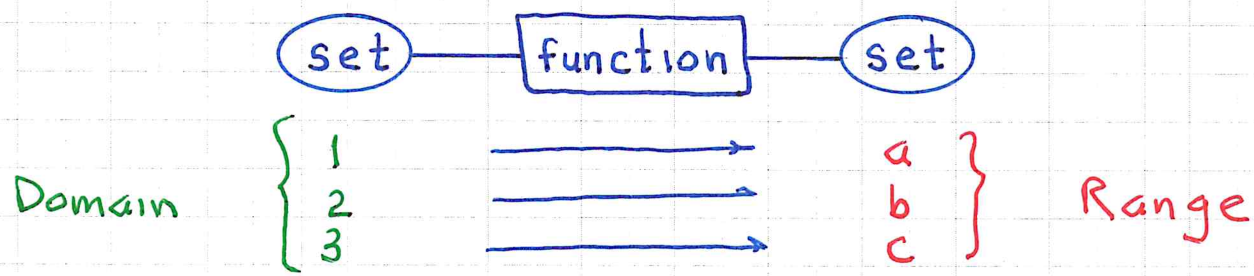
1.2 Functions

In English, the word "function" refers to what something does.

In math, a function is a rule that connects one set to another set in a particular way.



A function takes each value from an input set (Domain) and relates it to a value in an output set (Range).

Ex. 1 Rule: Output the number of sides.

Input	Output
Triangle	3
Pentagon	5
Octagon	8

Domain

Range

Most functions that you'll encounter in calculus will be a little more abstract than that. They'll usually just relate one variable to another in the form of an equation.

Ex. 2 Complete the table.

Rule: $y = x^2 - 4$	
Input	Output
x	y
-3	5
0	-4
2	0

There is an important limitation about functions that we need to know.

Definition: A function relates each element of one set (Domain) with exactly one element of another set (Range).

Ex. 3 Consider the equation: $x^2 + y^2 = 4$

Solve for y: $y^2 = 4 - x^2 \Rightarrow y = \pm\sqrt{4-x^2}$

Rule: $y = \pm\sqrt{4-x^2}$

Input	Output
0	± 2
1	$\pm\sqrt{3}$
$\sqrt{3}$	± 1

Note:

For each value of x we get two values of y.

So, this rule is NOT a function.

Because certain operations in math are undefined (eg. division by zero) the domain of a function may need to be restricted.

Ex. 4 Find the domain of the function.

Function	Domain	
$y = x^2 - 4$	All real numbers.	$(-\infty, \infty)$
$y = 1/t$	All real numbers except $t = 0$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{4-x^2}$	$[-2, 2]$	

Function Notation

We write a function like $y=x^2-4$ in this form: $\overset{\text{Name}}{f}(\underset{\text{Input}}{x}) = \underbrace{x^2-4}_{\text{Output}}$

We read this as: "f of x equals x^2-4"

Note that x is just a placeholder. It shows us where the input goes and what happens to it. Any letter will do.

Ex. 5 Given that $\boxed{f(x) = x^2 + x,}$ find the following:

a) $f(2) = 2^2 + 2 = 4 + 2 = 6$

b) $f(-3) = (-3)^2 + (-3) = 9 - 3 = 6$

c) $f(a) = a^2 + a$

d) $f(2x) = (2x)^2 + 2x = 4x^2 + 2x$

e) $f(☺) = ☺^2 + ☺$

f) $f(x+4) = (x+4)^2 + (x+4) = x^2 + 8x + 16 + x + 4 = x^2 + 9x + 20$

Note: There is nothing special about the letter f. It serves as a name.

We could have written:

Definition: The Difference Quotient for a function f is given by the formula: $DQ = \dfrac{f(x+h) - f(x)}{h}$ (where $h \neq 0$)

Ex. 6 Find the DQ for each of the following functions.

a) $f(x) = 1 - 2x$

Careful

$$DQ = \frac{f(x+h) - f(x)}{h} = \frac{\overbrace{1-2(x+h)}^{f(x+h)} - (1-2x)}{h}$$

$$= \frac{\cancel{1} - \cancel{2x} - 2h - \cancel{1} + \cancel{2x}}{h} = \frac{-2\cancel{h}}{\cancel{h}} = \boxed{-2}$$

b) $g(x) = x^2 - x$

$$DQ = \frac{g(x+h) - g(x)}{h} = \frac{\overbrace{(x+h)^2 - (x+h)}^{g(x+h)} - (x^2 - x)}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x} - h - \cancel{x^2} + \cancel{x}}{h}$$

$$= \frac{2xh + h^2 - h}{h} = \frac{\cancel{h}(2x + h - 1)}{\cancel{h}} = \boxed{2x + h - 1}$$