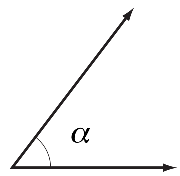
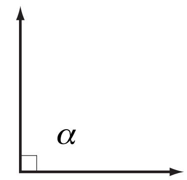
Geometry Facts Handout

As you begin your study of trigonometry, you will find it useful to know the following basic definitions and facts from plane geometry.

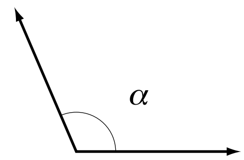
I. Lines and angles



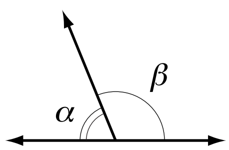
Acute angle
$0° < \alpha < 90°$



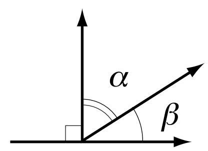
Right angle
$\alpha = 90°$



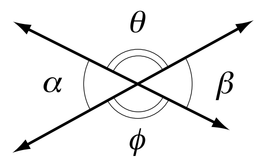
Obtuse angle
$90° < \alpha < 180°$



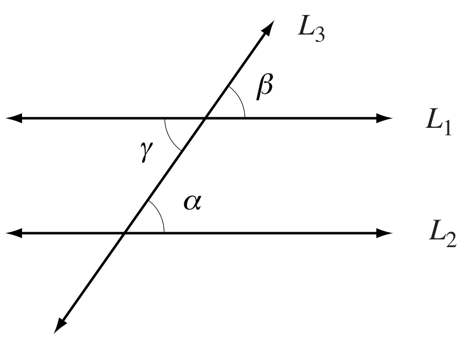
Supplementary angles
$\alpha + \beta = 180°$



Complementary angles
$\alpha + \beta = 90°$



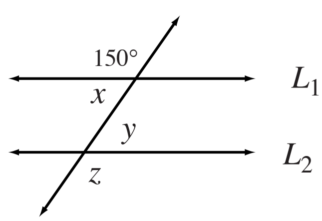
Intersecting lines
$\alpha = \beta$ and $\phi = \theta$



If two parallel lines L_1 and L_2 are cut by a third line L_3, then

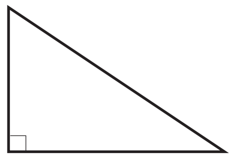
$\alpha = \beta$ (corresponding angles are equal)
$\alpha = \gamma$ (alternate interior angles are equal)

Example 1 Given that $L_1 \parallel L_2$ (this means that L_1 is parallel to L_2), find the measures of the angles marked x, y, and z.

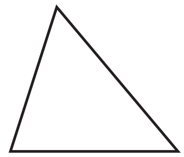


Solution Note that x and 150° are supplementary, so $150° + x = 180° \quad \Rightarrow \quad x = 30°$. Since x and y are alternate interior angles, $y = x = 30°$. Angles y and z are supplementary, so $30° + z = 180° \quad \Rightarrow \quad z = 150°$.

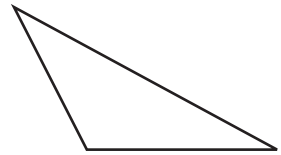
II. Triangles



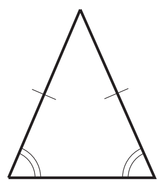
Right Triangle
One right angle.



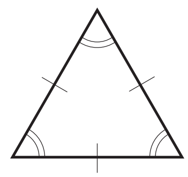
Acute Triangle
All acute angles.



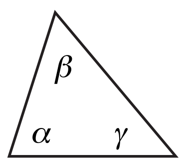
Obtuse Triangle
One obtuse angle.



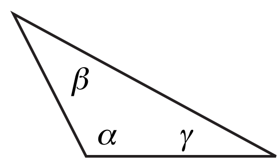
Isosceles Triangle
At least two sides equal.
At least two angles equal.



Equilateral Triangle
All sides equal.
All angles equal.

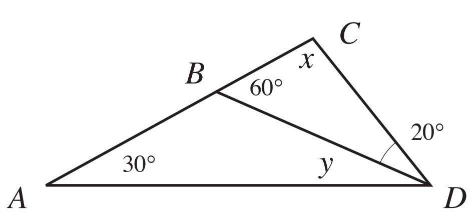


$$\alpha + \beta + \gamma = 180°$$



Theorem: The sum of the measures of the angles in any triangle is 180°.

Example 2 Find the measures of the angles marked x and y.



Solution Note that $x + 60° + 20° = 180° \quad \Rightarrow \quad x = 100°$.

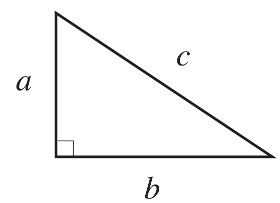
Also $x + 30° + 20° + y = 180° \quad \Rightarrow \quad 150° + y = 180° \quad \Rightarrow \quad y = 30°$

The following theorem is perhaps the most famous theorem in all of mathematics. It is known as the Pythagorean theorem in honor of the Greek mathematician Pythagoras (ca. 580 B.C.) who is believed to be the first person to have proved the theorem. However, this result about right triangles was known to the Babylonians and the Chinese long before Pythagoras lived.

Theorem: In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

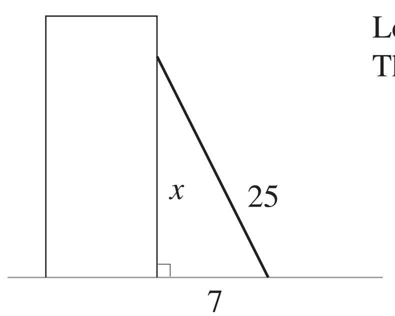
In symbols:

$$a^2 + b^2 = c^2$$



Example 3 A ladder of length 25 feet leans against a building so that the base of the ladder is 7 feet from the base of the building. How high does the ladder reach on the building?

Solution



Let x be the unknown height.
Then by the Pythagorean theorem,

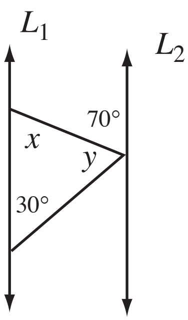
$$\begin{aligned} x^2 + 7^2 &= 25^2 \\ x^2 + 49 &= 625 \\ x^2 &= 576 \\ x &= 24 \end{aligned}$$

Thus, the ladder reaches 24 feet up the side of the building.

EXERCISES   NOTE: THESE PROBLEMS ARE TO BE DONE ON ENGINEERING PAPER.

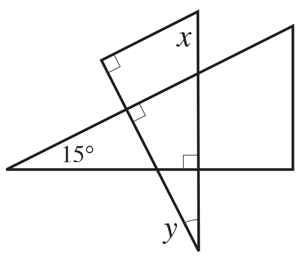
For exercises 1–5, find the measures of the angles marked x and y in each figure.

1.

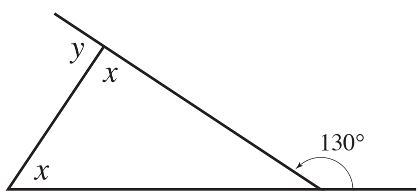


Given: $L_1 \parallel L_2$

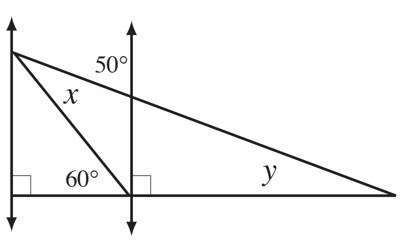
2.



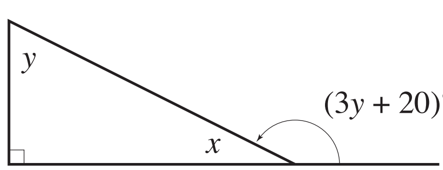
3.



4.



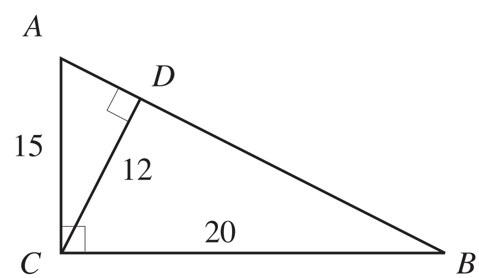
5.



6. A square has a side length of a. Determine the length of the diagonal of the square in terms of a.

7. An equilateral triangle has a side length of s. Determine the area of the triangle in terms of s. [*Hint*: Draw an altitude and use the Pythagorean theorem to find its length.]

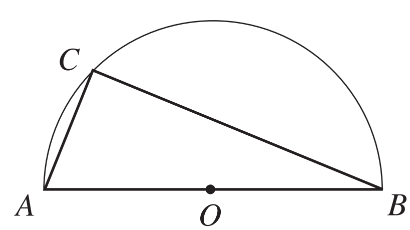
8. In the figure below, $AC = 15$, $BC = 20$ and $CD = 12$. Find the measures of AB and AD.



9. A rectangular box has length 6, width 3 and height 2. Find the length of the longest item that can fit in the box.

10. Suppose that triangle ABC is drawn so that AB lies along the diameter of a semicircle with center O and that C is a point on the semicircle. (See the figure below.) Use a fact about isosceles triangles and the fact about the sum of the angles in a triangle to show that angle C is a right angle.

[*Hint*: Draw OC.]



Answers to odd numbered exercises.

1. $x = 70°$ $y = 80°$

3. $x = 65°$ $y = 115°$

5. $x = 55°$ $y = 35°$

7. Area $= \dfrac{s^2\sqrt{3}}{4}$

9. 7 ft.