Math 1B **The Dot Product (§12.3 Day 2)**

To find the vector projection of **u** onto **v**, multiply a unit vector in the direction of **v** by $\text{comp}_{\mathbf{v}}\mathbf{u}$.

Rule: To find $\text{proj}_{\mathbf{v}}\mathbf{u}$ (the vector projection of **u** on **v**), note that

$$\overrightarrow{\text{proj}_{\mathbf{v}}\mathbf{u}} \Rightarrow \text{comp}_{\mathbf{v}}\mathbf{u}\left(\frac{\mathbf{v}}{\|\mathbf{v}\|}\right) = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|}\left(\frac{\vec{v}}{\|\vec{v}\|}\right) = \left(\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\right)\vec{v} = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v}$$

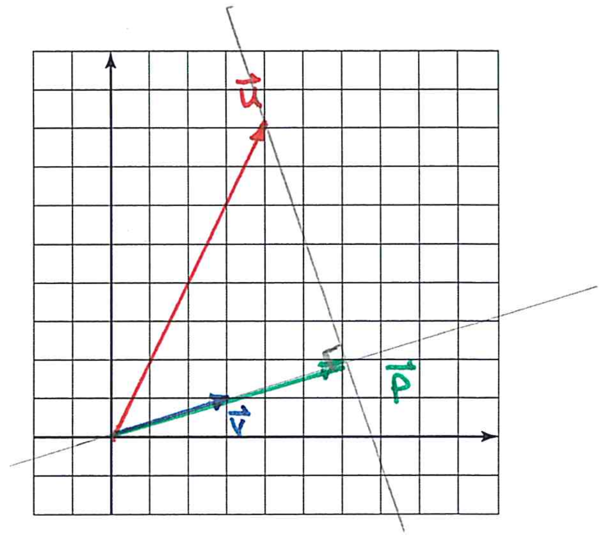
Example #1

Given that $\mathbf{u} = \langle 4,8\rangle$ and $\mathbf{v} = \langle 3,1\rangle$, find $\text{proj}_{\mathbf{v}}\mathbf{u}$ and illustrate.

$$\text{proj}_{\vec{v}}\vec{u} = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v}$$

$$= \left(\frac{12+8}{9+1}\right)\vec{v}$$

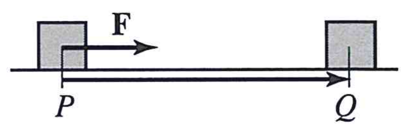
$$= 2\vec{v} = \langle 6,2\rangle = \vec{P}$$



Application (III): Work

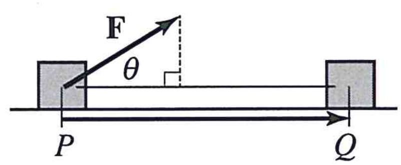
The work W done by a constant force **F** acting along the line of motion of an object is defined to be

$$W = (\text{magnitude of force}) \times (\text{distance}) = \|\mathbf{F}\| \, \|\overrightarrow{PQ}\|$$



If the constant force is not directed along the line of motion, then the work done by the force is given by:

$$W = \left(\|\mathbf{F}\|\cos\theta\right)\|\overrightarrow{PQ}\| = \|\mathbf{F}\| \, \|\overrightarrow{PQ}\|\cos\theta = \vec{F}\cdot\overrightarrow{PQ}$$



Example #2 Find the work done in each of the following situations.

a) A wagon is pulled a distance of 100 m along a horizontal path by a constant force of 70 N. The handle of the wagon is held at an angle of 35° above the horizontal.

$$W = \vec{F}\cdot\overrightarrow{PQ} = \|\vec{F}\|\,\|\overrightarrow{PQ}\|\cos\theta = 70\cdot 100\cos 35^\circ$$

$$\approx 5734 \text{ N-m} = 5734 \text{ J} \quad (\text{J} = \text{joule})$$

b) The force vector $\mathbf{F} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ moves a particle from point the $P(2,1,0)$ to the point $Q(4,6,2)$.

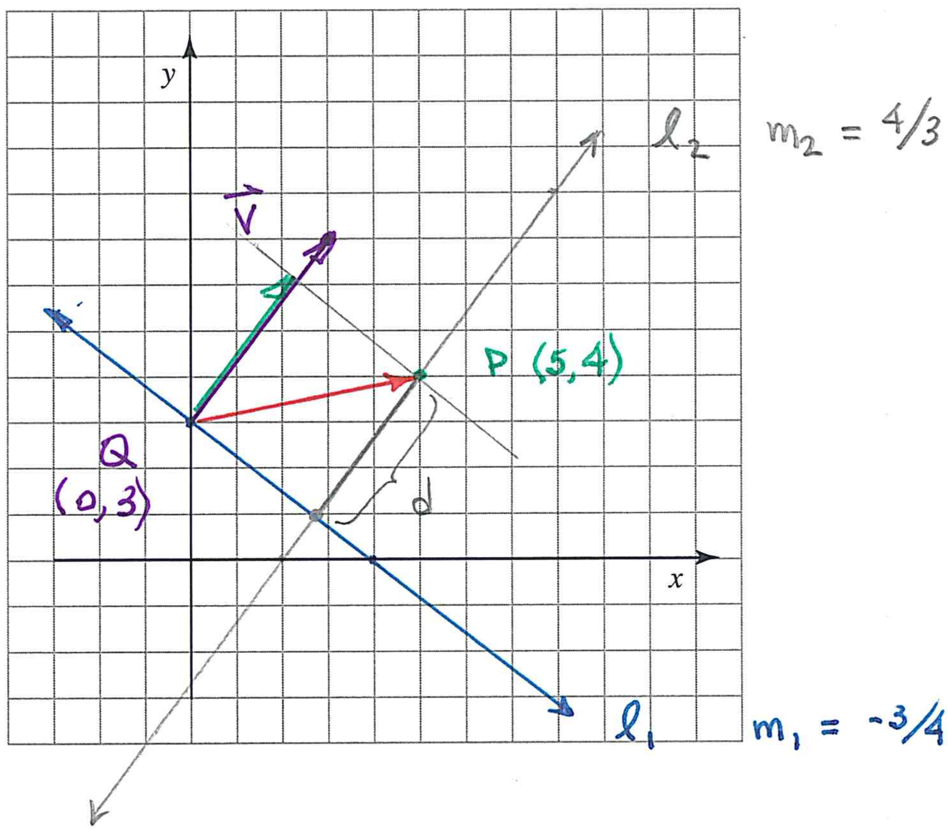
$$\overrightarrow{PQ} = \langle 4-2, 6-1, 2-0\rangle = \langle 2,5,2\rangle = 2\vec{i} + 5\vec{j} + 2\vec{k}$$

$$\text{So } W = \vec{F}\cdot\overrightarrow{PQ} = 6 + 20 + 10 = \boxed{36}$$

Application (IV): Distance From Point to Line

Example #3 Find the distance between the point $(5, 4)$ and the line $3x + 4y - 12 = 0$.

Now, $3x + 4y - 12 = 0 \Rightarrow y = -3/4\, x + 3$



To find d:

1) Find a vector $\parallel$ to ℓ_2 : $\vec{v} = \langle 3, 4 \rangle$
2) Find $\overrightarrow{QP} = \langle 5-0,\ 4-3 \rangle = \langle 5, 1 \rangle$
3) Find $d = \left| \text{comp}_{\vec{v}}\, \overrightarrow{QP} \right| = \left| \dfrac{\vec{v} \cdot \overrightarrow{QP}}{\| \vec{v} \|} \right| = \left| \dfrac{15 + 4}{\sqrt{9 + 16}} \right| = \boxed{\dfrac{19}{5}}$

In General, the distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is given by

$$\boxed{d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}}$$