

Appendix B: Review of Graphs, Equations, and Inequalities

B.1 The Cartesian Plane

The Cartesian Plane

Just as you can represent real numbers by points on a real number line, you can represent ordered pairs of real numbers by points in a plane called the **rectangular coordinate system**, or the **Cartesian plane**, after the French mathematician René Descartes (1596–1650).

The Cartesian plane is formed by using two real number lines intersecting at right angles, as shown in Figure B.1. The horizontal real number line is usually called the **x-axis**, and the vertical real number line is usually called the **y-axis**. The point of intersection of these two axes is the **origin**, and the two axes divide the plane into four parts called **quadrants**.

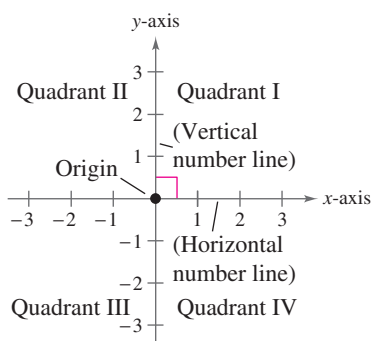


Figure B.1 The Cartesian Plane

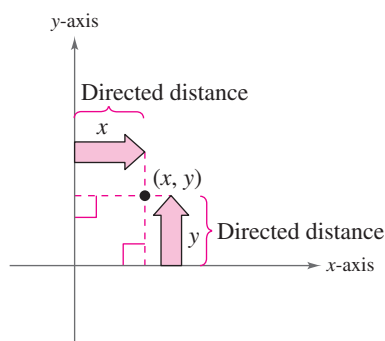
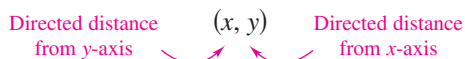


Figure B.2 Ordered Pair (x, y)

Each point in the plane corresponds to an **ordered pair** (x, y) of real numbers x and y , called **coordinates** of the point. The **x-coordinate** represents the directed distance from the y-axis to the point, and the **y-coordinate** represents the directed distance from the x-axis to the point, as shown in Figure B.2.



The notation (x, y) denotes both a point in the plane and an open interval on the real number line. The context will tell you which meaning is intended.

Example 1 Plotting Points in the Cartesian Plane

Plot the points $(-1, 2)$, $(3, 4)$, $(0, 0)$, $(3, 0)$, and $(-2, -3)$ in the Cartesian plane.

Solution

To plot the point $(-1, 2)$, imagine a vertical line through -1 on the x -axis and a horizontal line through 2 on the y -axis. The intersection of these two lines is the point $(-1, 2)$. This point is one unit to the left of the y -axis and two units up from the x -axis. The other four points can be plotted in a similar way, as shown in Figure B.3.

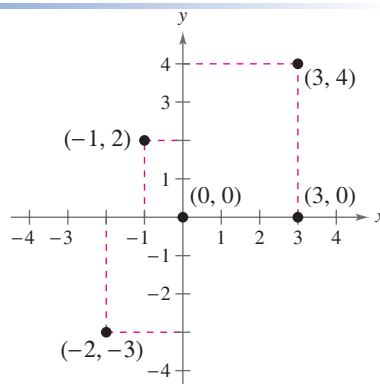


Figure B.3

What you should learn

- Plot points in the Cartesian plane and sketch scatter plots.
- Use the Distance Formula to find the distance between two points.
- Use the Midpoint Formula to find the midpoint of a line segment.
- Find the equation of a circle.
- Translate points in the plane.

Why you should learn it

The Cartesian plane can be used to represent relationships between two variables. For instance, Exercise 93 on page B10 shows how to represent graphically the numbers of performers elected to the Rock and Roll Hall of Fame from 1991 through 2010.


CHECKPOINT Now try Exercise 13.

The beauty of a rectangular coordinate system is that it enables you to see relationships between two variables. It would be difficult to overestimate the importance of Descartes’s introduction of coordinates in the plane. Today, his ideas are in common use in virtually every scientific and business-related field.

In the next example, data are represented graphically by points plotted in a rectangular coordinate system. This type of graph is called a **scatter plot**.

Example 2 Sketching a Scatter Plot 

The numbers of employees E (in thousands) in dentist offices in the United States from 1999 through 2009 are shown in the table, where t represents the year. Sketch a scatter plot of the data by hand. (Source: U.S. Bureau of Labor Statistics)

|  Year, t | Employees, E |
|---|----------------|
| 1999 | 667 |
| 2000 | 688 |
| 2001 | 705 |
| 2002 | 725 |
| 2003 | 744 |
| 2004 | 760 |
| 2005 | 774 |
| 2006 | 786 |
| 2007 | 808 |
| 2008 | 818 |
| 2009 | 818 |

Solution

Before you sketch the scatter plot, it is helpful to represent each pair of values by an ordered pair (t, E) as follows.

- $(1999, 667), (2000, 688), (2001, 705), (2002, 725), (2003, 744), (2004, 760),$
- $(2005, 774), (2006, 786), (2007, 808), (2008, 818), (2009, 818)$

To sketch a scatter plot of the data shown in the table, first draw a vertical axis to represent the number of employees (in thousands) and a horizontal axis to represent the year. Then plot the resulting points, as shown in Figure B.4. Note that the break in the t -axis indicates that the numbers 0 through 1998 have been omitted.

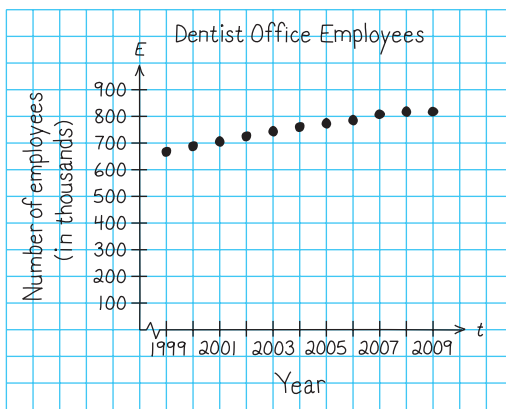


Figure B.4

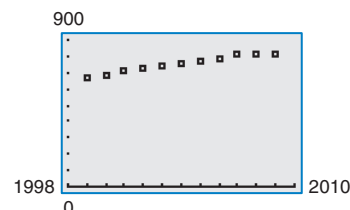
 **CHECKPOINT** Now try Exercise 31.

In Example 2, you could have let $t = 1$ represent the year 1999. In that case, the horizontal axis of the graph would not have been broken, and the tick marks would have been labeled 1 through 11 (instead of 1999 through 2009).

Technology Tip



You can use a graphing utility to graph the scatter plot in Example 2. For instructions on how to use the *list editor* and the *statistical plotting* feature, see Appendix A; for specific keystrokes, go to this textbook’s *Companion Website*.



The Distance Formula

Recall from the Pythagorean Theorem that, for a right triangle with hypotenuse of length c and sides of lengths a and b , you have

$$a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem}$$

as shown in Figure B.5. (The converse is also true. That is, if $a^2 + b^2 = c^2$, then the triangle is a right triangle.)

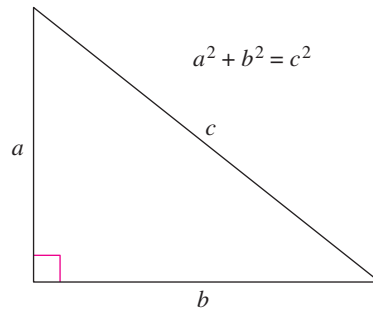


Figure B.5

Suppose you want to determine the distance d between two points (x_1, y_1) and (x_2, y_2) in the plane. With these two points, a right triangle can be formed, as shown in Figure B.6.

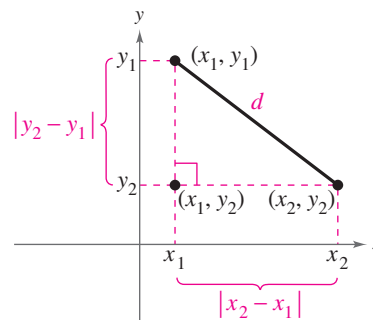


Figure B.6

The length of the vertical side of the triangle is

$$|y_2 - y_1| \quad \text{Length of vertical side}$$

and the length of the horizontal side is

$$|x_2 - x_1|. \quad \text{Length of horizontal side}$$

By the Pythagorean Theorem,

$$\begin{aligned} d^2 &= |x_2 - x_1|^2 + |y_2 - y_1|^2 \\ d &= \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} \\ d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \end{aligned}$$

This result is called the **Distance Formula**.

The Distance Formula

The distance d between the points (x_1, y_1) and (x_2, y_2) in the plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Example 3 Finding a Distance

Find the distance between the points

$$(-2, 1) \text{ and } (3, 4).$$

Algebraic Solution

Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (3, 4)$. Then apply the Distance Formula as follows.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{[3 - (-2)]^2 + (4 - 1)^2} && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2. \\ &= \sqrt{(5)^2 + (3)^2} && \text{Simplify.} \\ &= \sqrt{34} && \text{Simplify.} \\ &\approx 5.83 && \text{Use a calculator.} \end{aligned}$$

So, the distance between the points is about 5.83 units.

You can use the Pythagorean Theorem to check that the distance is correct.

$$\begin{aligned} d^2 &\stackrel{?}{=} 3^2 + 5^2 && \text{Pythagorean Theorem} \\ (\sqrt{34})^2 &\stackrel{?}{=} 3^2 + 5^2 && \text{Substitute for } d. \\ 34 &= 34 && \text{Distance checks. } \checkmark \end{aligned}$$

CHECKPOINT Now try Exercise 33.

When the Distance Formula is used, it does not matter which point is (x_1, y_1) and which is (x_2, y_2) , because the result will be the same. For instance, in Example 3, let $(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (-2, 1)$. Then

$$d = \sqrt{(-2 - 3)^2 + (1 - 4)^2} = \sqrt{(-5)^2 + (-3)^2} = \sqrt{34} \approx 5.83.$$

Example 4 Verifying a Right Triangle

Show that the points

$$(2, 1), (4, 0), \text{ and } (5, 7)$$

are the vertices of a right triangle.

Solution

The three points are plotted in Figure B.8. Using the Distance Formula, you can find the lengths of the three sides as follows.

$$\begin{aligned} d_1 &= \sqrt{(5 - 2)^2 + (7 - 1)^2} = \sqrt{9 + 36} = \sqrt{45} \\ d_2 &= \sqrt{(4 - 2)^2 + (0 - 1)^2} = \sqrt{4 + 1} = \sqrt{5} \\ d_3 &= \sqrt{(5 - 4)^2 + (7 - 0)^2} = \sqrt{1 + 49} = \sqrt{50} \end{aligned}$$

Because

$$(d_1)^2 + (d_2)^2 = 45 + 5 = 50 = (d_3)^2$$

you can conclude that the triangle must be a right triangle.

CHECKPOINT Now try Exercise 47.

Graphical Solution

Use centimeter graph paper to plot the points $A(-2, 1)$ and $B(3, 4)$. Carefully sketch the line segment from A to B . Then use a centimeter ruler to measure the length of the segment.

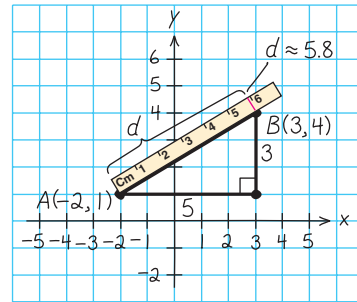


Figure B.7

The line segment measures about 5.8 centimeters, as shown in Figure B.7. So, the distance between the points is about 5.8 units.

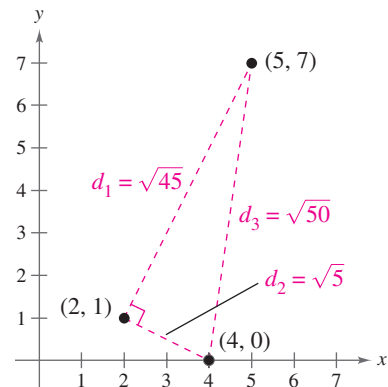


Figure B.8

The Midpoint Formula

To find the **midpoint** of the line segment that joins two points in a coordinate plane, find the average values of the respective coordinates of the two endpoints using the **Midpoint Formula**.

The Midpoint Formula

The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by the Midpoint Formula

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

Example 5 Finding a Line Segment's Midpoint

Find the midpoint of the line segment joining the points $(-5, -3)$ and $(9, 3)$.

Solution

Let $(x_1, y_1) = (-5, -3)$ and $(x_2, y_2) = (9, 3)$.

$$\begin{aligned} \text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) && \text{Midpoint Formula} \\ &= \left(\frac{-5 + 9}{2}, \frac{-3 + 3}{2} \right) && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2. \\ &= (2, 0) && \text{Simplify.} \end{aligned}$$

The midpoint of the line segment is $(2, 0)$, as shown in Figure B.9.

 **CHECKPOINT** Now try Exercise 59.

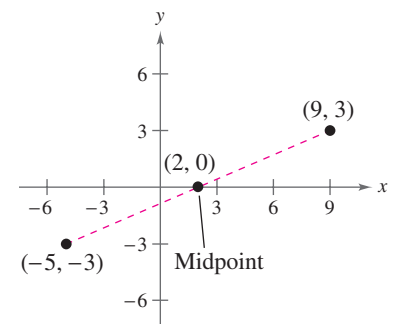


Figure B.9

Example 6 Estimating Annual Revenues



Verizon Communications had annual revenues of \$88.1 billion in 2006 and \$97.4 billion in 2008. Without knowing any additional information, what would you estimate the 2007 revenue to have been? (Source: Verizon Communications)

Solution

One solution to the problem is to assume that revenue followed a linear pattern. With this assumption, you can estimate the 2007 revenue by finding the midpoint of the line segment connecting the points $(2006, 88.1)$ and $(2008, 97.4)$.

$$\begin{aligned} \text{Midpoint} &= \left(\frac{2006 + 2008}{2}, \frac{88.1 + 97.4}{2} \right) \\ &\approx (2007, 92.8) \end{aligned}$$

So, you would estimate the 2007 revenue to have been about \$92.8 billion, as shown in Figure B.10. (The actual 2007 revenue was \$93.5 billion.)

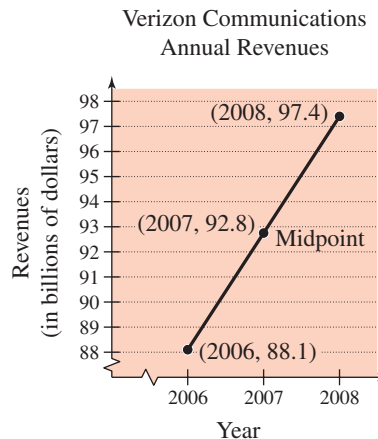


Figure B.10

 **CHECKPOINT** Now try Exercise 65.

The Equation of a Circle

The Distance Formula provides a convenient way to define circles. A **circle of radius r** with center at the point (h, k) is shown in Figure B.11. The point (x, y) is on this circle if and only if its distance from the center (h, k) is r . This means that a **circle** in the plane consists of all points (x, y) that are a given positive distance r from a fixed point (h, k) . Using the Distance Formula, you can express this relationship by saying that the point (x, y) lies on the circle if and only if

$$\sqrt{(x - h)^2 + (y - k)^2} = r.$$

By squaring each side of this equation, you obtain the **standard form of the equation of a circle**.

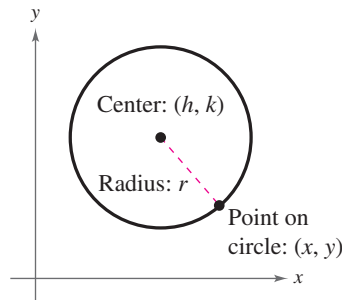


Figure B.11

Standard Form of the Equation of a Circle

The **standard form of the equation of a circle** is

$$(x - h)^2 + (y - k)^2 = r^2.$$

The point (h, k) is the **center** of the circle, and the positive number r is the **radius** of the circle. The standard form of the equation of a circle whose center is the origin, $(h, k) = (0, 0)$, is $x^2 + y^2 = r^2$.

Example 7 Writing an Equation of a Circle

The point $(3, 4)$ lies on a circle whose center is at $(-1, 2)$, as shown in Figure B.12. Write the standard form of the equation of this circle.

Solution

The radius r of the circle is the distance between $(-1, 2)$ and $(3, 4)$.

$$\begin{aligned} r &= \sqrt{[3 - (-1)]^2 + (4 - 2)^2} && \text{Substitute for } x, y, h, \text{ and } k. \\ &= \sqrt{4^2 + 2^2} && \text{Simplify.} \\ &= \sqrt{16 + 4} && \text{Simplify.} \\ &= \sqrt{20} && \text{Radius} \end{aligned}$$

Using $(h, k) = (-1, 2)$ and $r = \sqrt{20}$, the equation of the circle is

$$\begin{aligned} (x - h)^2 + (y - k)^2 &= r^2 && \text{Equation of circle} \\ [x - (-1)]^2 + (y - 2)^2 &= (\sqrt{20})^2 && \text{Substitute for } h, k, \text{ and } r. \\ (x + 1)^2 + (y - 2)^2 &= 20. && \text{Standard form} \end{aligned}$$

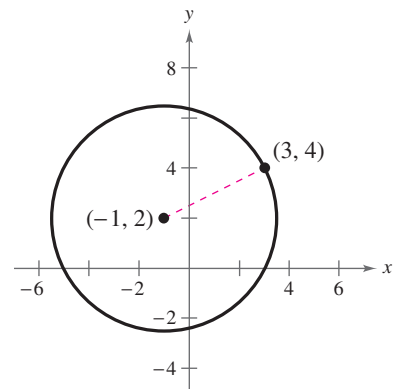


Figure B.12

CHECKPOINT Now try Exercise 69.

Application

Much of computer graphics consists of transformations of points in a coordinate plane. One type of transformation, a translation, is illustrated in Example 8. Other types of transformations include reflections, rotations, and stretches.

Example 8 Translating Points in the Plane

The triangle in Figure B.13 has vertices at the points

$$(-1, 2), (1, -4), \text{ and } (2, 3).$$

Shift the triangle three units to the right and two units upward and find the vertices of the shifted triangle, as shown in Figure B.14.

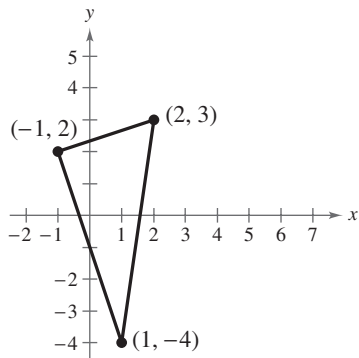


Figure B.13

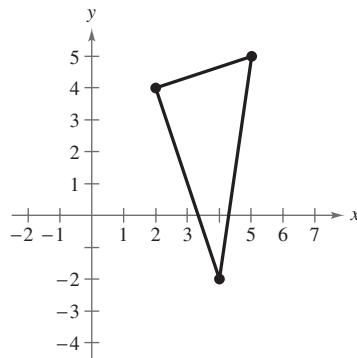


Figure B.14

Solution

To shift the vertices three units to the right, add 3 to each of the x -coordinates. To shift the vertices two units upward, add 2 to each of the y -coordinates.

| <i>Original Point</i> | <i>Translated Point</i> |
|-----------------------|-----------------------------|
| $(-1, 2)$ | $(-1 + 3, 2 + 2) = (2, 4)$ |
| $(1, -4)$ | $(1 + 3, -4 + 2) = (4, -2)$ |
| $(2, 3)$ | $(2 + 3, 3 + 2) = (5, 5)$ |

Plotting the translated points and sketching the line segments between them produces the shifted triangle shown in Figure B.14.

CHECKPOINT Now try Exercise 87.

Example 8 shows how to translate points in a coordinate plane. The following transformed points are related to the original points as follows.

| <i>Original Point</i> | <i>Transformed Point</i> | |
|-----------------------|--------------------------|--|
| (x, y) | $(-x, y)$ | $(-x, y)$ is a reflection of the original point in the y -axis. |
| (x, y) | $(x, -y)$ | $(x, -y)$ is a reflection of the original point in the x -axis. |
| (x, y) | $(-x, -y)$ | $(-x, -y)$ is a reflection of the original point through the origin. |

The figures provided with Example 8 were not really essential to the solution. Nevertheless, it is strongly recommended that you develop the habit of including sketches with your solutions because they serve as useful problem-solving tools.

B.1 Exercises

For instructions on how to use a graphing utility, see Appendix A.

Vocabulary and Concept Check

In Exercises 1–4, fill in the blank(s).

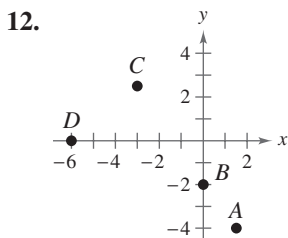
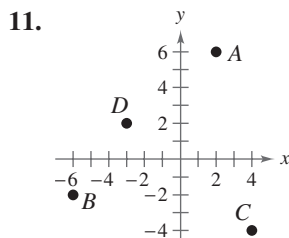
- An ordered pair of real numbers can be represented in a plane called the rectangular coordinate system or the _____ plane.
- The _____ is a result derived from the Pythagorean Theorem.
- Finding the average values of the respective coordinates of the two endpoints of a line segment in a coordinate plane is also known as using the _____.
- The standard form of the equation of a circle is _____, where the point (h, k) is the _____ of the circle and the positive number r is the _____ of the circle.

In Exercises 5–10, match each term with its definition.

- | | |
|---------------------|--|
| 5. x -axis | (a) point of intersection of vertical axis and horizontal axis |
| 6. y -axis | (b) directed distance from the x -axis |
| 7. origin | (c) horizontal real number line |
| 8. quadrants | (d) four regions of the coordinate plane |
| 9. x -coordinate | (e) directed distance from the y -axis |
| 10. y -coordinate | (f) vertical real number line |

Procedures and Problem Solving

Approximating Coordinates of Points In Exercises 11 and 12, approximate the coordinates of the points.



Plotting Points in the Cartesian Plane In Exercises 13–16, plot the points in the Cartesian plane.

- $(-4, 2), (-3, -6), (0, 5), (1, -4)$
- $(4, -2), (0, 0), (-4, 0), (-5, -5)$
- $(3, 8), (0.5, -1), (5, -6), (-2, -2.5)$
- $(1, -\frac{1}{2}), (-\frac{5}{2}, 2), (3, -3), (\frac{3}{2}, 1)$

Finding Coordinates of Points In Exercises 17–20, find the coordinates of the point.

- The point is located five units to the left of the y -axis and four units above the x -axis.
- The point is located three units below the x -axis and two units to the right of the y -axis.
- The point is on the y -axis and six units below the x -axis.
- The point is on the x -axis and 11 units to the left of the y -axis.

Determining Quadrants In Exercises 21–30, determine the quadrant(s) in which (x, y) is located so that the condition(s) is (are) satisfied.

- | | |
|--------------------------|--------------------------|
| 21. $x > 0$ and $y < 0$ | 22. $x < 0$ and $y < 0$ |
| 23. $x = -4$ and $y > 0$ | 24. $x > 2$ and $y = 3$ |
| 25. $y < -5$ | 26. $x > 4$ |
| 27. $x < 0$ and $-y > 0$ | 28. $-x > 0$ and $y < 0$ |
| 29. $xy > 0$ | 30. $xy < 0$ |


Sketching a Scatter Plot In Exercises 31 and 32, sketch a scatter plot of the data shown in the table.

- ✓ 31. **Accounting** The table shows the sales y (in millions of dollars) for Apple for the years 2000 through 2009. (Source: Apple Inc.)



| Year | Sales, y (in millions of dollars) |
|------|--|
| 2000 | 7,983 |
| 2001 | 5,363 |
| 2002 | 5,742 |
| 2003 | 6,207 |
| 2004 | 8,279 |
| 2005 | 13,931 |
| 2006 | 19,315 |
| 2007 | 24,006 |
| 2008 | 32,479 |
| 2009 | 36,537 |

32. **Meteorology** The table shows the lowest temperature on record y (in degrees Fahrenheit) in Flagstaff, Arizona for each month x , where $x = 1$ represents January. (Source: U.S. National Oceanic and Atmospheric Administration)

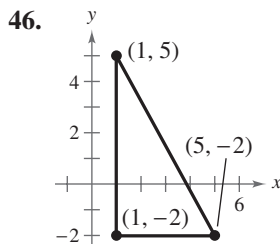
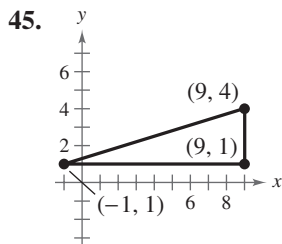
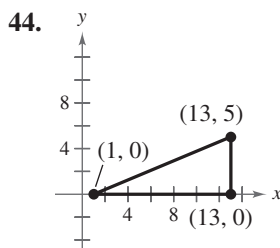
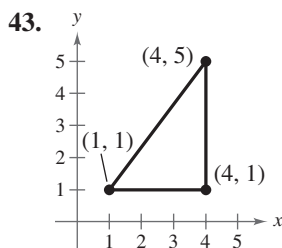


| Month, x | Temperature, y |
|------------|------------------|
| 1 | -22 |
| 2 | -23 |
| 3 | -16 |
| 4 | -2 |
| 5 | 14 |
| 6 | 22 |
| 7 | 32 |
| 8 | 24 |
| 9 | 23 |
| 10 | -2 |
| 11 | -13 |
| 12 | -23 |

Finding a Distance In Exercises 33–42, find the distance between the points algebraically and confirm graphically by using centimeter graph paper and a centimeter ruler.

- ✓ 33. $(6, -3), (6, 5)$ 34. $(-3, 0), (-3, -7)$
 35. $(-3, -1), (2, -1)$ 36. $(-11, 4), (-1, 4)$
 37. $(-2, 6), (3, -6)$
 38. $(8, 5), (0, 20)$
 39. $(\frac{1}{2}, \frac{4}{3}), (2, -1)$
 40. $(-\frac{2}{3}, 3), (-1, \frac{5}{4})$
 41. $(-4.2, 3.1), (-12.5, 4.8)$
 42. $(9.5, -2.6), (-3.9, 8.2)$

Verifying a Right Triangle In Exercises 43–46, (a) find the length of each side of the right triangle and (b) show that these lengths satisfy the Pythagorean Theorem.



Verifying a Polygon In Exercises 47–54, show that the points form the vertices of the polygon.


- ✓ 47. Right triangle: $(4, 0), (2, 1), (-1, -5)$
 48. Right triangle: $(-1, 3), (3, 5), (5, 1)$
 49. Isosceles triangle: $(1, -3), (3, 2), (-2, 4)$
 50. Isosceles triangle: $(2, 3), (4, 9), (-2, 7)$
 51. Parallelogram: $(2, 5), (0, 9), (-2, 0), (0, -4)$
 52. Parallelogram: $(0, 1), (3, 7), (4, 4), (1, -2)$
 53. Rectangle: $(-5, 6), (0, 8), (-3, 1), (2, 3)$ (Hint: Show that the diagonals are of equal length.)
 54. Rectangle: $(2, 4), (3, 1), (1, 2), (4, 3)$ (Hint: Show that the diagonals are of equal length.)

Finding a Line Segment's Midpoint In Exercises 55–64, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points.

55. $(0, 0), (8, 6)$
 56. $(1, 12), (6, 0)$
 57. $(-4, 10), (4, -5)$
 58. $(-7, -4), (2, 8)$
 ✓ 59. $(-1, 2), (5, 4)$
 60. $(2, 10), (10, 2)$
 61. $(\frac{1}{2}, 1), (-\frac{5}{2}, \frac{4}{3})$
 62. $(-\frac{1}{3}, -\frac{1}{3}), (-\frac{1}{6}, -\frac{1}{2})$
 63. $(6.2, 5.4), (-3.7, 1.8)$
 64. $(-16.8, 12.3), (5.6, 4.9)$


Estimating Annual Revenues In Exercises 65 and 66, use the Midpoint Formula to estimate the annual revenues (in millions of dollars) for Texas Roadhouse and Papa John's in 2006. The revenues for the two companies in 2003 and 2009 are shown in the tables. Assume that the revenues followed a linear pattern. (Sources: Texas Roadhouse, Inc.; Papa John's International)

- ✓ 65. Texas Roadhouse



| Year | Annual revenue (in millions of dollars) |
|------|---|
| 2003 | 287 |
| 2009 | 942 |

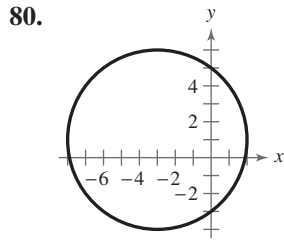
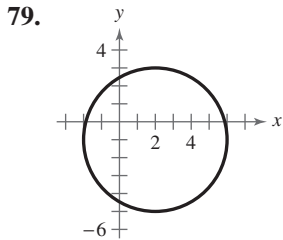
66. Papa John's Intl.



| Year | Annual revenue (in millions of dollars) |
|------|---|
| 2003 | 917 |
| 2009 | 1106 |

Writing an Equation of a Circle In Exercises 67–80, write the standard form of the equation of the specified circle.

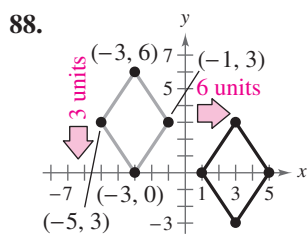
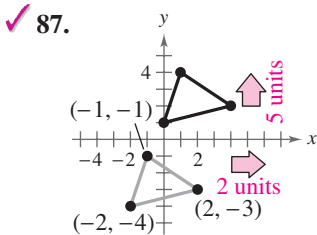
- 67. Center: (0, 0); radius: 5
- 68. Center: (0, 0); radius: 6
- ✓ 69. Center: (2, -1); radius: 4
- 70. Center: (-5, 3); radius: 2
- 71. Center: (-1, 2); solution point: (0, 0)
- 72. Center: (3, -2); solution point: (-1, 1)
- 73. Endpoints of a diameter: (0, 0), (6, 8)
- 74. Endpoints of a diameter: (-4, -1), (4, 1)
- 75. Center: (-2, 1); tangent to the x -axis
- 76. Center: (3, -2); tangent to the y -axis
- 77. The circle inscribed in the square with vertices (7, -2), (-1, -2), (-1, -10), and (7, -10)
- 78. The circle inscribed in the square with vertices (-12, 10), (8, 10), (8, -10), and (-12, -10)



Sketching a Circle In Exercises 81–86, find the center and radius, and sketch the circle.

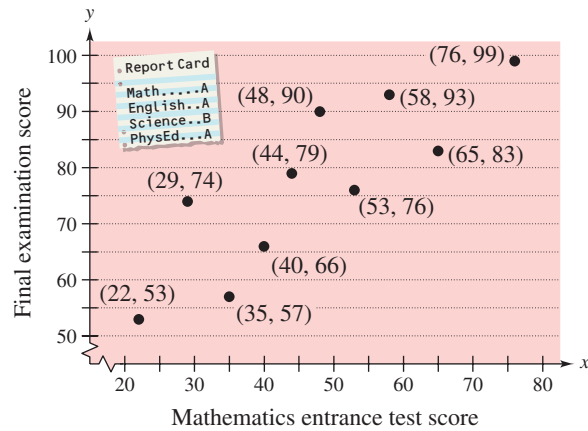
- 81. $x^2 + y^2 = 9$
- 82. $x^2 + y^2 = 16$
- 83. $(x - 1)^2 + (y + 3)^2 = 4$
- 84. $x^2 + (y - 1)^2 = 49$
- 85. $(x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 = \frac{9}{4}$
- 86. $(x - \frac{2}{3})^2 + (y + \frac{1}{4})^2 = \frac{25}{36}$

Translating Points in the Plane In Exercises 87–90, the polygon is shifted to a new position in the plane. Find the coordinates of the vertices of the polygon in the new position.

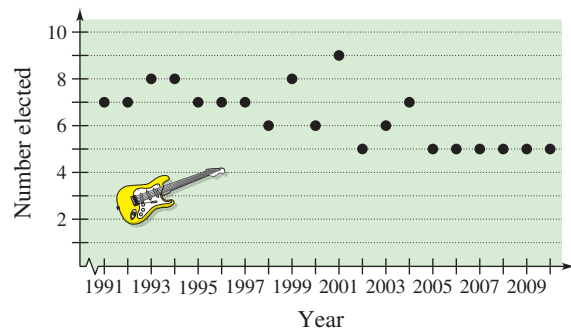


- 89. Original coordinates of vertices: (0, 2), (-3, 5), (-5, 2), (-2, -1)
Shift: three units upward, one unit to the left
- 90. Original coordinates of vertices: (1, -1), (3, 2), (1, -2)
Shift: two units downward, three units to the left

Education In Exercises 91 and 92, refer to the scatter plot, which shows the mathematics entrance test scores x and the final examination scores y in an algebra course for a sample of 10 students.

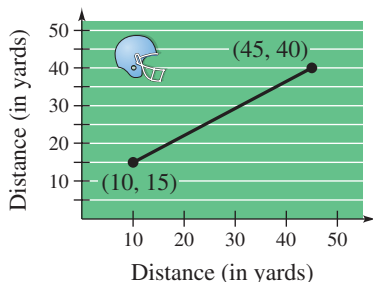


- 91. Find the entrance exam score of any student with a final exam score in the 80s.
- 92. Does a higher entrance exam score necessarily imply a higher final exam score? Explain.
- 93. **Why you should learn it** (p. B1) The graph shows the numbers of performers who were elected to the Rock and Roll Hall of Fame from 1991 through 2010. Describe any trends in the data. From these trends, predict the number of performers elected in 2011. (Source: rockhall.com)

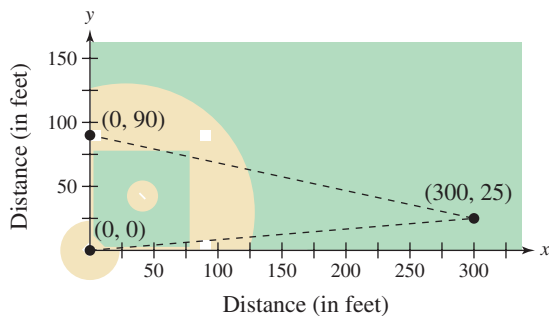


- 94. **Aviation** A jet plane flies from Naples, Italy in a straight line to Rome, Italy, which is 120 kilometers north and 150 kilometers west of Naples. How far does the plane fly?

95. **Physical Education** In a football game, a quarterback throws a pass from the 15-yard line, 10 yards from the sideline, as shown in the figure. The pass is caught on the 40-yard line, 45 yards from the same sideline. How long is the pass?



96. **Physical Education** A major league baseball diamond is a square with 90-foot sides. In the figure, home plate is at the origin and the first base line lies on the positive x -axis. The right fielder fields the ball at the point $(300, 25)$. How far does the right fielder have to throw the ball to get a runner out at home plate? How far does the right fielder have to throw the ball to get a runner out at third base? (Round your answers to one decimal place.)



Conclusions

True or False? In Exercises 97–99, determine whether the statement is true or false. Justify your answer.

97. The points $(-8, 4)$, $(2, 11)$, and $(-5, 1)$ represent the vertices of an isosceles triangle.
98. If four points represent the vertices of a polygon, and the four sides are equal, then the polygon must be a square.
99. In order to divide a line segment into 16 equal parts, you would have to use the Midpoint Formula 16 times.
100. **Think About It** What is the y -coordinate of any point on the x -axis? What is the x -coordinate of any point on the y -axis?
101. **Think About It** When plotting points on the rectangular coordinate system, is it true that the scales on the x - and y -axes must be the same? Explain.

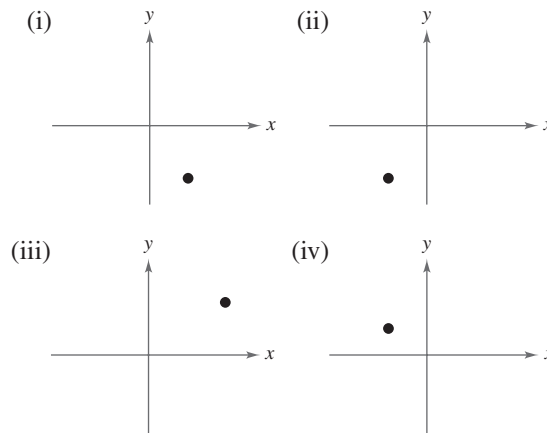
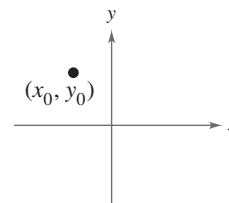
102. **Exploration** A line segment has (x_1, y_1) as one endpoint and (x_m, y_m) as its midpoint. Find the other endpoint (x_2, y_2) of the line segment in terms of $x_1, y_1, x_m,$ and y_m . Use the result to find the coordinates of the endpoint of a line segment when the coordinates of the other endpoint and midpoint are, respectively,

(a) $(1, -2), (4, -1)$. (b) $(-5, 11), (2, 4)$.

103. **Exploration** Use the Midpoint Formula three times to find the three points that divide the line segment joining (x_1, y_1) and (x_2, y_2) into four parts. Use the result to find the points that divide the line segment joining the given points into four equal parts.

(a) $(1, -2), (4, -1)$ (b) $(-2, -3), (0, 0)$

104. **Think About It** Use the plot of the point (x_0, y_0) in the figure. Match the transformation of the point with the correct plot. Explain your reasoning. [The plots are labeled (i), (ii), (iii), and (iv).]



(a) $(x_0, -y_0)$ (b) $(-2x_0, y_0)$
 (c) $(x_0, \frac{1}{2}y_0)$ (d) $(-x_0, -y_0)$

105. **Proof** Prove that the diagonals of the parallelogram in the figure intersect at their midpoints.

