MATH 27
Exam #1
Fall 2022

Name: Key

Roll #: _____ Score: _______/100

Work each problem in the space provided on the exam page. Full or partial credit will be given only if all work has been shown and the answers are clearly indicated. Each numbered problem is worth 10 points.

1. Answer **T** if always true, **F** if otherwise.

F a) The distance between the origin and the point with coordinates (a,b) is $\sqrt{a+b}$.

T b) The solutions to the inequality $y \le 0$ are the values of x such that the graph of $y = f(x)$ lies on or below the x-axis.

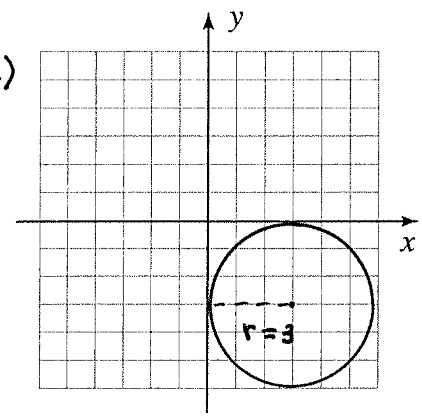
T c) A relative maximum occurs at the point where the function changes from increasing to decreasing.

T d) If $f(x)$ is a one-to-one function, then $f(-x)$ is also one-to-one.

T e) If $g(x)$ is an even function, then $g^{-1}(x)$ does not exist.

2. a) Draw a picture of a circle that is centered at the point (3, –3) and is tangent to the y-axis.
   b) Write the equation of the circle from part (a) in standard form.

a)



b) $(x-h)^2 + (y-k)^2 = r^2$

C: $(h,k) = (3,-3)$ and $r = 3$

So,

EQ: $\boxed{(x-3)^2 + (y+3)^2 = 9}$

3. Solve the equation: $x^2 - 6x = -4$ in two ways.

a) **Algebraic Solution**
[*Hint*: It's a quadratic equation.]
Leave your answer in simplified radical form.

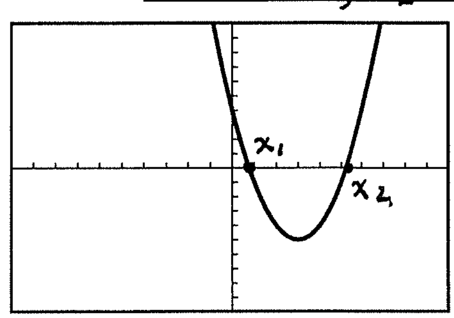
Zero Form: $x^2 - 6x + 4 = 0$

By the QF: $x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 4}}{2}$

$= \dfrac{6 \pm \sqrt{36-16}}{2}$

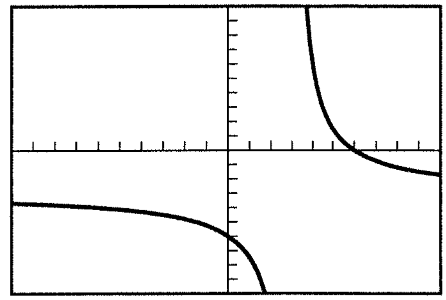
$= \dfrac{6 \pm \sqrt{20}}{2} = \dfrac{6 \pm 2\sqrt{5}}{2} = \boxed{3 \pm \sqrt{5}}$

b) **Graphical Solution**
Y1 = $x^2 - 6x + 4$
Solutions: $x_1 = 0.764$, $x_2 = 5.236$



Window: [–10, 10] x [–10, 10].
Label the solutions on the graph.
Approximate each solution to three decimal place accuracy.

4. Solve the inequality **graphically**: $\frac{x+6}{x-3} < 4$.



a) Zero Form: $\frac{x+6}{x-3} - 4 < 0$

b) Y1 = $(x+6)/(x-3) - 4$

c) Sketch the graph in the standard viewing window: [–10, 10] x [–10, 10].

d) Write the solution using either inequality or interval notation. (below the x-axis)

Inequality: $x < 3$ or $x > 6$

or Interval: $(-\infty, 3) \cup (6, \infty)$

5. For the function $f(x) = 4x - x^2$ evaluate the expression $\frac{f(x+h) - f(x)}{h}$. Simplify completely.

$$\frac{f(x+h) - f(x)}{h} = \frac{4(x+h) - (x+h)^2 - (4x - x^2)}{h}$$

$$= \frac{4x + 4h - (x^2 + 2xh + h^2) - 4x + x^2}{h}$$

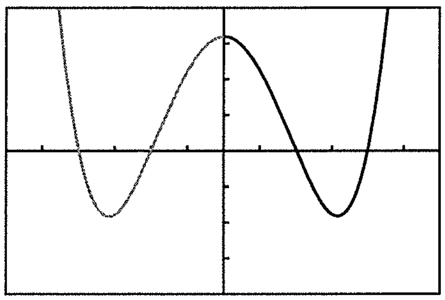
$$= \frac{4h - x^2 - 2xh - h^2 + x^2}{h}$$

$$= \frac{4h - 2xh - h^2}{h}$$

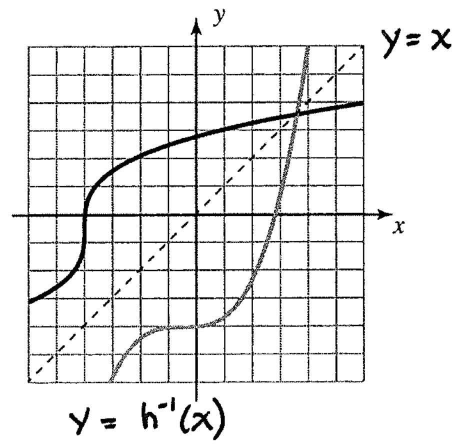
$$= \frac{h(4 - 2x - h)}{h}$$

$$= \boxed{4 - 2x - h}$$

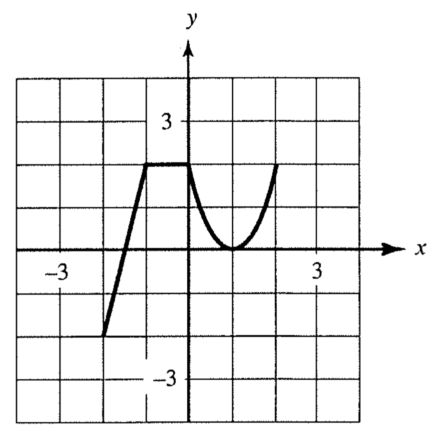
6. a) The graph shown below is of the function $g(x)$ for $x \geq 0$. Complete the graph if it is known that h is an **even** function.



b) The graph of a one-to-one function $y = h(x)$ is shown. Sketch the graph of the inverse function $y = h^{-1}(x)$.



7. Let $f(x)$ be defined by the adjacent graph. Describe, using symbols, the effect of each transformation and sketch the graphs of the resulting functions.

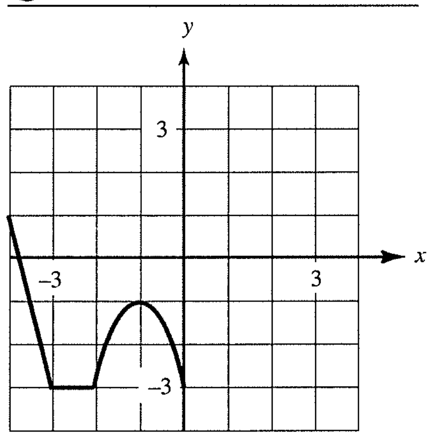


a) $g(x) = -f(x+2) - 1$

① HS 2 ←

② Rx

③ VS 1 ↓

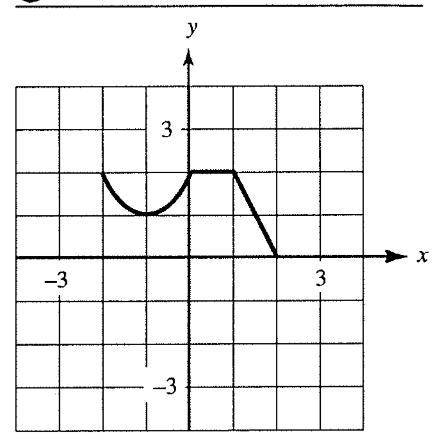


b) $h(x) = \frac{1}{2} f(-x) + 1$

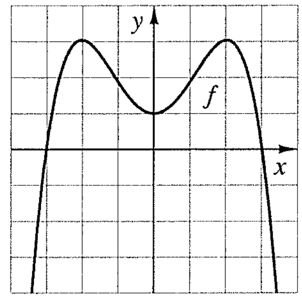
① Ry

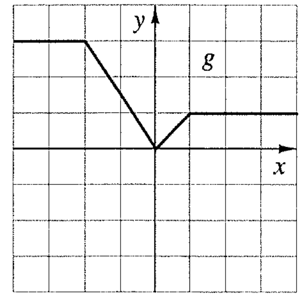
② VShrink c = 1/2

③ VS 1 ↑



8. Refer to the graphs of the functions f and g to compute the required quantities.





6 a) $(f+g)(-2) = f(-2) + g(-2)$

3 b) $(fg)(2) = f(2) \cdot g(2)$

0 c) $\left(\frac{g}{f}\right)(0) = \frac{g(0)}{f(0)}$

1 d) $(g \circ f)(2) = g(f(2)) = g(3) = 1$

1 e) $(f \circ f)(3) = f(f(3)) = f(0) = 1$

9. Let $f(x) = \dfrac{3x}{2x+1}$. Find $f^{-1}(x)$ algebraically.

1) $y = \dfrac{3x}{2x+1}$

2) $x = \dfrac{3y}{2y+1}$

3) $x(2y+1) = 3y$

$2xy + x = 3y$

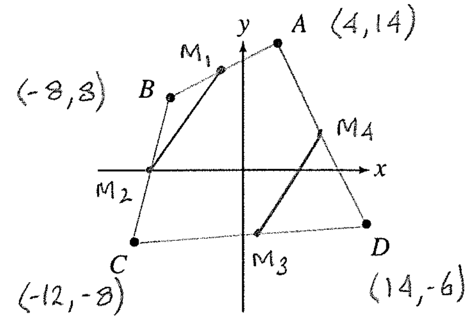
$2xy - 3y = -x$

$y(2x-3) = -x$

$y = \dfrac{-x}{2x-3}$

4) $\boxed{f^{-1}(x) = \dfrac{-x}{2x-3}}$

10. A theorem from geometry states that joining the midpoints of the sides of any quadrilateral results in a new quadrilateral that is a parallelogram.



The adjacent figure shows the graphs of the points:
$A(4,14)$, $B(-8,8)$, $C(-12,-8)$, and $D(14,-6)$.

a) Find the coordinates of the midpoints of the sides of quadrilateral $ABCD$.

Midpoint of $\overline{AB} = M_1 = \left(\frac{-8+4}{2}, \frac{8+14}{2}\right) = (-2, 11)$

Midpoint of $\overline{BC} = M_2 = \left(\frac{-12-8}{2}, \frac{-8+8}{2}\right) = (-10, 0)$

Midpoint of $\overline{CD} = M_3 = \left(\frac{-12+14}{2}, \frac{-8-6}{2}\right) = (1, -7)$

Midpoint of $\overline{DA} = M_4 = \left(\frac{14+4}{2}, \frac{-6+14}{2}\right) = (9, 4)$

b) One way to prove that a quadrilateral is a parallelogram is to demonstrate that one pair of opposite sides is parallel and equal in length. Use this fact to show that $M_1M_2M_3M_4$ is a parallelogram.

Show that $\overline{M_1M_2} \parallel \overline{M_3M_4}$ and $d(\overline{M_1M_2}) = d(\overline{M_3M_4})$

Now $m(\overline{M_1M_2}) = \dfrac{11-0}{-2+10} = \dfrac{11}{8}$

and $m(\overline{M_3M_4}) = \dfrac{4+7}{9-1} = \dfrac{11}{8}$

$\overline{M_1M_2} \parallel \overline{M_3M_4}$

Also, $d(\overline{M_1M_2}) = \sqrt{8^2+11^2} = \sqrt{185}$

and $d(\overline{M_3M_4}) = \sqrt{8^2+11^2} = \sqrt{185}$

$d(\overline{M_1M_2}) = d(\overline{M_3M_4})$

Thus $M_1M_2M_3M_4$ is a parallelogram.

[Note: $m(\overline{M_2M_3}) = \dfrac{-7}{11}$ and $d(\overline{M_2M_3}) = \sqrt{11^2+7^2} = \sqrt{170}$]