Similar Triangles Handout

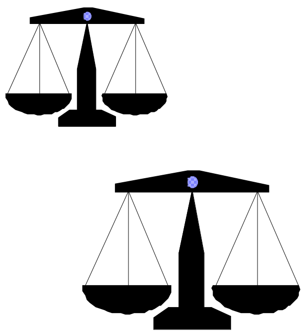


FIGURE 1

When an architect draws plans for a house or an engineer makes a drawing of a machine part, the result is drawn to scale showing the same objects in reduced sizes. Two figures with the same shape are said to be **similar**.

The silhouettes of the scales shown in Figure 1 are similar.

> **Definition of Similar Figures**
>
> Two figures are **similar** if one of the figures can be uniformly enlarged so that it is identical with the other figure.

This just means that if two figures are similar then one can be "blown up" to match the other.

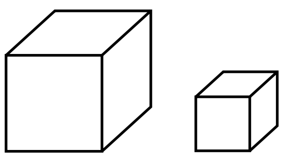


FIGURE 2

The term similar may be applied to three-dimensional objects as well as plane objects. The two boxes in Figure 2 are similar.

In this handout, we will be concerned with similar triangles. You could probably guess that two triangles will have the same shape if three angles of one triangle have the same measure as three angles of the other triangle.

> **Theorem**
>
> If two angles of one triangle have the same measure as two angles of another triangle, then the triangles are similar.

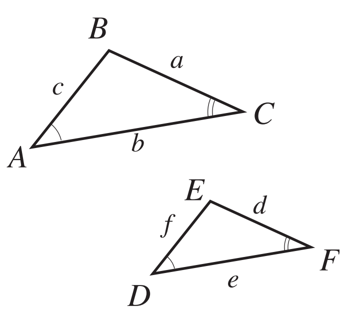


FIGURE 3

Note: The arcs drawn in the angles indicate which angles are equal. The same letter names the angle and the side opposite from it.

In Figure 3, ΔABC is similar to ΔDEF since $\angle A = \angle D$ and $\angle C = \angle F$. In symbols, we write $\Delta ABC \sim \Delta DEF$. It is important to write the names of the triangles so that the letters of the corresponding angles are in the same order.

The benefit that comes from having similar triangles is found in the following result.

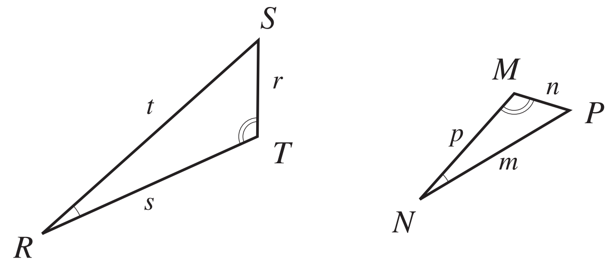
> **Theorem**
>
> If two triangles are similar, then their corresponding sides are in proportion.

For the triangles in Figure 3, this means that $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$.

This is called an **extended proportion** since it involves three fractions.

EXAMPLE 1

Referring to the triangles shown below, (**a**) name the similar triangles, (**b**) write an extended proportion that is true for these triangles and (**c**) if $r = 4$, $n = 3$, and $t = 8$, then find m.



Solution

(**a**) Note from the figure that $\angle R = \angle N$ and $\angle T = \angle M$. Thus $\angle S = \angle P$ and $\Delta RST \sim \Delta NPM$.

(**b**) Comparing the corresponding sides gives: $\frac{r}{n} = \frac{s}{p} = \frac{t}{m}$.

Note that the corresponding sides r and n are opposite from the equal angles R and N.

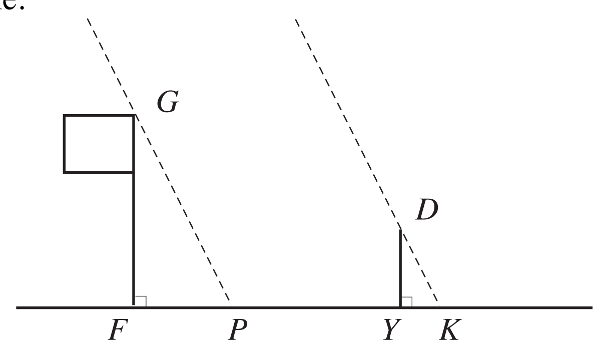
(**c**) Using the proportion from part (b) we have

$$\frac{r}{n} = \frac{t}{m} \quad \Rightarrow \quad \frac{4}{3} = \frac{8}{m} \quad \Rightarrow \quad 4m = 24 \quad \Rightarrow \quad m = 6 .$$

Consider the following example.

EXAMPLE 2

The shadow of a flagpole has length 20 feet. At the same time, the length of the shadow of a yardstick is 6 inches. Find the height of the flagpole.



Solution

If we assume that the flagpole and yardstick are perpendicular to the ground and that the rays from the sun are parallel, then $\angle F = \angle Y$ and $\angle P = \angle K$, and we have $\Delta FGP \sim \Delta YDK$.

This means that $\frac{f}{y} = \frac{g}{d} = \frac{p}{k}$.

We know that $g = 20$ ft, $k = 1$ yd $= 3$ ft and $d = 6$ in $= \frac{1}{2}$ ft.

Since p is the height of the flagpole,

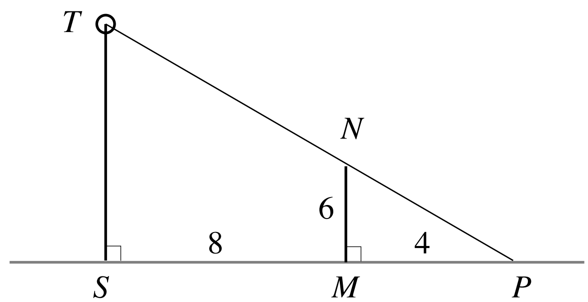
$$\frac{p}{3} = \frac{20}{\frac{1}{2}} \quad \Rightarrow \quad \frac{p}{3} = 40 \quad \Rightarrow \quad p = 120 .$$

The height of the flagpole is 120 feet.

In some situations, one similar triangle will overlap another. This usually means that one angle is shared by both triangles.

EXAMPLE 3

A man, 6 feet tall, is walking away from a street light. If the length of the man's shadow is 4 feet when he is 8 feet from the light, then how high is the light?



Solution

Let segment ST represent the street light and MN represent the man. Note that $\angle S = \angle M$ and that $\angle P = \angle P$. Thus $\Delta STP \sim \Delta MNP$.

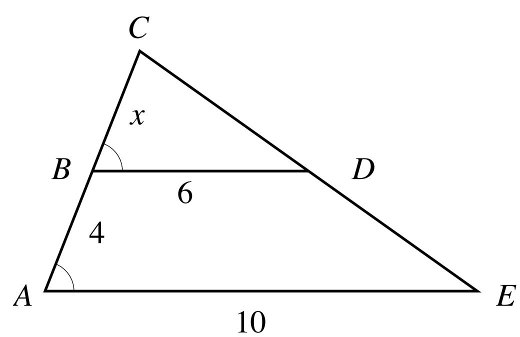
This means that $\dfrac{s}{m} = \dfrac{t}{n} = \dfrac{ST}{MN}$. Now we know that $t = 12$, $n = 4$ and $MN = 6$. Thus, $\dfrac{12}{4} = \dfrac{ST}{6} \quad \Rightarrow \quad ST = 18$.

The street light is 18 feet high.

There are certain problems in which the unknown side of one triangle may also be part of a side of the other triangle.

EXAMPLE 4

Solve for x.



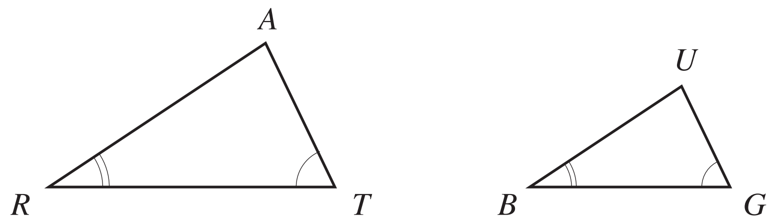
Solution

Note that $\Delta ACE \sim \Delta BCD$ and that the length of side $AC = x + 4$.

Thus, $\dfrac{x}{x+4} = \dfrac{6}{10} = \dfrac{3}{5} \quad \Rightarrow \quad 5x = 3x + 12 \quad \Rightarrow \quad x = 6$.

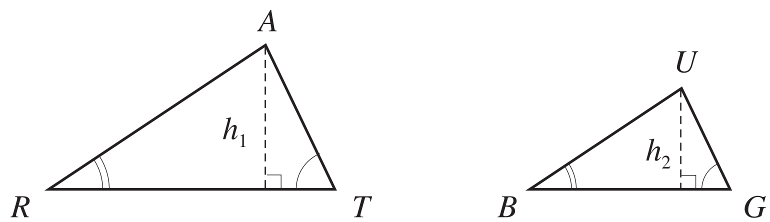
We know that when two triangles are similar, the corresponding sides are in proportion. In fact, there are many other corresponding parts of the triangles that are in the same proportion. Two of these are shown below.

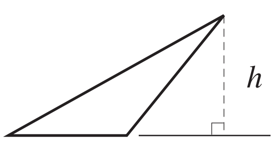
For example, suppose that $\Delta RAT \sim \Delta BUG$.



Then $\frac{r}{b} = \frac{a}{u} = \frac{t}{g}$.

An altitude of a triangle is the segment from a vertex perpendicular to the side opposite that vertex. Let h_1 be the altitude of ΔRAT from *A* and h_2 be the altitude of ΔBUG from *U*.

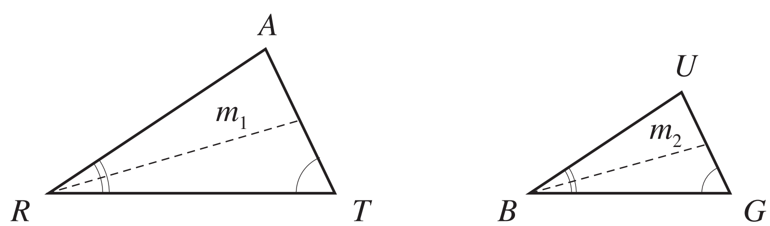




Note: In some triangles, an altitude may lie outside of the triangle.

Then $\frac{h_1}{h_2} = \frac{r}{b} = \frac{a}{u} = \frac{t}{g}$.

A median of a triangle is the segment joining a vertex to the midpoint of the side opposite that vertex. Let m_1 be the median of ΔRAT from *R* and m_2 be the median of ΔBUG from *B*.



Then $\frac{m_1}{m_2} = \frac{r}{b} = \frac{a}{u} = \frac{t}{g}$

The study of similar triangles has many practical applications. Determining distances that are difficult or impossible to measure is one such application. This is how the distance between the Earth and Moon was first found. In addition, similar triangles are the basis for right triangle trigonometry and its many important applications.

EXERCISES

NOTE: THESE PROBLEMS ARE TO BE DONE ON ENGINEERING PAPER.

For exercises 1–4, state whether the two triangles are *always*, *sometimes*, or *never* similar.

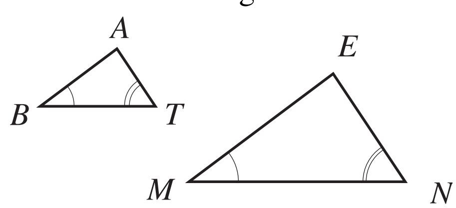
1. Two equilateral triangles.

2. Two right triangles.

3. A right triangle and an equilateral triangle.

4. Two right triangles, one with an acute angle of measure 60° and the other with an acute angle of measure 30°.

5. Consider the triangles shown below.

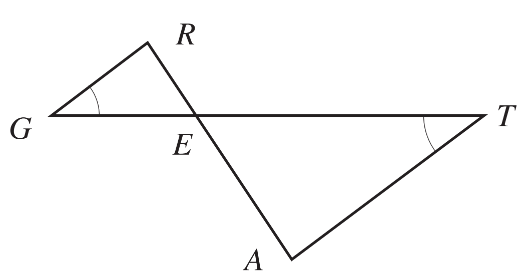


a) Name the similar triangles.

b) Write an extended proportion that is true for these triangles.

c) If $t = 2$, $a = 3$, and $n = 6$, find e.

6. Consider the triangles shown below.

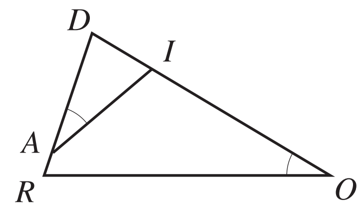


a) Name the similar triangles.

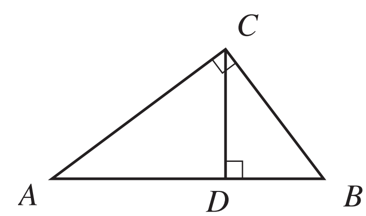
b) Write an extended proportion that is true for these triangles.

c) If $g = 30$, $AT = 36$, and $t = 45$, find GR.

7. Name the similar triangles in the figure.

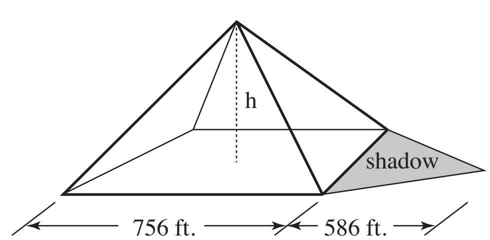


8. Name the similar triangles in the figure. [*Hint*: There are three.]

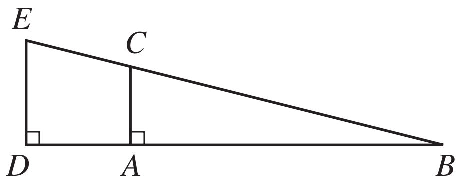


9. A flagpole casts a shadow of 28 feet at the same time the shadow of a person 6 feet tall is 2 feet long. How tall is the flagpole?

10. The Greek mathematician Thales was known to have calculated the height of the great pyramid of Cheops by measuring the shadow of a pole. He found that the base of the pyramid has sides that are 756 ft. long. If his 3 ft. pole cast a shadow that was 6 ft. long at the same time that the shadow of the pyramid was 586 ft. long, determine the height of the pyramid **to the nearest foot.**

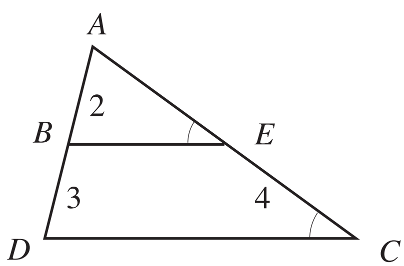


11. To measure the distance between points A and B on two opposite sides of a canyon, a man takes the following measurements: $AC = 5$ m, $DE = 6$ m, and $AD = 15$ m. What is the distance between the sides of the canyon?

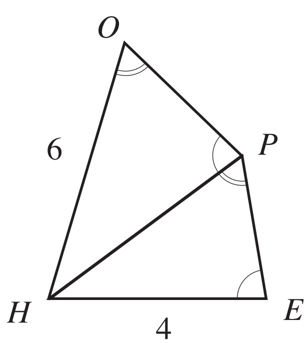


12. A girl, 5 feet tall, notices that when she is 2 feet from her boyfriend the line of sight to the top of a building just passes over the top of his head. Her boyfriend is 6 feet tall and is standing 100 feet from the building. How tall is the building?

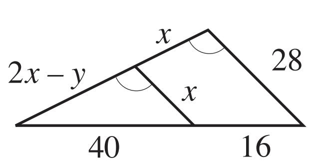
13. Find AE in the figure below.



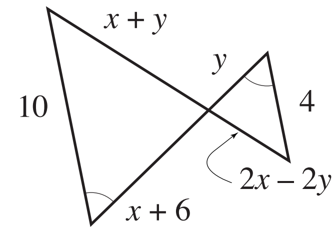
14. Find HP in the figure below.



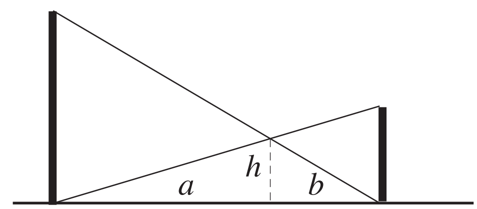
15. Find the value of each variable in the figure below.



16. Find the value of each variable in the figure below.

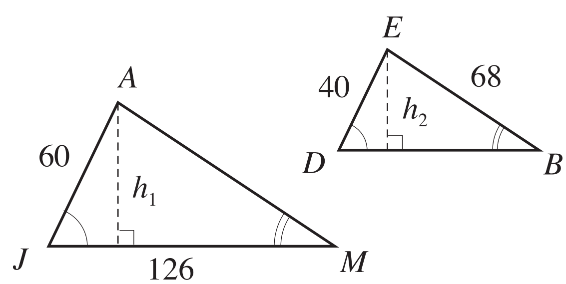


17. Wires are stretched from the top of each of two vertical poles to the bottom of the other as shown. If one pole is 4 feet tall and the other 12 feet tall, how far above the ground do the wires cross?



[*Hint*: Look for two pairs of similar triangles.]

For Exercises 18 and 19, use the triangles shown below.



18. **a)** Find the missing side lengths.

b) Find the ratio of the perimeter of ΔJAM to the perimeter of ΔDEB.

19. Suppose that $h_1 = 48$.

a) Find h_2.

b) Find the ratio of the area of ΔJAM to the area of ΔDEB.

20. Suppose that $\Delta ABC \sim \Delta DEF$ and let h_1 be the altitude from A and h_2 be the altitude from D. Find the ratio of the area of ΔABC to the area of ΔDEF in terms of a and d.

Answers to odd numbered exercises.

1. Always

3. Never

5. a) $\Delta BAT \sim \Delta MEN$ b) $\dfrac{b}{m} = \dfrac{a}{e} = \dfrac{t}{n}$ c) 9

7. $\Delta ADI \sim \Delta ODR$

9. 84 feet

11. $AB = 75$ m

13. $AE = \dfrac{8}{3}$

15. $x = 20$, $y = -10$

17. $h = 3$

19. a) $h_2 = 32$ b) $\dfrac{9}{4}$