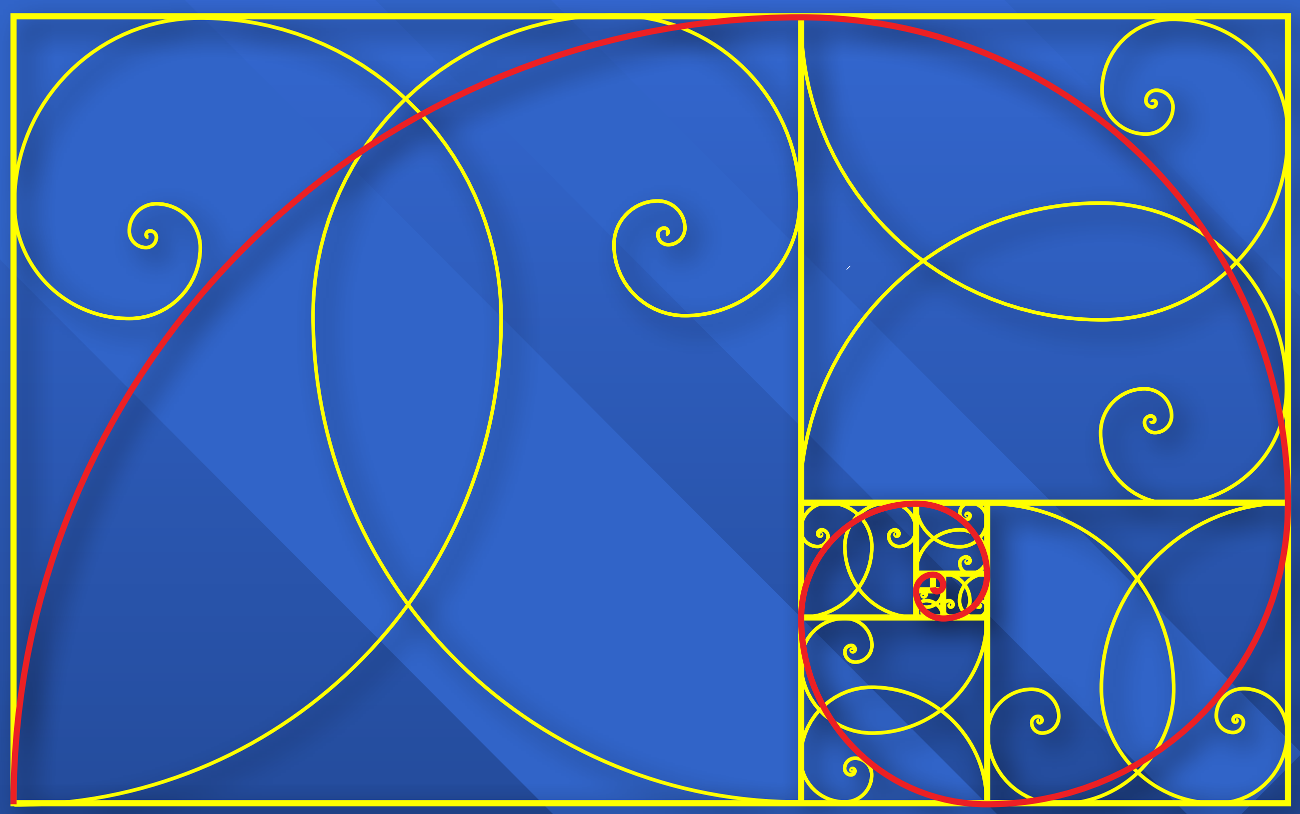
Digging for Gold

Discovering the Golden Ratio

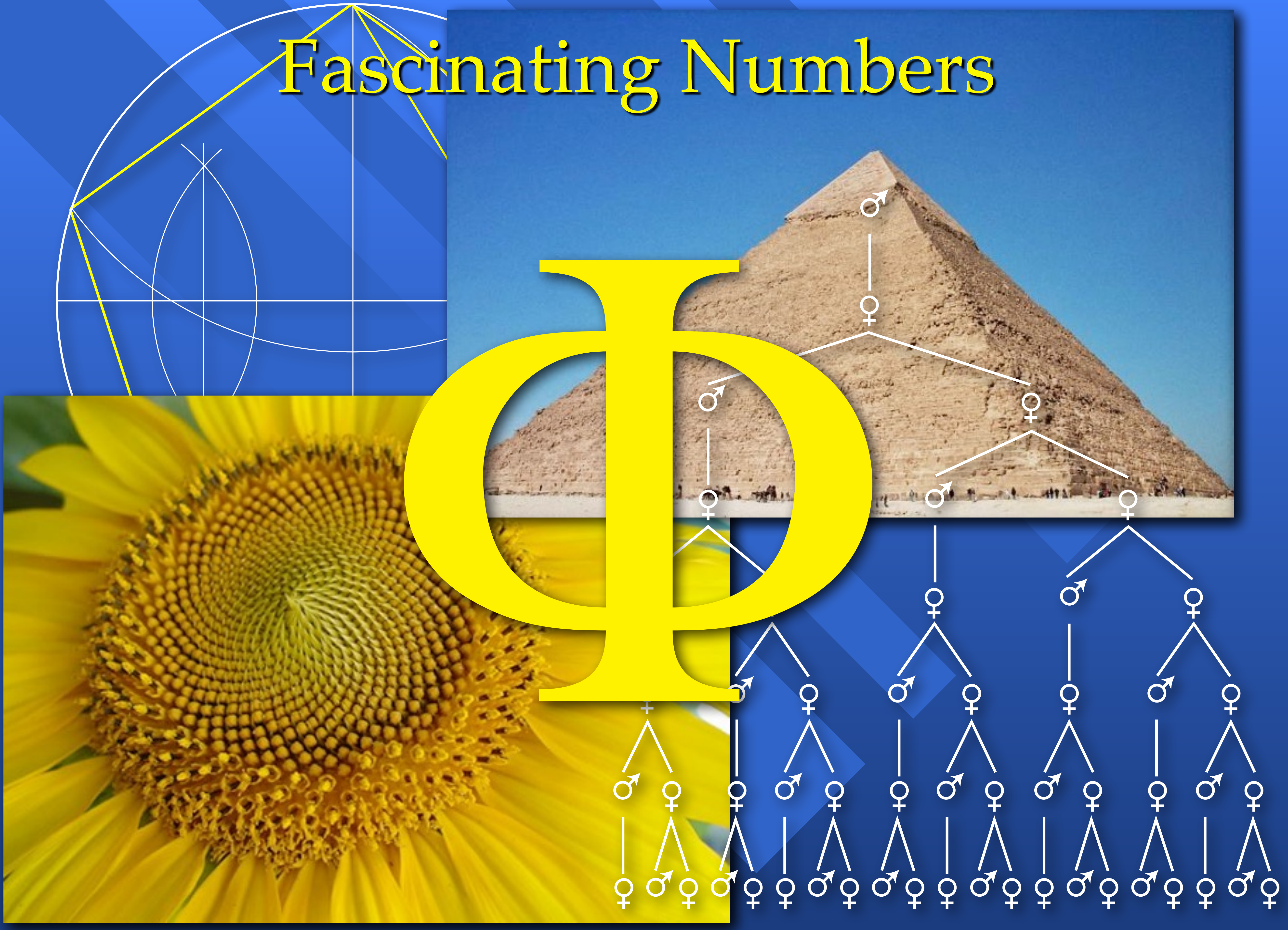


John Martin
Santa Rosa Junior College

AMATYC Conference
November 10, 2017

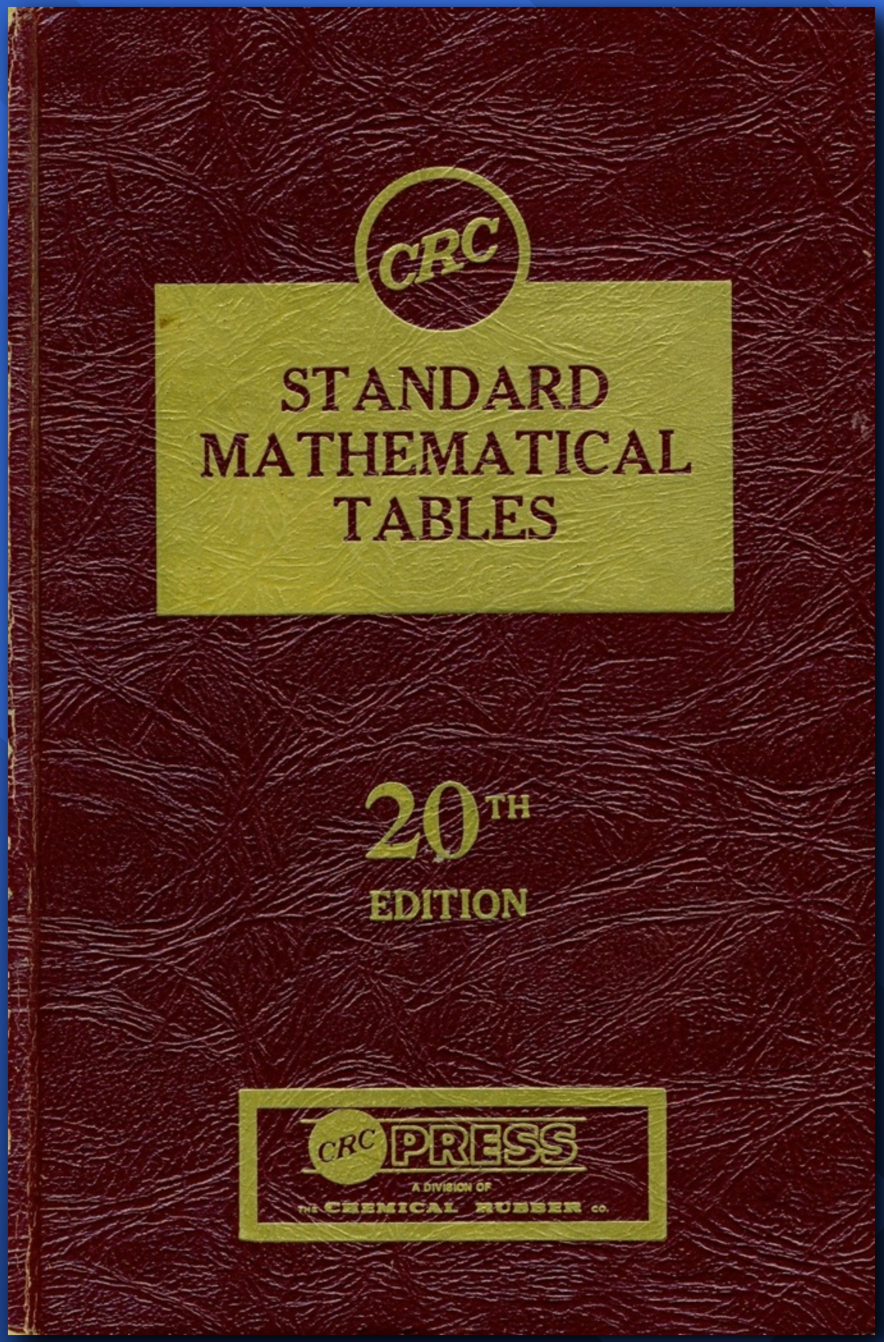
Fascinating Numbers

Fascinating Numbers



Fascinating Numbers

Fascinating Numbers



Fascinating Numbers

Numerical Constants

MISCELLANEOUS CONSTANTS

π CONSTANTS

$\pi = 3.14159\ 26535\ 89793\ 23846\ 26433\ 83279\ 50288\ 41971\ 69399\ 37510$

$1/\pi = 0.31830\ 98861\ 83790\ 67153\ 77675\ 26745\ 02872\ 40689\ 19291\ 48091$

$\pi^2 = 9.86960\ 44010\ 89358\ 61883\ 44909\ 99876\ 15113\ 53136\ 99407\ 24079$

$\log_e \pi = 1.14472\ 98858\ 49400\ 17414\ 34273\ 51353\ 05871\ 16472\ 94812\ 91531$

$\log_{10} \pi = 0.49714\ 98726\ 94133\ 85435\ 12682\ 88290\ 89887\ 36516\ 78324\ 38044$

$\log_{10} \sqrt{2\pi} = 0.39908\ 99341\ 79057\ 52478\ 25035\ 91507\ 69595\ 02099\ 34102\ 92127$

CONSTANTS INVOLVING e

$e = 2.71828\ 18284\ 59045\ 23536\ 02874\ 71352\ 66249\ 77572\ 47093\ 69995$

$1/e = 0.36787\ 94411\ 71442\ 32159\ 55237\ 70161\ 46086\ 74458\ 11131\ 03176$

$e^2 = 7.38905\ 60989\ 30650\ 22723\ 04274\ 60575\ 00781\ 31803\ 15570\ 55184$

$M = \log_{10} e = 0.43429\ 44819\ 03251\ 82765\ 11289\ 18916\ 60508\ 22943\ 97005\ 80366$

$1/M = \log_e 10 = 2.30258\ 50929\ 94045\ 68401\ 79914\ 54684\ 36420\ 76011\ 01488\ 62877$

$\log_{10} M = 9.63778\ 43113\ 00536\ 78912\ 29674\ 98565 - 10$

π^e AND e^π CONSTANTS

$\pi^e = 22.45915\ 77183\ 61045\ 47342\ 71522$

$e^\pi = 23.14069\ 26327\ 79269\ 00572\ 90864$

$e^{-\pi} = 0.04321\ 39182\ 63772\ 24977\ 44177$

$e^{\frac{1}{2}\pi} = 4.81047\ 73809\ 65351\ 65547\ 30357$

$i^i = e^{-\frac{1}{2}\pi} = 0.20787\ 95763\ 50761\ 90854\ 69556$

NUMERICAL CONSTANTS

$\sqrt{2} = 1.41421\ 35623\ 73095\ 04880\ 16887\ 24209\ 69807\ 85696\ 71875\ 37694$

$\sqrt[3]{2} = 1.25992\ 10498\ 94873\ 16476\ 72106\ 07278\ 22835\ 05702\ 51464\ 70150$

$\log_e 2 = 0.69314\ 71805\ 59945\ 30941\ 72321\ 21458\ 17656\ 80755\ 00134\ 36025$

$\log_{10} 2 = 0.30102\ 99956\ 63981\ 19521\ 37388\ 94724\ 49302\ 67681\ 89881\ 46210$

$\sqrt{3} = 1.73205\ 08075\ 68877\ 29352\ 74463\ 41505\ 87236\ 69428\ 05253\ 81038$

$\sqrt[3]{3} = 1.44224\ 95703\ 07408\ 38232\ 16383\ 10780\ 10958\ 83918\ 69253\ 49935$

$\log_e 3 = 1.09861\ 22886\ 68109\ 69139\ 52452\ 36922\ 52570\ 46474\ 90557\ 82274$

$\log_{10} 3 = 0.47712\ 12547\ 19662\ 43729\ 50279\ 03255\ 11530\ 92001\ 28864\ 19069$

OTHER CONSTANTS

Euler's Constant $\gamma = 0.57721\ 56649\ 01532\ 86061$

$\log_e \gamma = -0.54953\ 93129\ 81644\ 82234$

Golden Ratio $\phi = 1.61803\ 39887\ 49894\ 84820\ 45868\ 34365\ 63811\ 77203\ 09180$

Fascinating Numbers

OTHER CONSTANTS

Euler's Constant γ	=	0.57721 56649 01532 86061
$\log_e \gamma$	=	−0.54953 93129 81644 82234
Golden Ratio ø	=	1.61803 39887 49894 84820 45868 34365 63811 77203 09180

Fascinating Numbers

OTHER CONSTANTS

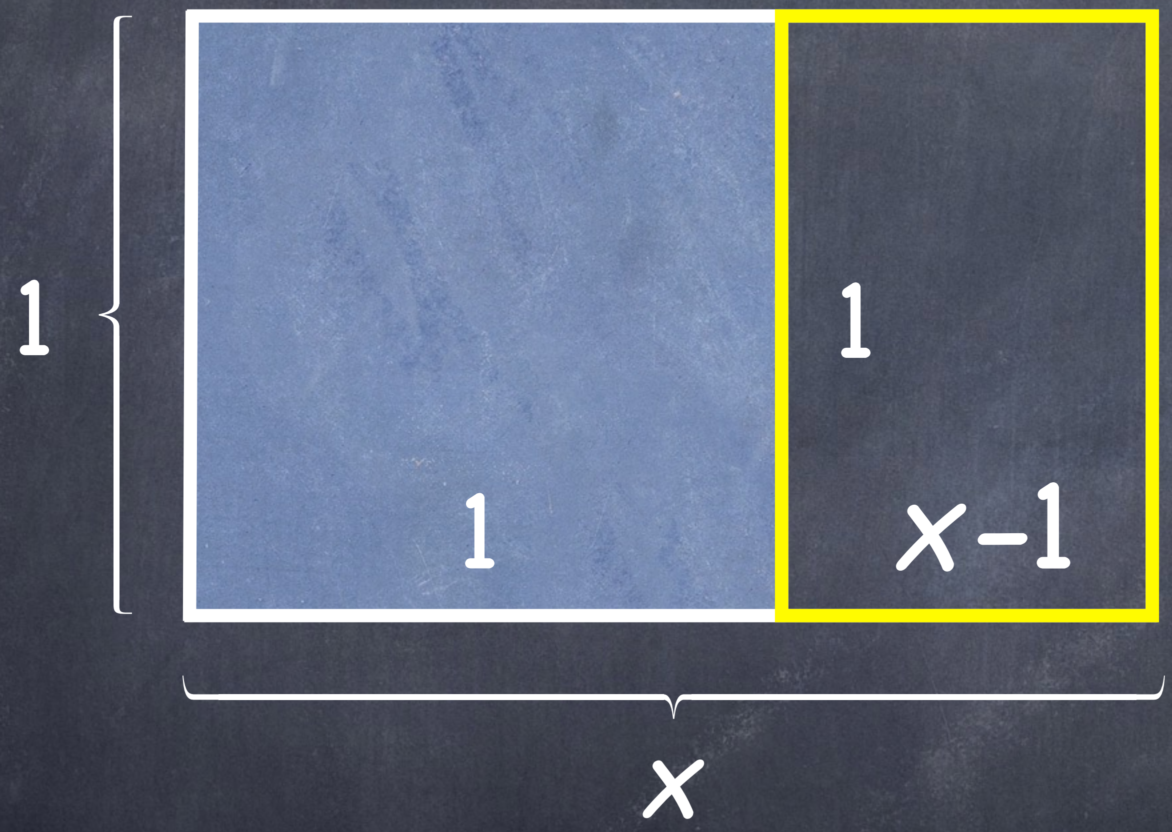
Euler's Constant	$\gamma =$	0.57721 56649 01532 86061
	$\log_e \gamma =$	−0.54953 93129 81644 82234
Golden Ratio	ø =	1.61803 39887 49894 84820 45868 34365 63811 77203 09180

The Golden Rectangle:

A rectangle with the property that the removal of a square results in a new rectangle that has the same length to width ratio as the original.

The Golden Rectangle:

A rectangle with the property that the removal of a square results in a new rectangle that has the same proportions as the original.



$$\frac{x}{1} = \frac{1}{x-1}$$

$$x^2 - x = 1$$

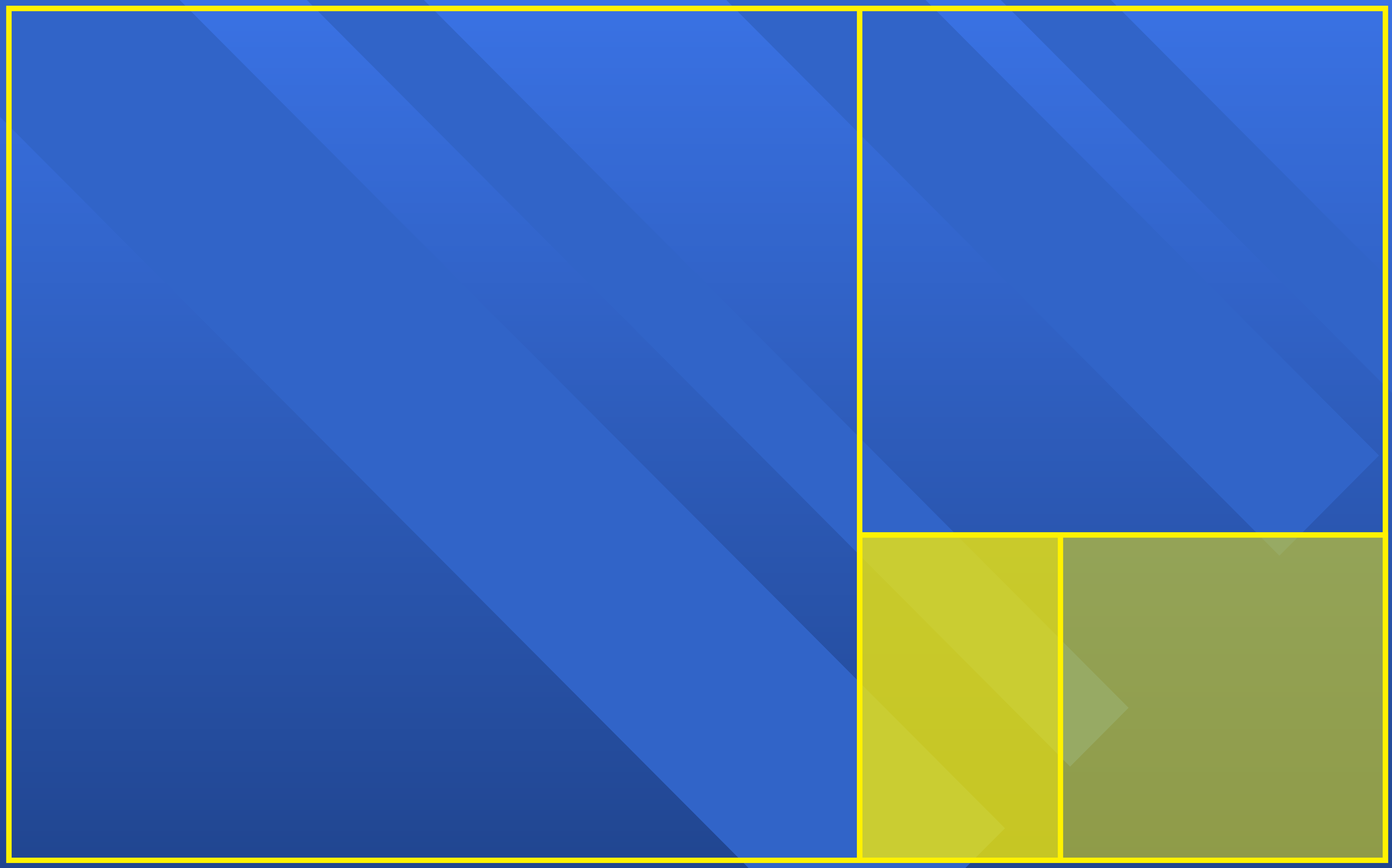
$$x^2 - x - 1 = 0$$

$$x = \frac{1 \pm \sqrt{1-4(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2} \approx 1.618 \text{ or } -.618$$

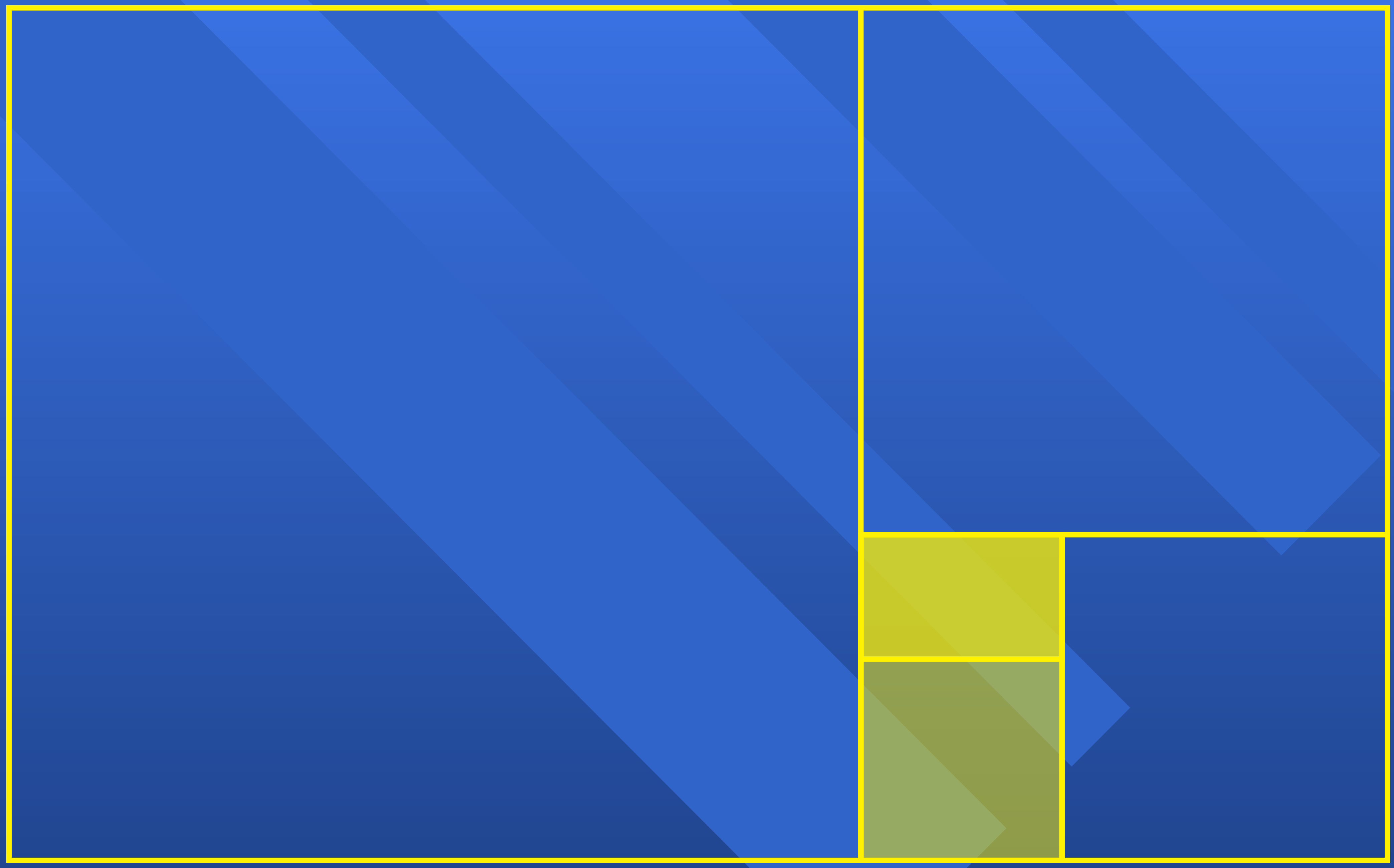
The Golden Rectangle

The Golden Rectangle

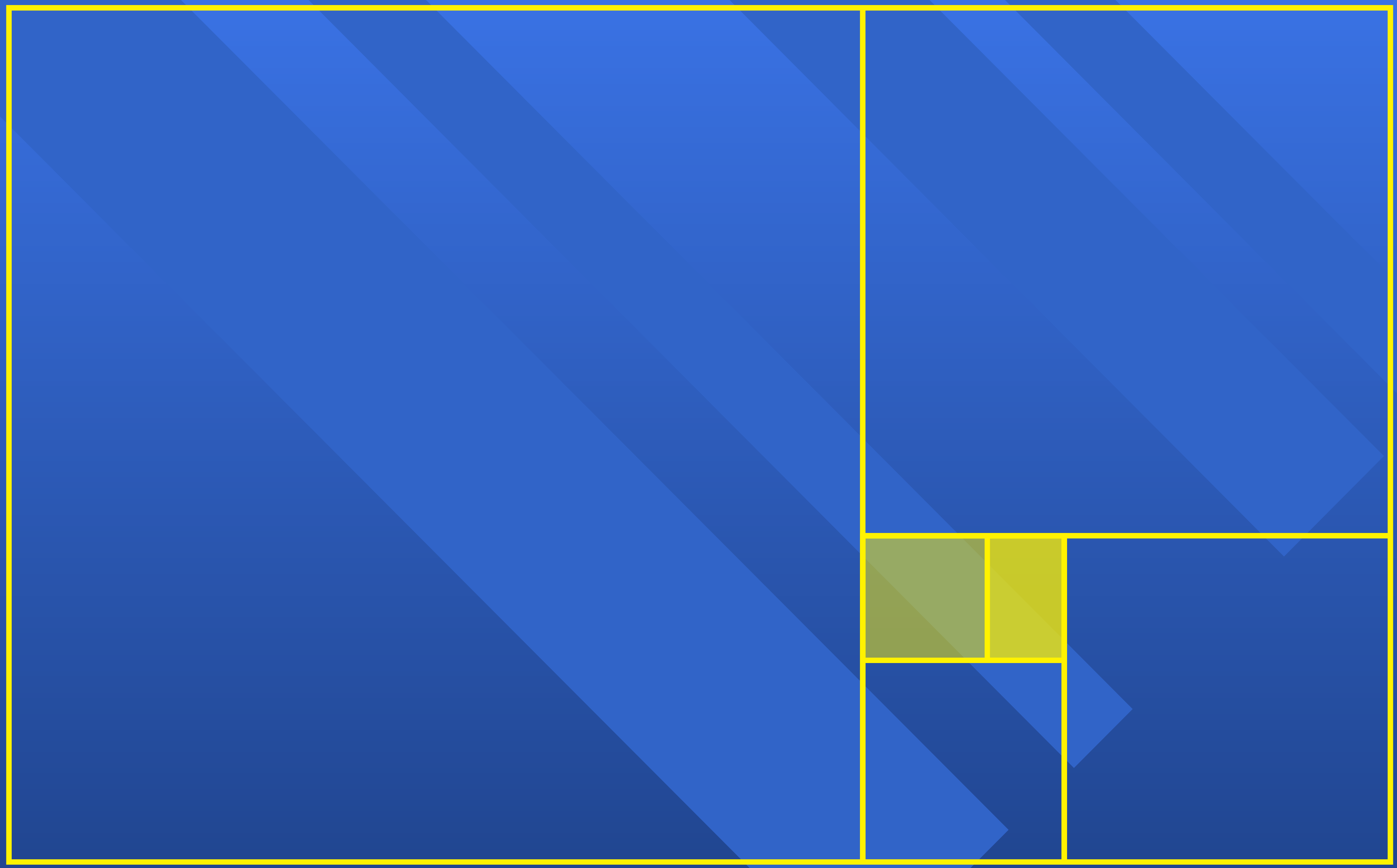
The Golden Rectangle



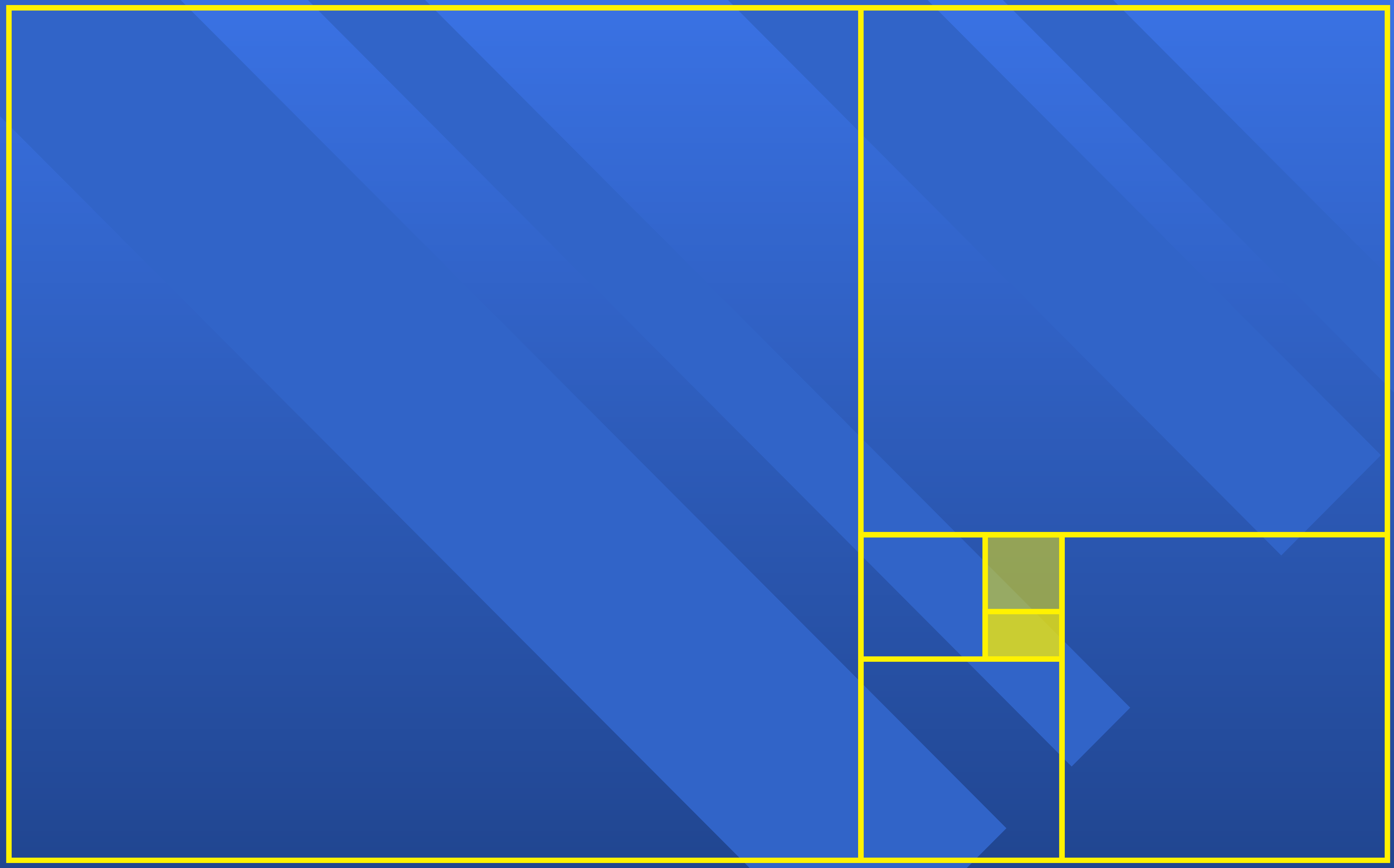
The Golden Rectangle



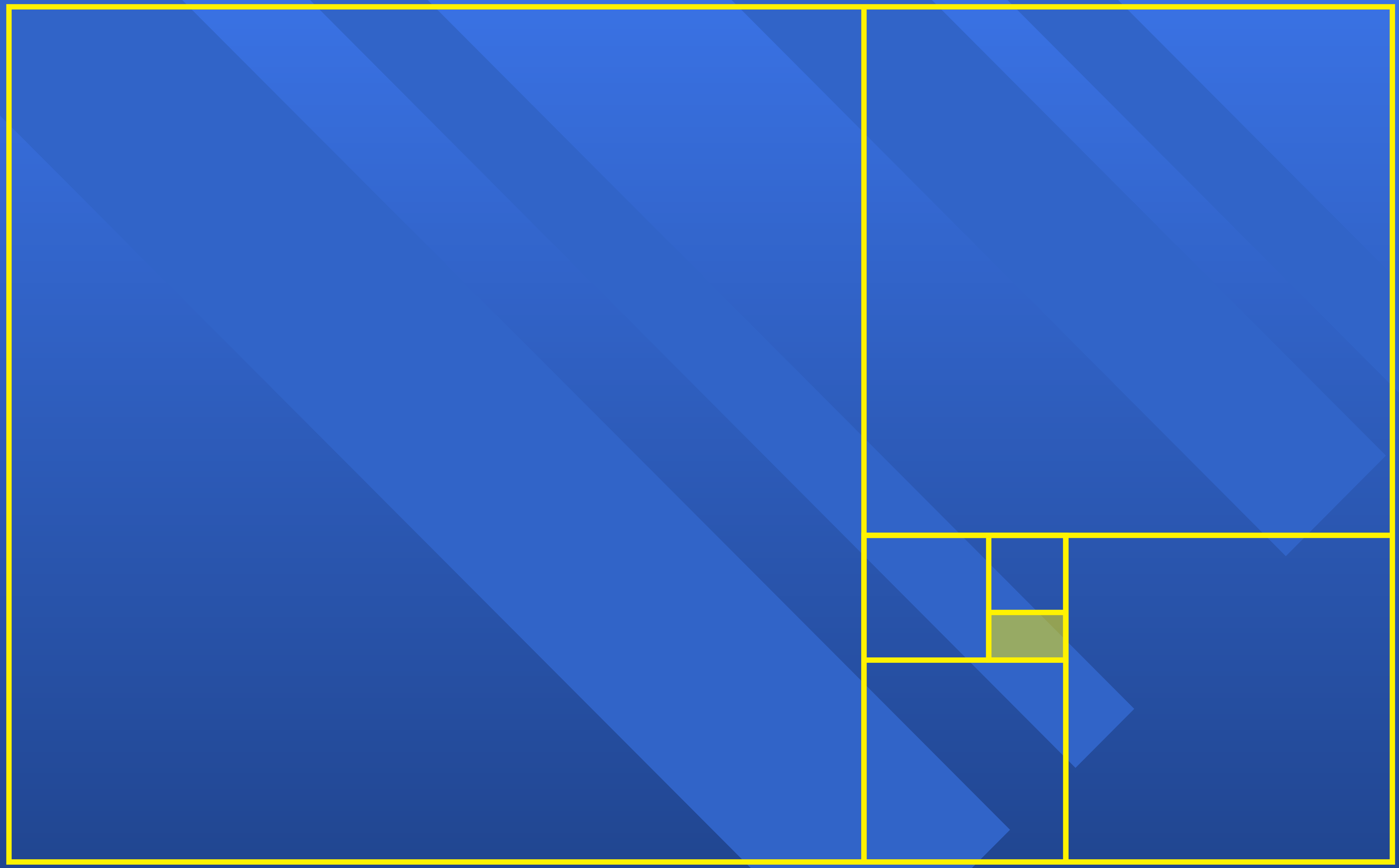
The Golden Rectangle



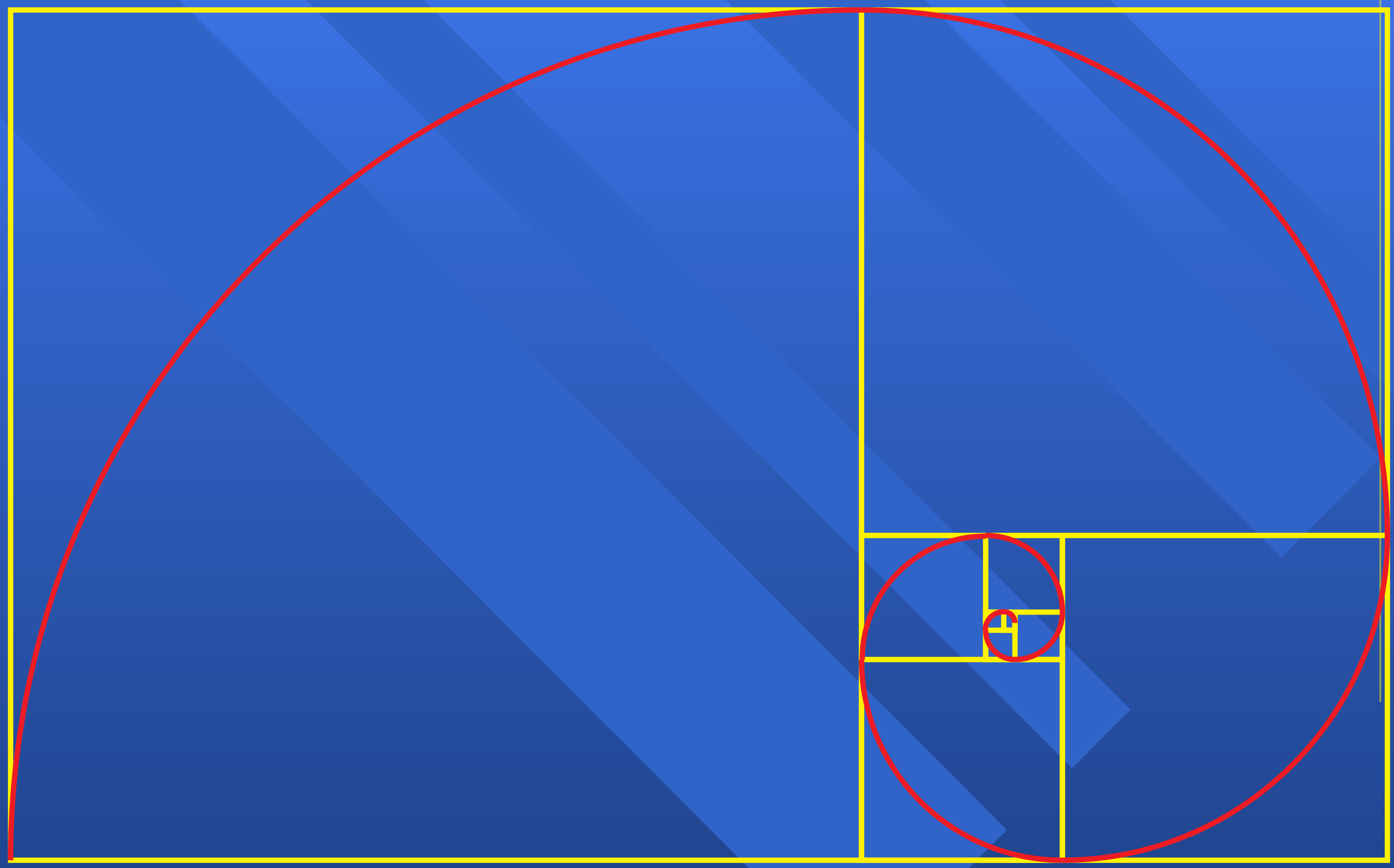
The Golden Rectangle



The Golden Rectangle



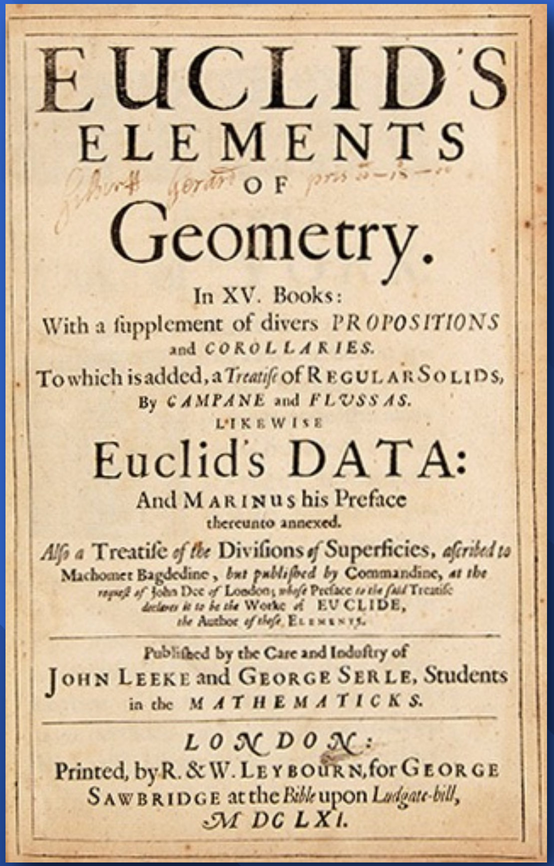
The Golden Rectangle



History of the Golden Ratio

Euclid of Alexandria

(ca. 325 B.C. – 265 B.C.)


EUCLID'S
ELEMENTS
OF
Geometry.
In XV. Books:
With a ſupplement of divers PROPOSITIONS
and COROLLARIES.
To which is added, a Treatiſe of REGULAR SOLIDS,
By CAMPANE and FLUSSAS.
LIKEWISE
Euclid's DATA:
And MARINUS his Preface
thereunto annexed.
Alſo a Treatiſe of the Diviſions of Superficies, aſcribed to
Machomet Bagdedine, but publiſhed by Commandine, at the
requeſt of John Dee of London; whoſe Preface to the ſaid Treatiſe
declares it to be the Worke of EUCLIDE,
the Author of theſe ELEMENTS.

Publiſhed by the Care and Induſtry of
JOHN LEEKE and GEORGE SERLE, Students
in the MATHEMATICKS.

LONDON:
Printed, by R. & W. LEYBOURN, for GEORGE
SAWBRIDGE at the Bible upon Ludgate-hill,
M DC LXI.

Consists of 13 books

Containing 465 propositions

History of the Golden Ratio

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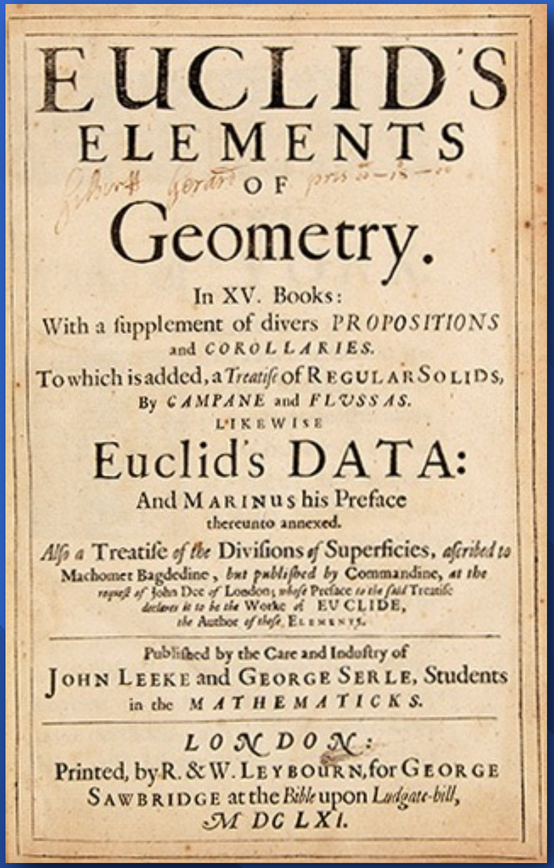
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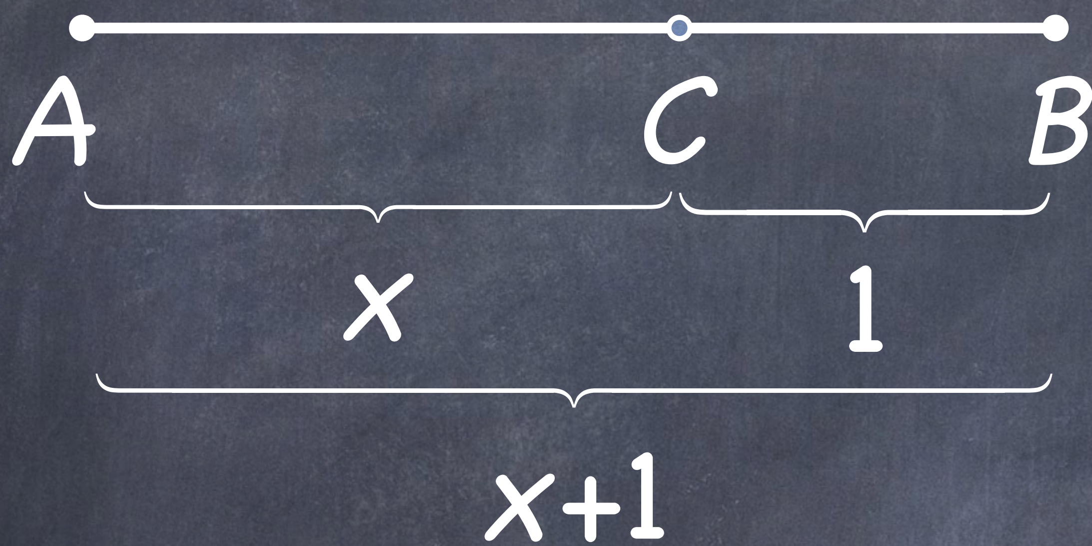
SAWBRIDGE at the Bible upon Ludgate-hill,
MDCLXI.

Book VI, Proposition 30

The Extreme and Mean Ratio

Given a segment $\overline{AB}$, find the point C such that



$$\frac{AC}{CB} = \frac{AB}{AC}$$

$$\frac{x}{1} = \frac{x+1}{x}$$

$$x^2 = x+1$$

$$x^2-x-1 = 0$$

$$x \approx 1.618$$

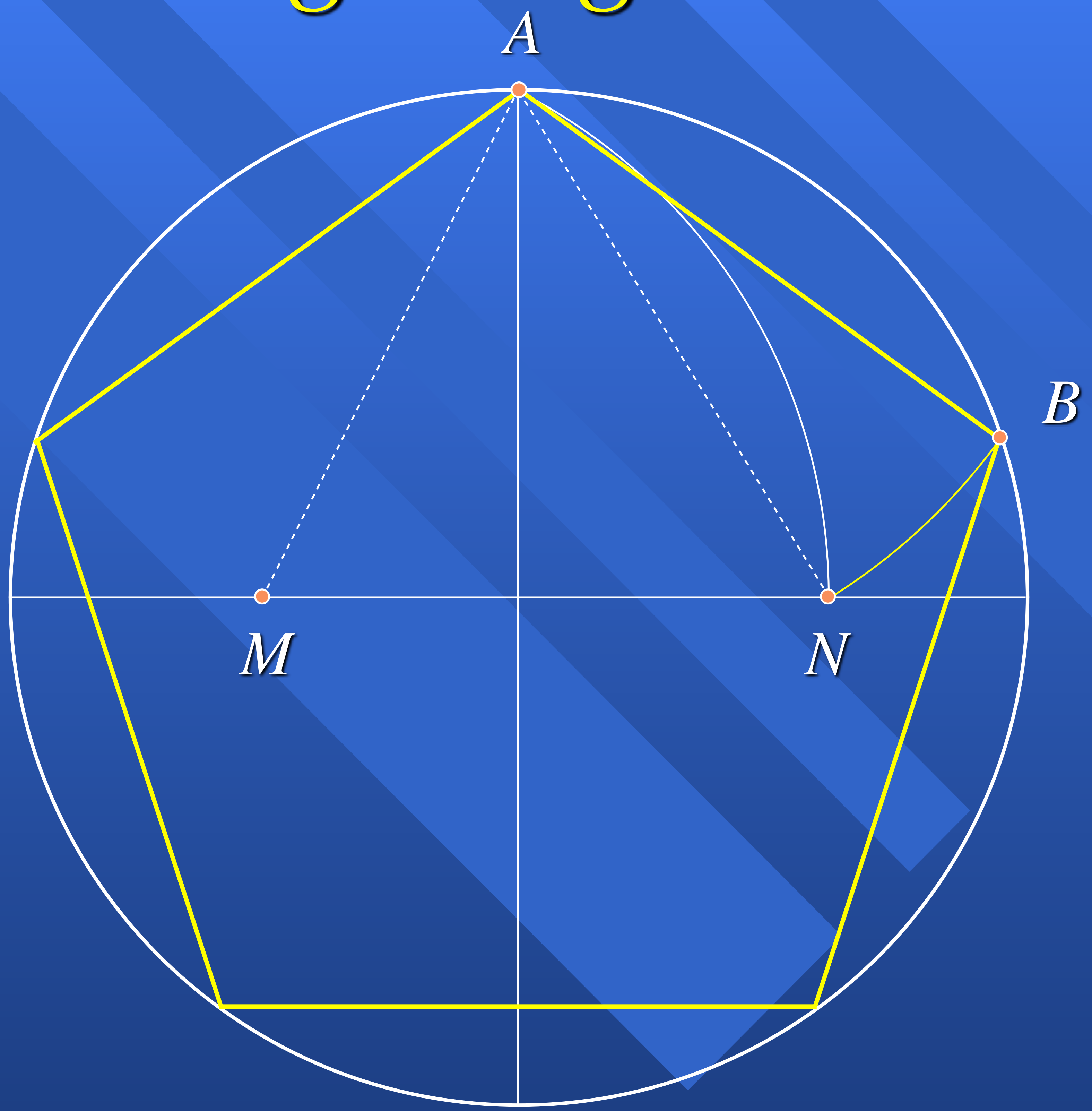
History of the Golden Ratio

Martin Ohm (1835) ⟶ "Golden Section"

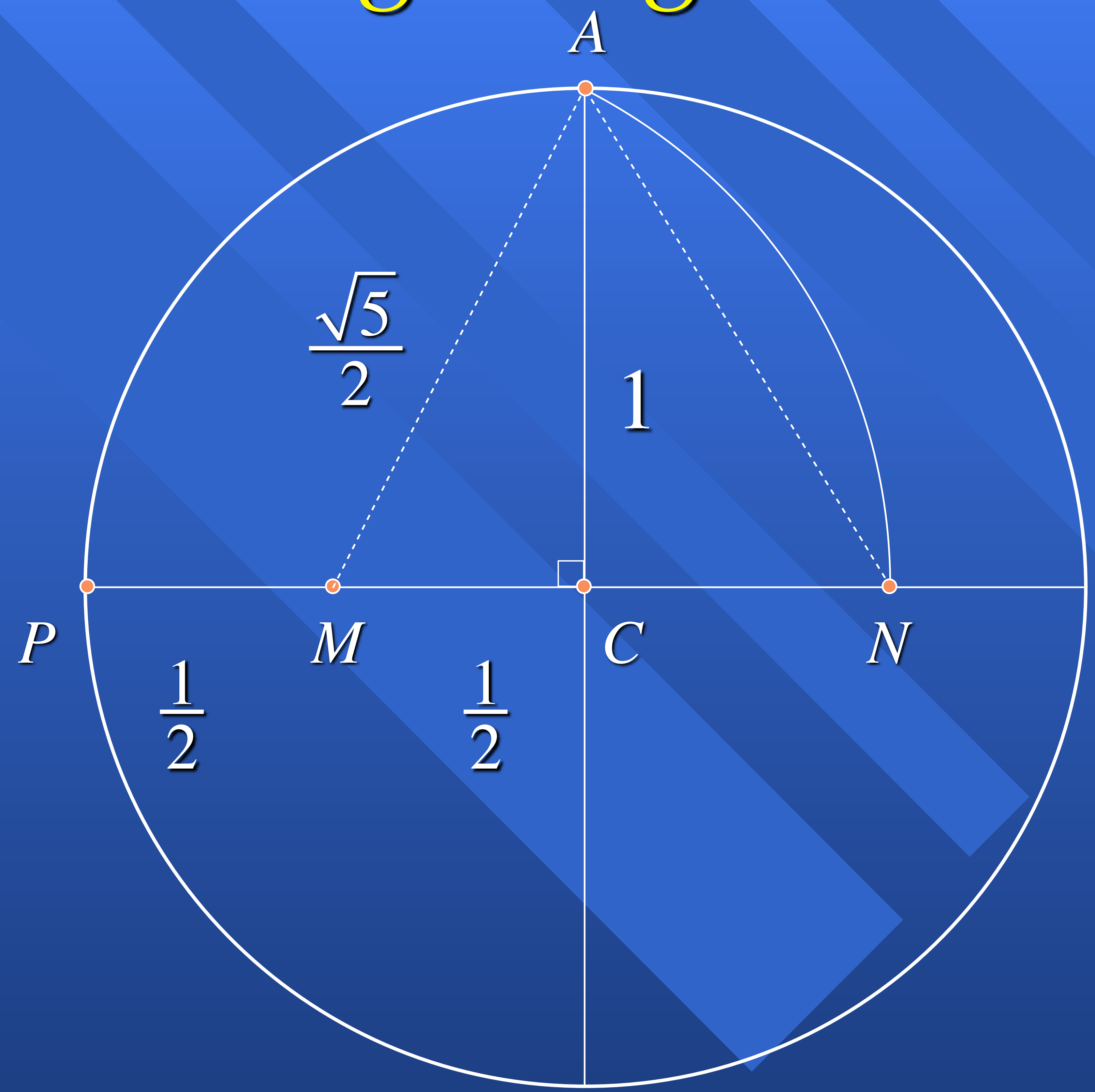
Mark Barr ⟶ $\Phi = \frac{1+\sqrt{5}}{2}$

Phidias ($\Phi\varepsilon\iota\delta\iota\alpha\varsigma$) (ca. 490 – 430 B.C.)

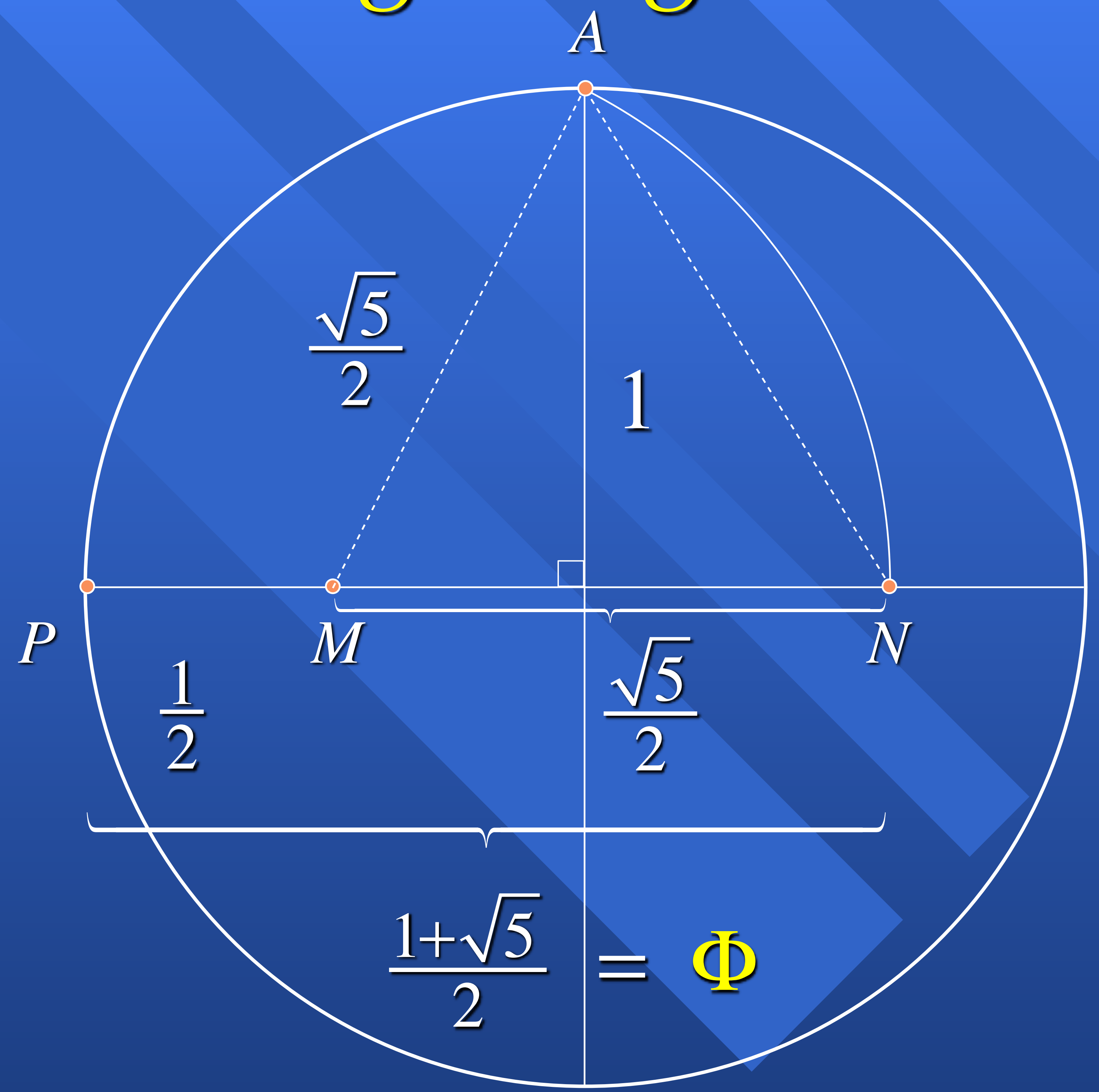
Constructing a Regular Pentagon



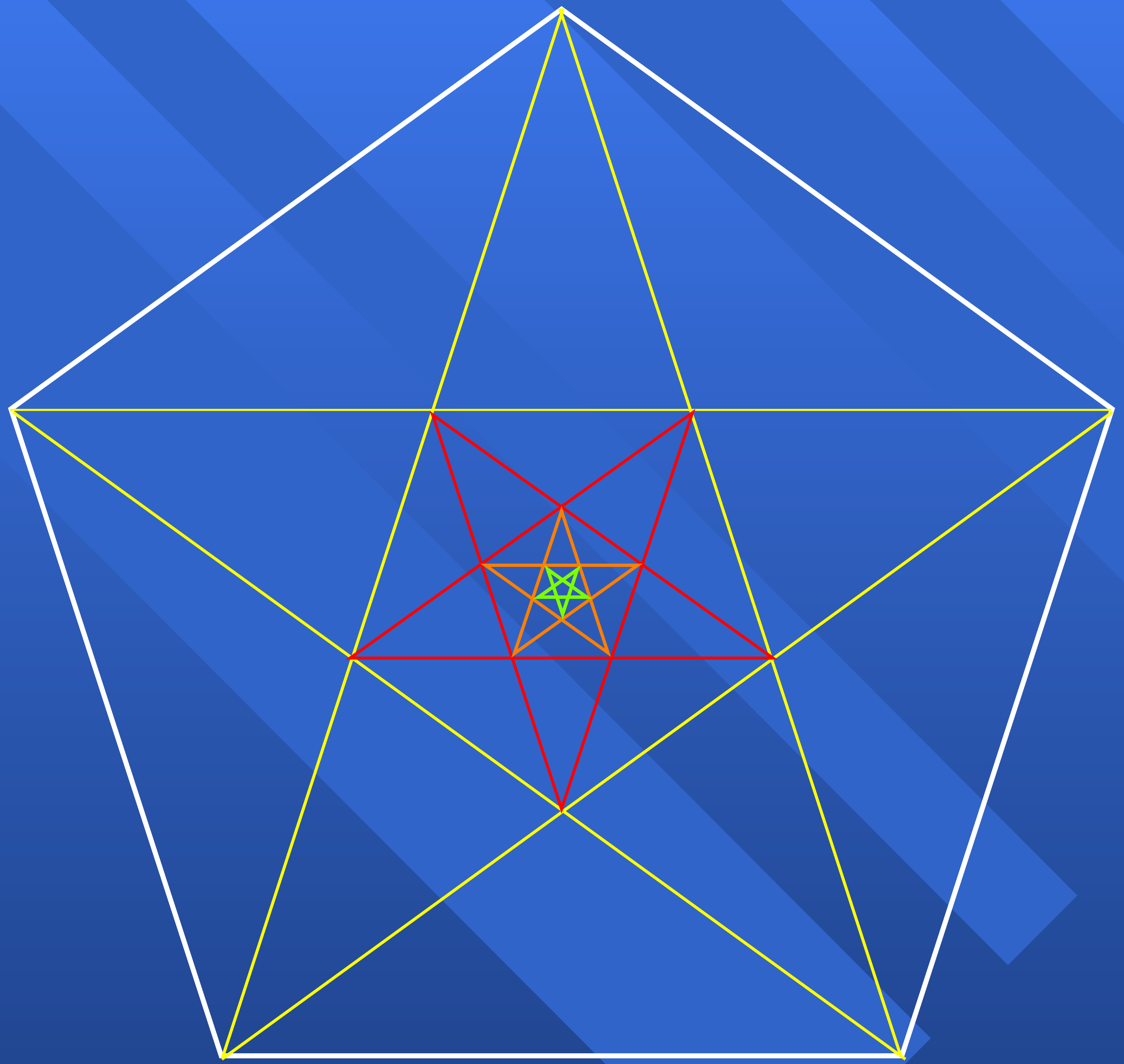
Constructing a Regular Pentagon



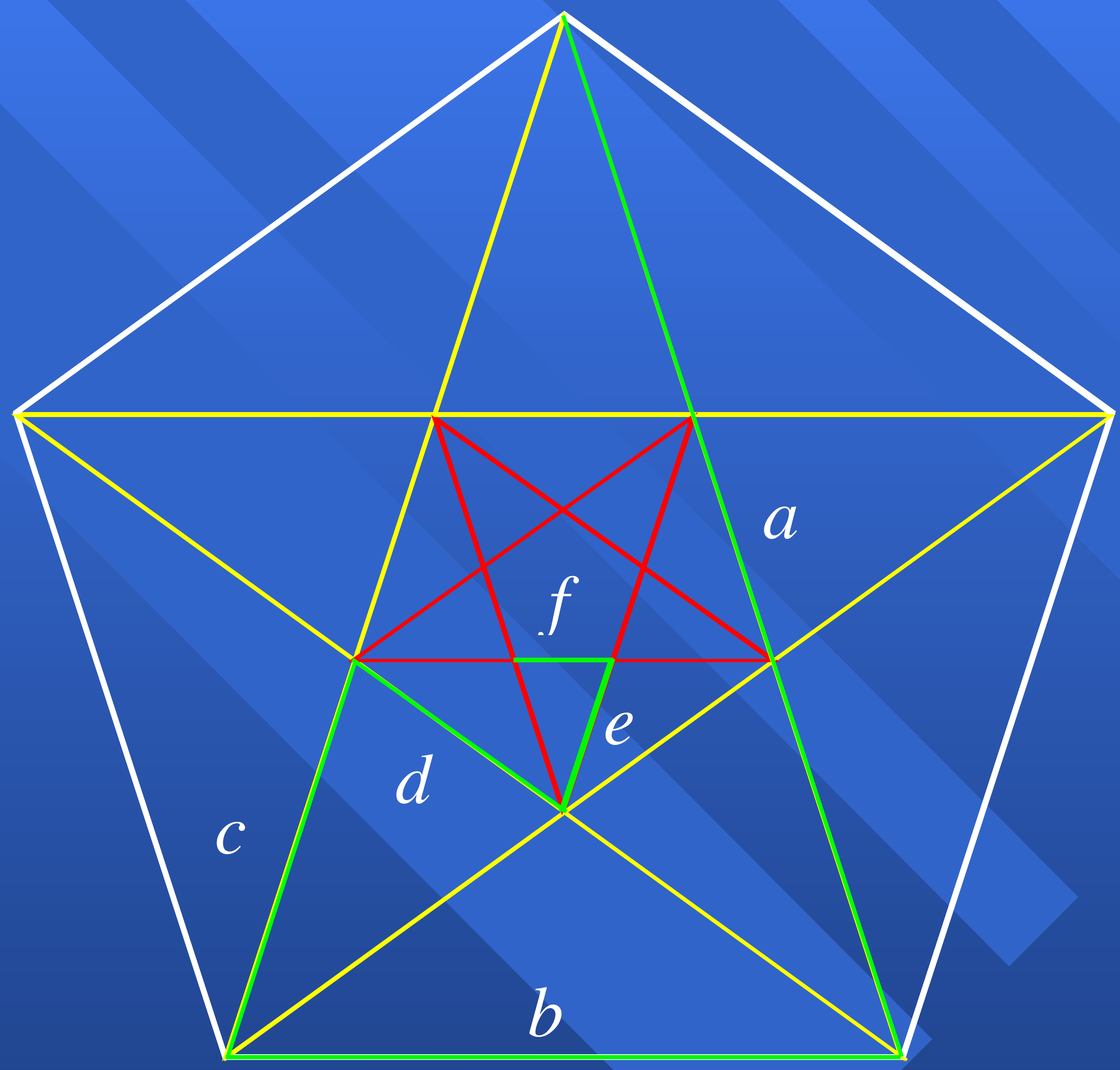
Constructing a Regular Pentagon



The Pentagram



The Pentagram

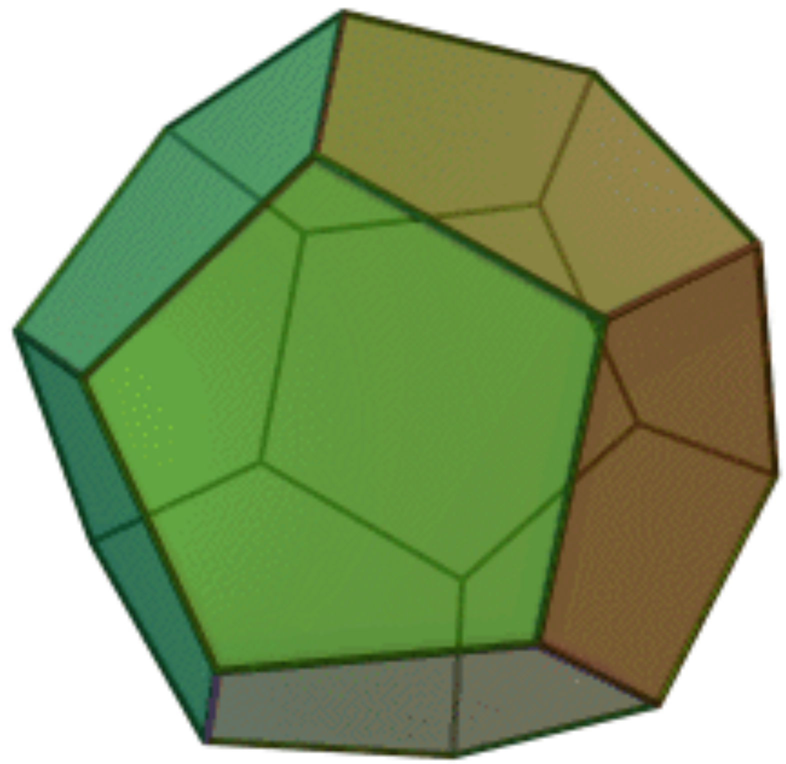


$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f} = \Phi$$

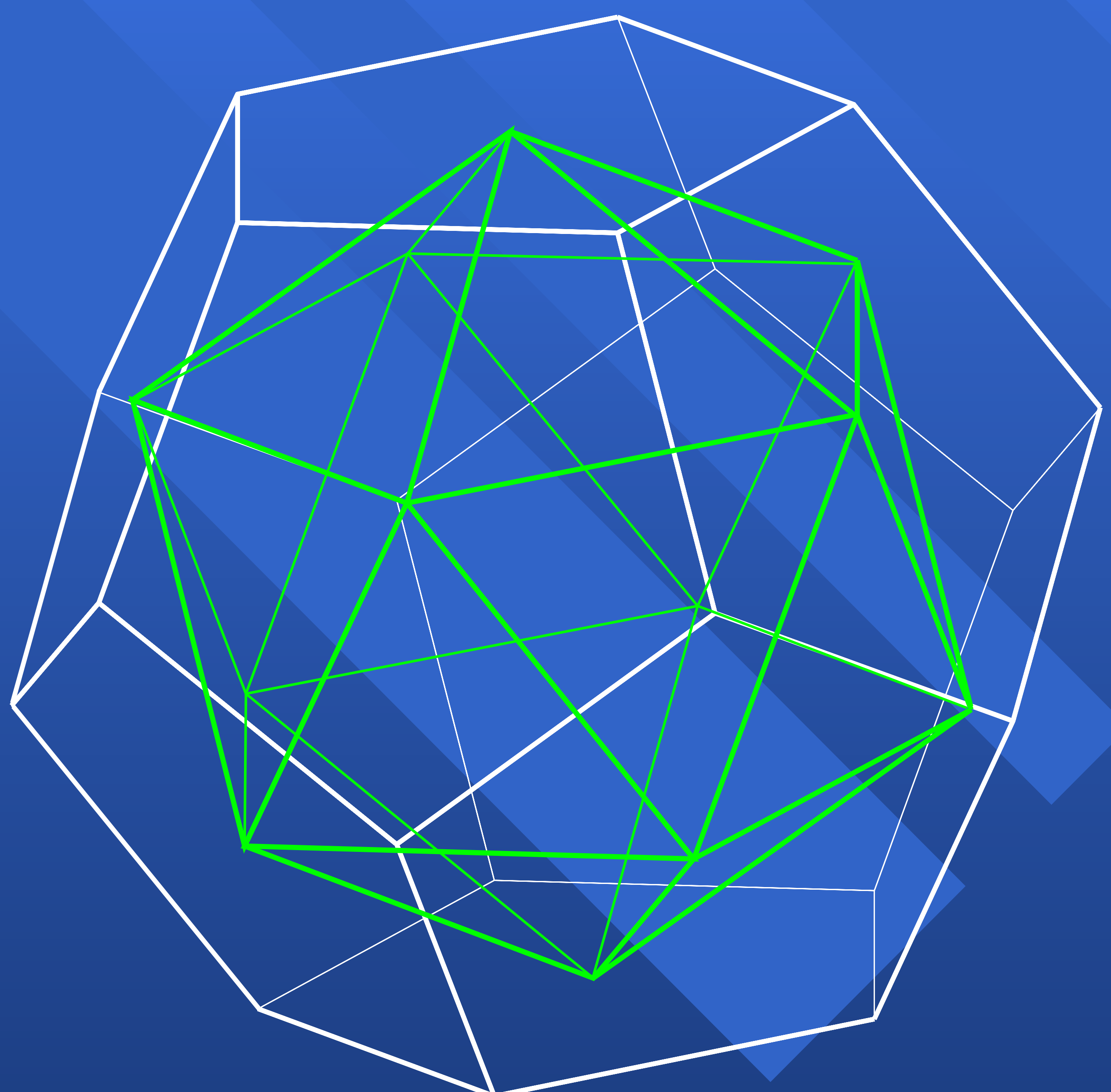
The Pythagoreans and the Pentagram

Pythagoras of Samos (ca. 569 B.C. – 475 B.C.)

THE DODECAHEDRON



The Golden Rectangle and the Dodecahedron



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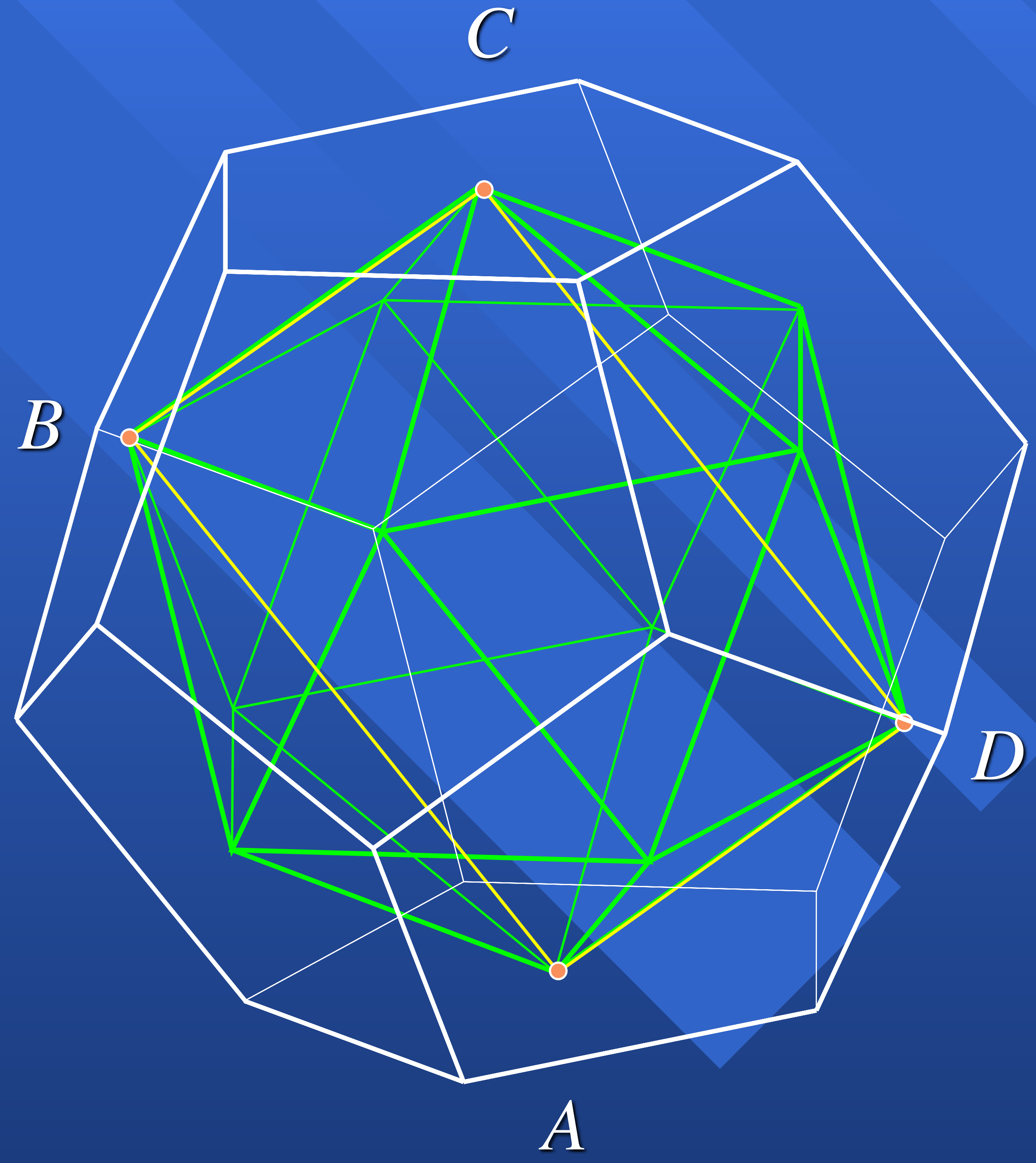

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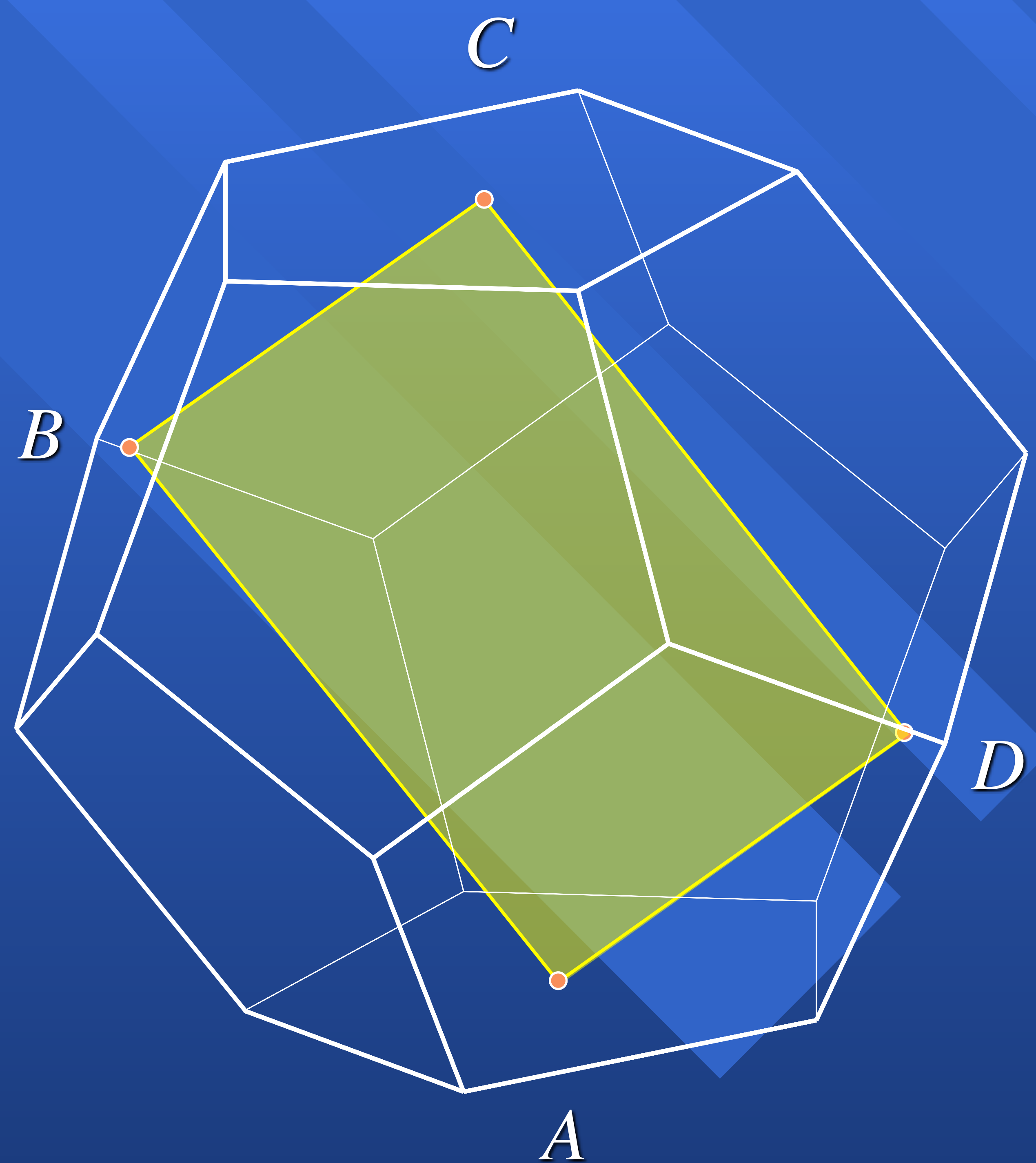


A Mathematical Sculpture

"Essence"

Richard Werner

Art and the Golden Ratio

"Sacrament of the Last Supper"

Salvador Dali

Art and the Golden Ratio

Leonardo

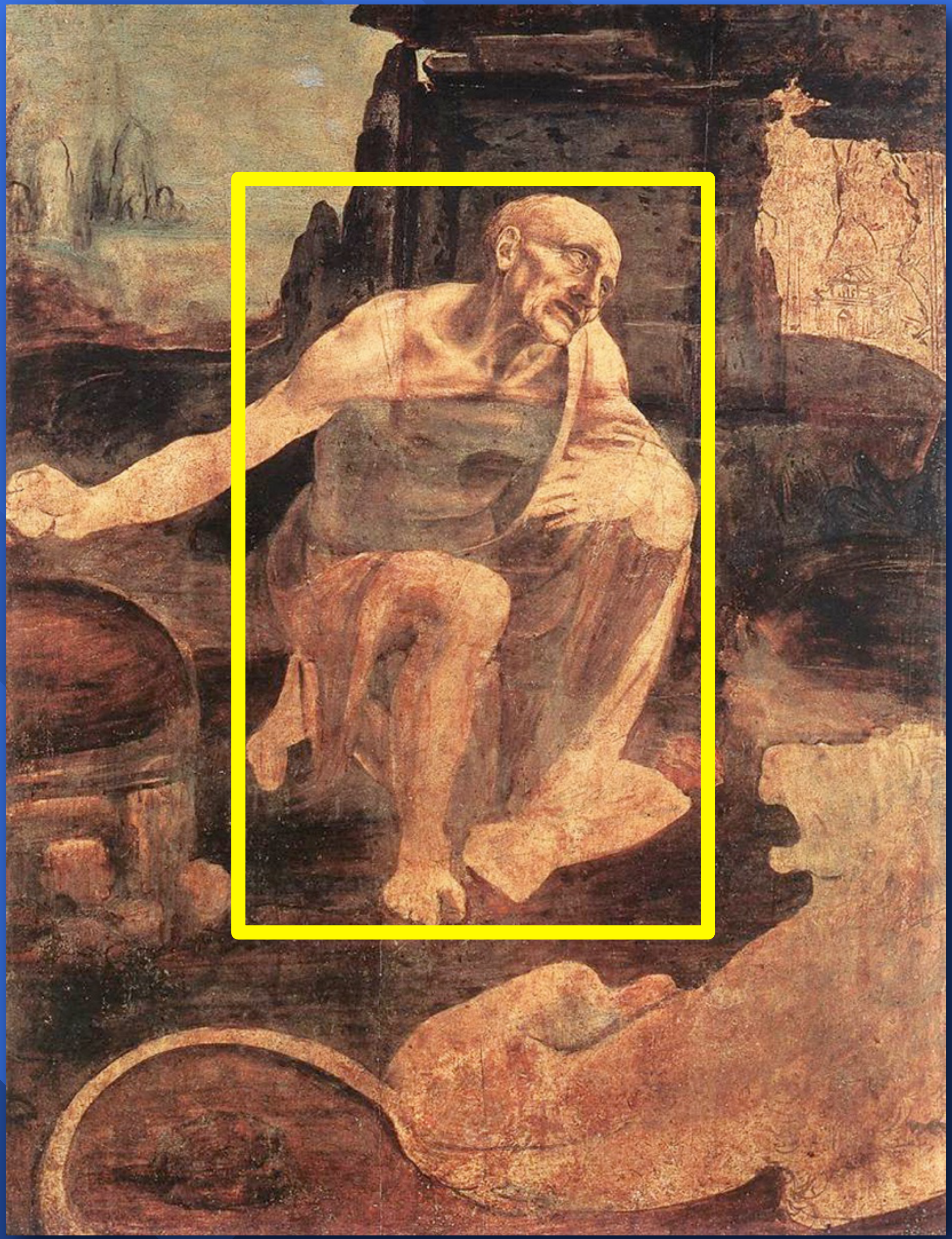
Dürer

Raphael

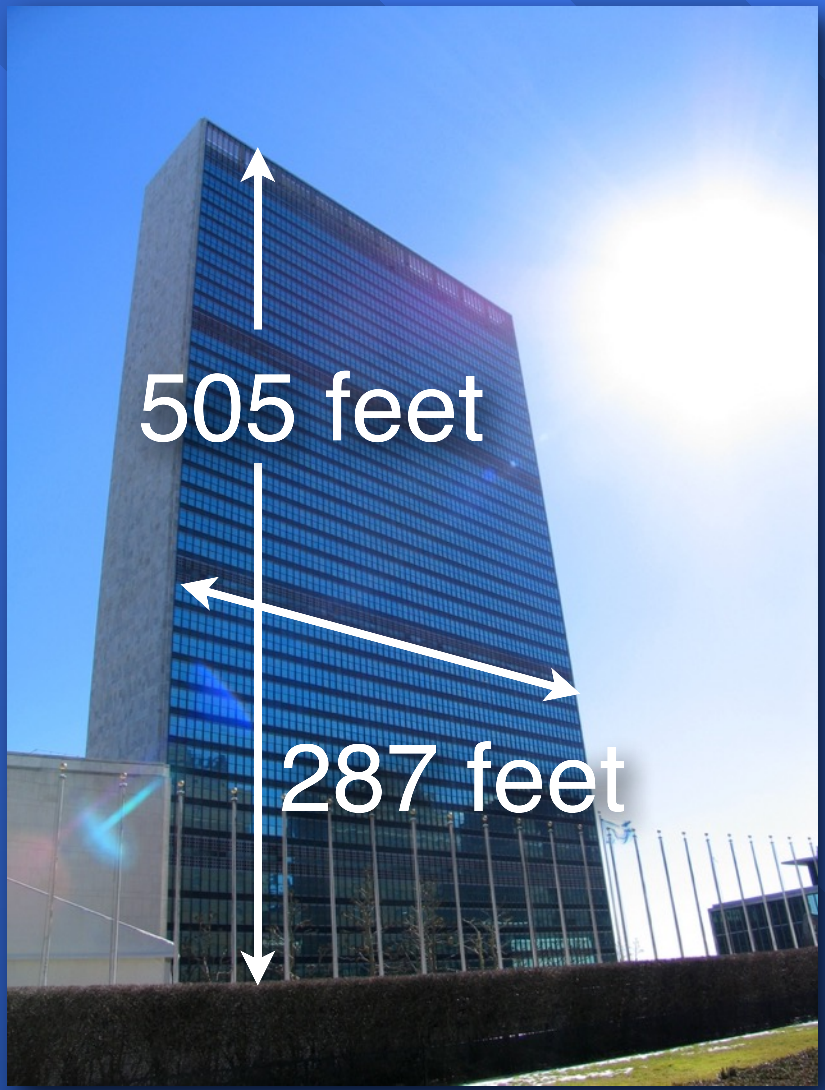
Art and the Golden Ratio

"A Golden Rectangle fits so neatly around St. Jerome that some experts believe Leonardo purposely painted the figure to conform to those proportions."

Mathematics
David Bergamini



All that Glitters?

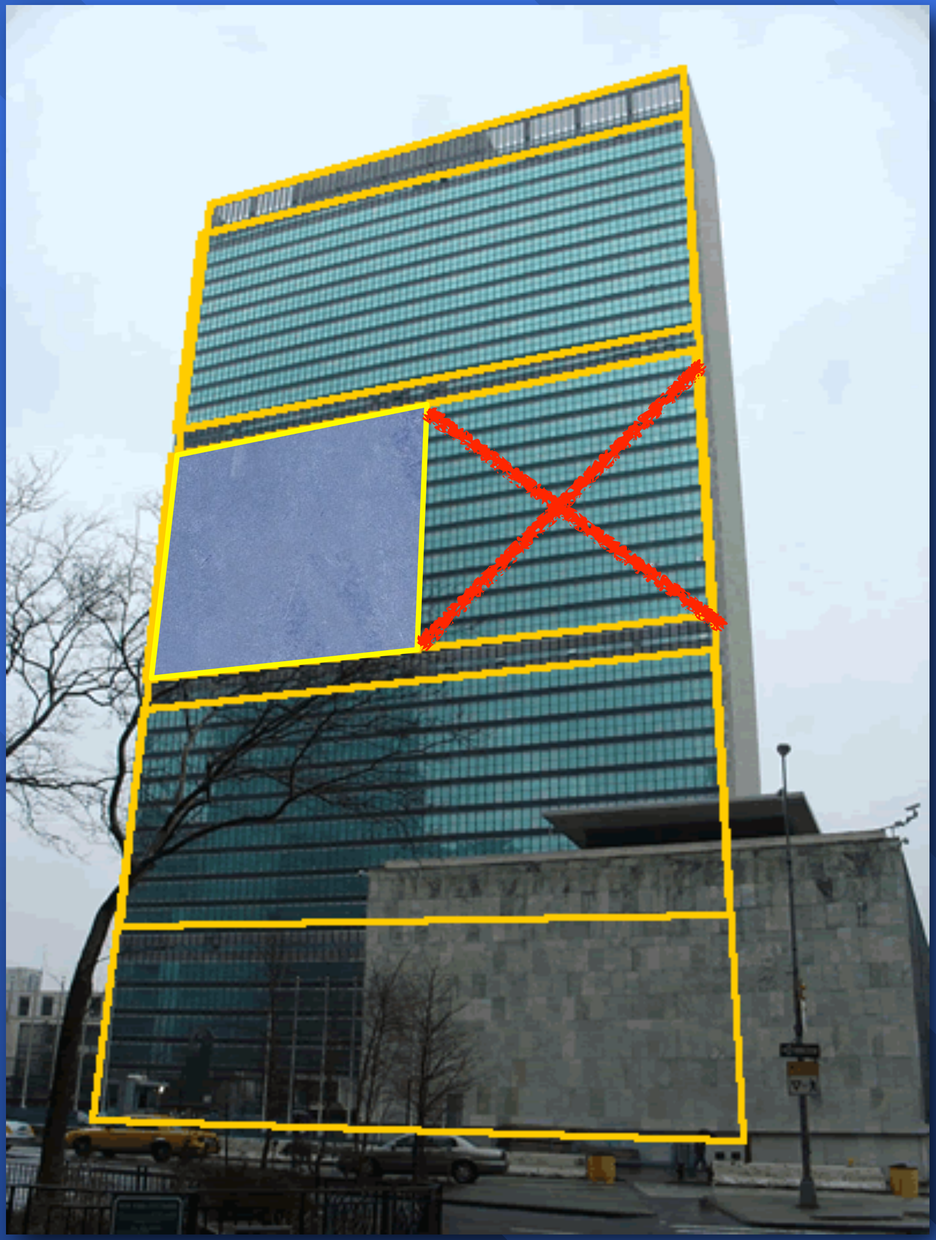


“The Greeks saw beauty in number and shape and their excitement with the golden ratio manifested itself in their art and architecture and has been echoed by later civilizations in such places as Notre Dame in Paris and in the UN building in New York.”

Random House Encyclopedia

$$\frac{H}{W} = \frac{505}{287} \approx 1.76 \neq \Phi \approx 1.618$$

All that Glitters?



http://library.thinkquest.org/trio/TTQ05063/phibeauty4.htm

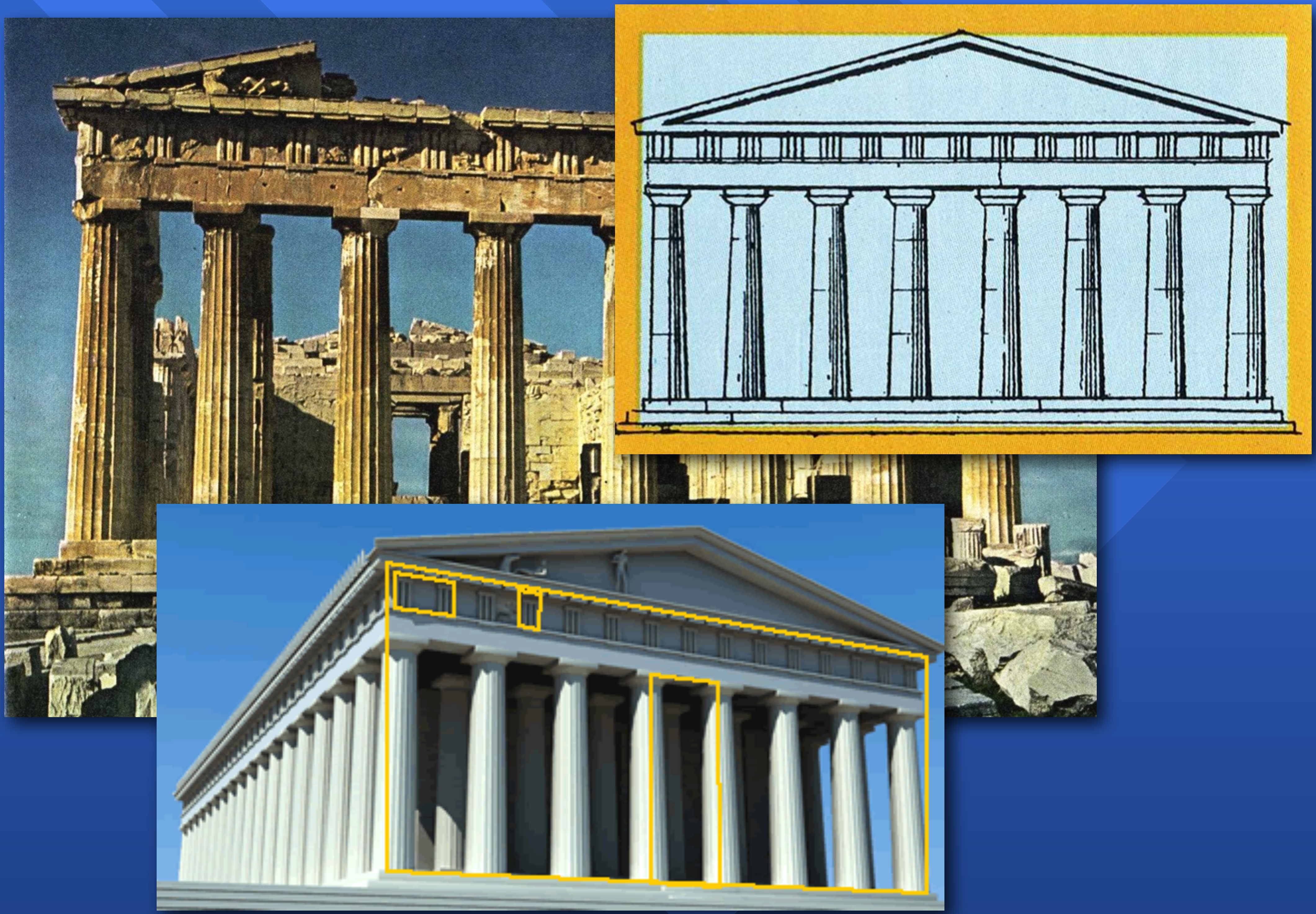
Good as Gold?

FEDERAL RESERVE NOTE
CB 20003000 D
B2
UNITED STATES FEDERAL RESERVE SYSTEM
THIS NOTE IS LEGAL TENDER FOR ALL DEBTS, PUBLIC AND PRIVATE
Treasurer of the United States.
SERIES 2001
THE UNITED STATES OF AMERICA
TWENTY
CB 20003000 D
Secretary of the Treasury.
JACKSON
TWENTY DOLLARS

Good as Gold?


RESERVE NOTE
20
THE
UNITED STATES
OF AMERICA
CB20003000D
Secretary of the Treasury.
20
JACKSON
TWENTY DOLLARS

The Parthenon



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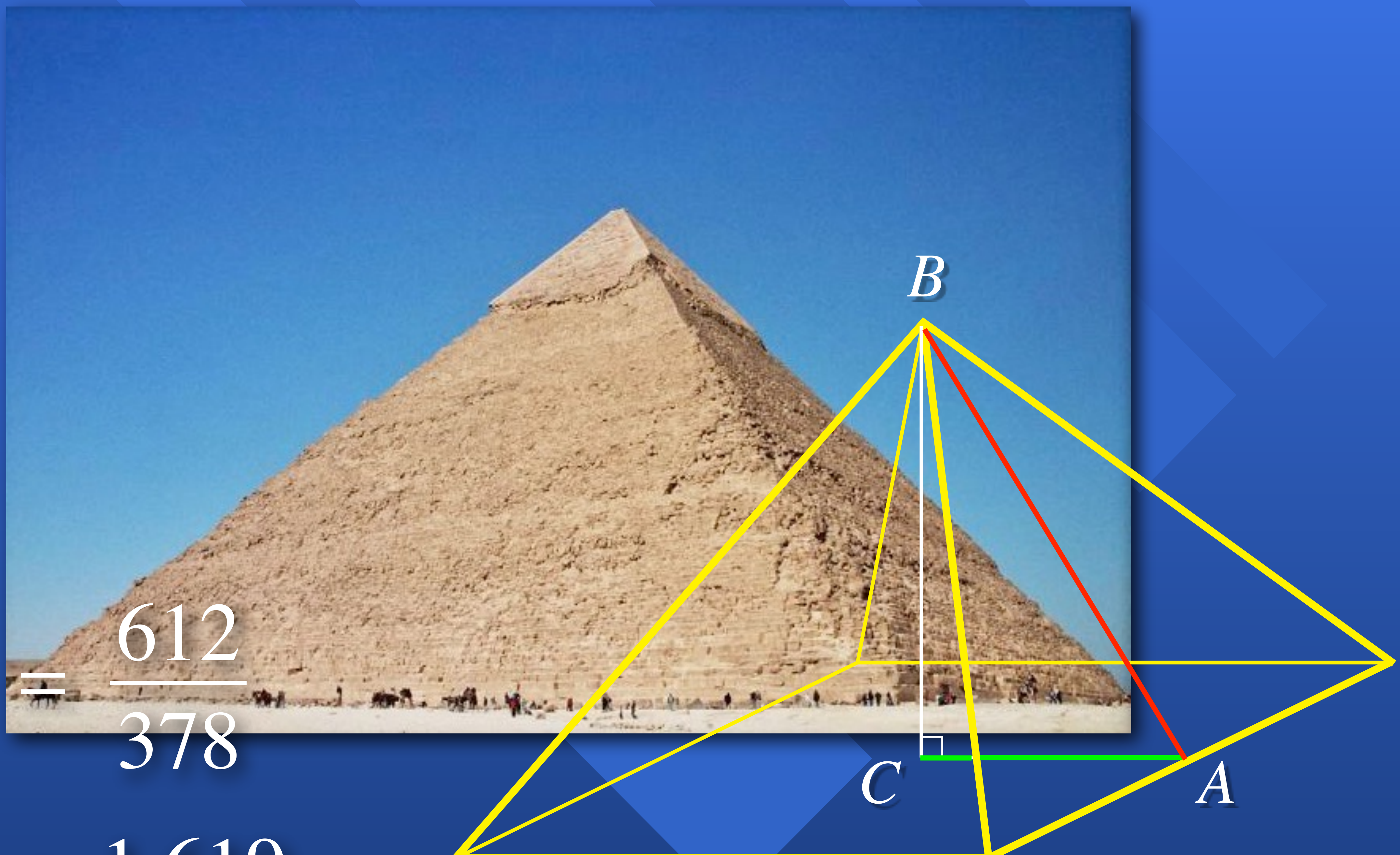
The Great Pyramid



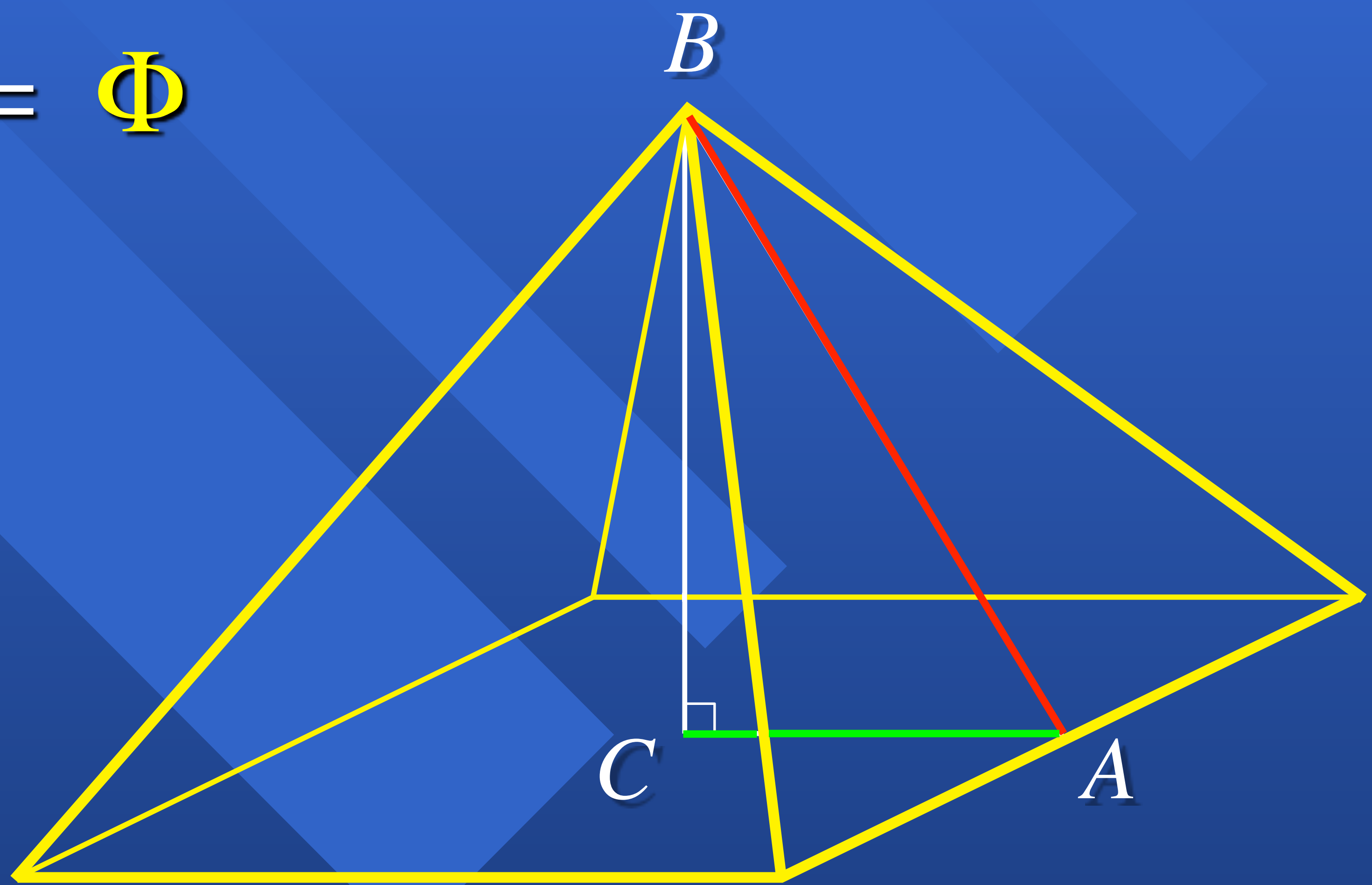
$$\frac{AB}{AC} = \frac{612}{378}$$

$$\approx 1.619$$

The Great Pyramid

"It was reported that the Greek historian Herodotus learned from the Egyptian priests that the square of the Great Pyramid's height is equal to the area of its triangular lateral side."

This implies that $$\frac{AB}{AC} = \Phi$$

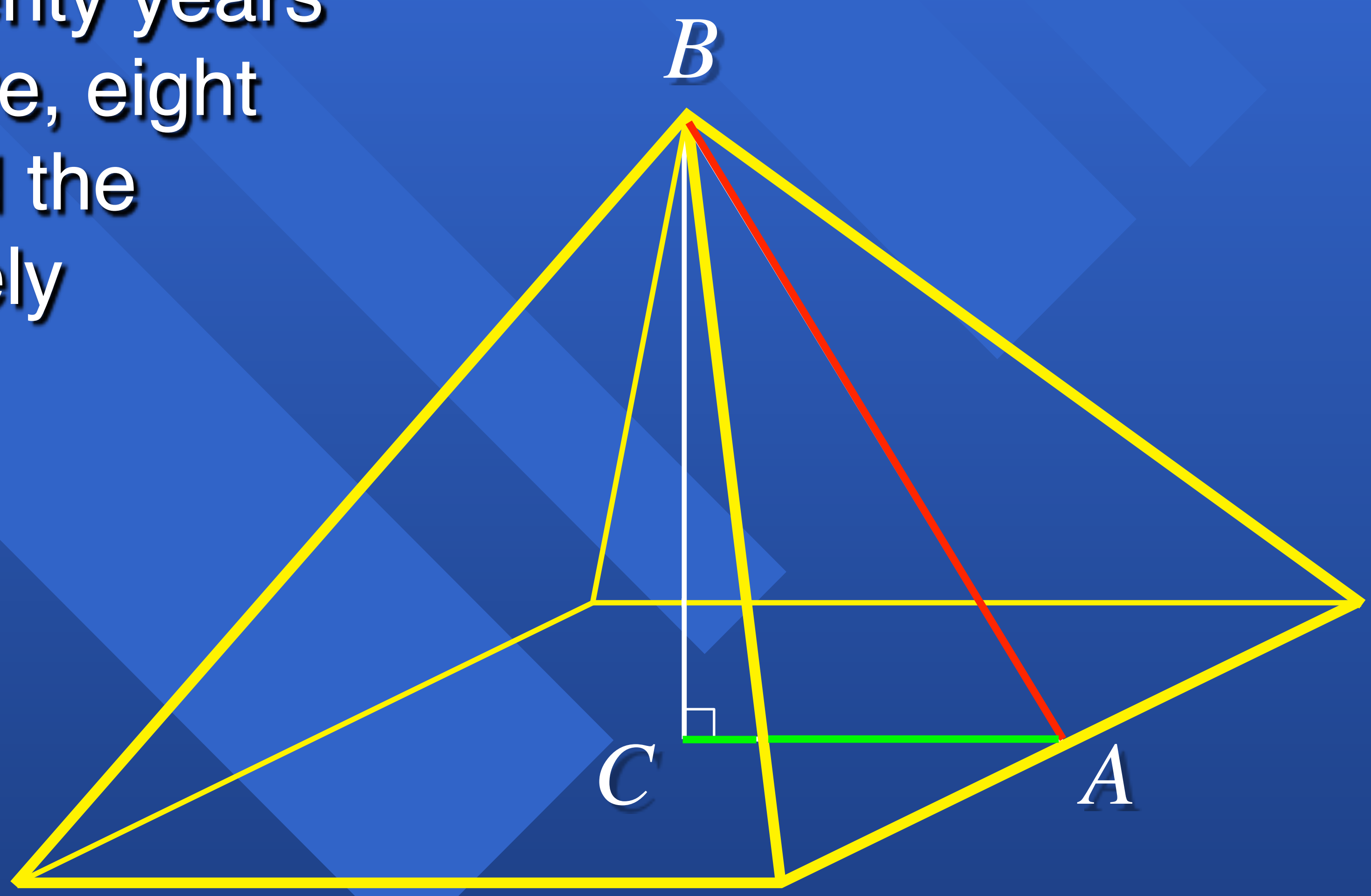


The Great Pyramid

"It was reported that the Greek historian Herodotus learned from the Egyptian priests that the square of the Great Pyramid's height is equal to the area of its triangular lateral side."

"The Pyramid itself was twenty years in the building. It is a square, eight hundred feet each way, and the height the same, built entirely of polished stone fitted together with the utmost care."

History, Book II, P. 24
Herodotus



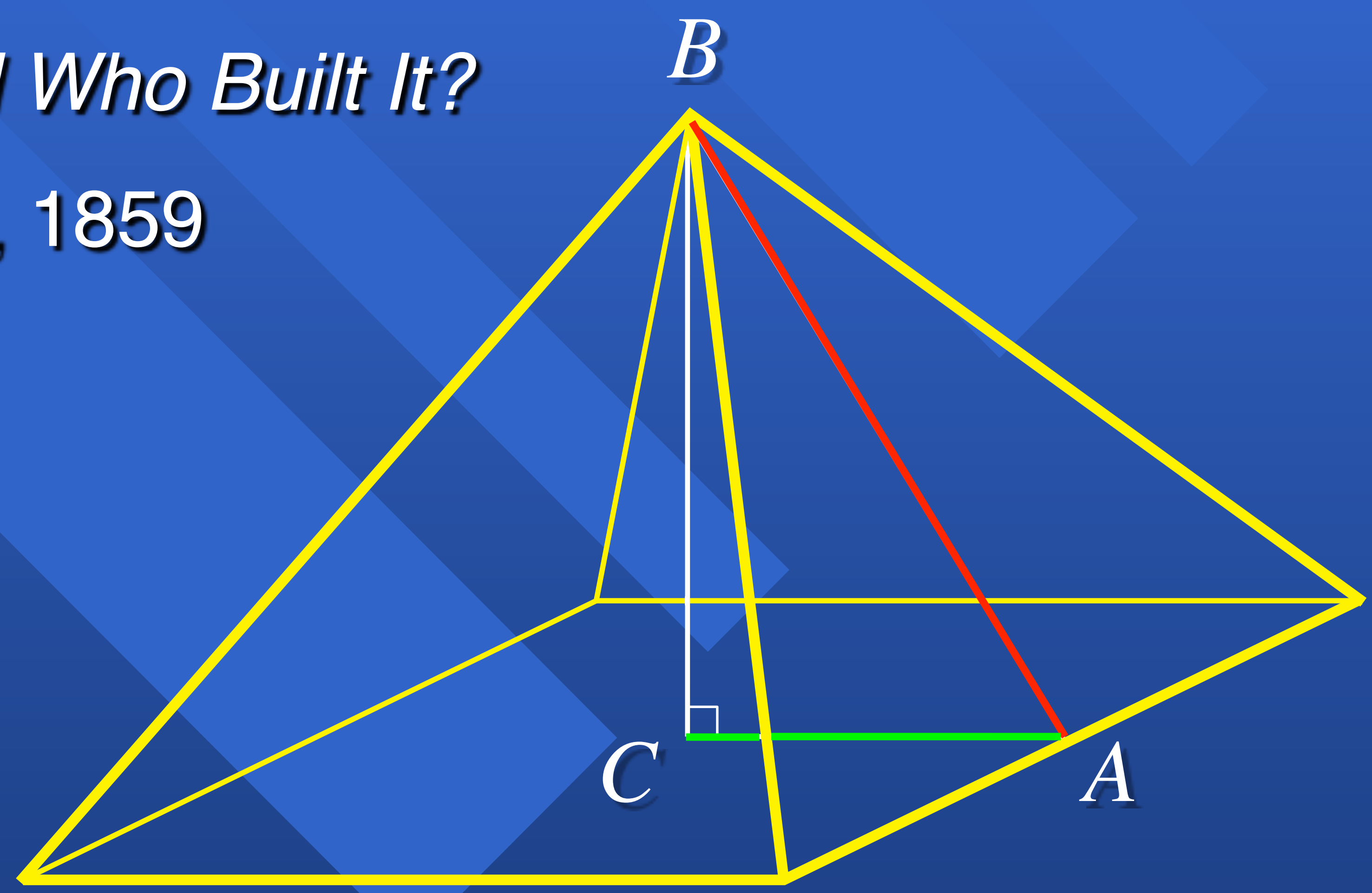
The Great Pyramid

"It was reported that the Greek historian Herodotus learned from the Egyptian priests that the square of the Great Pyramid's height is equal to the area of its triangular lateral side."

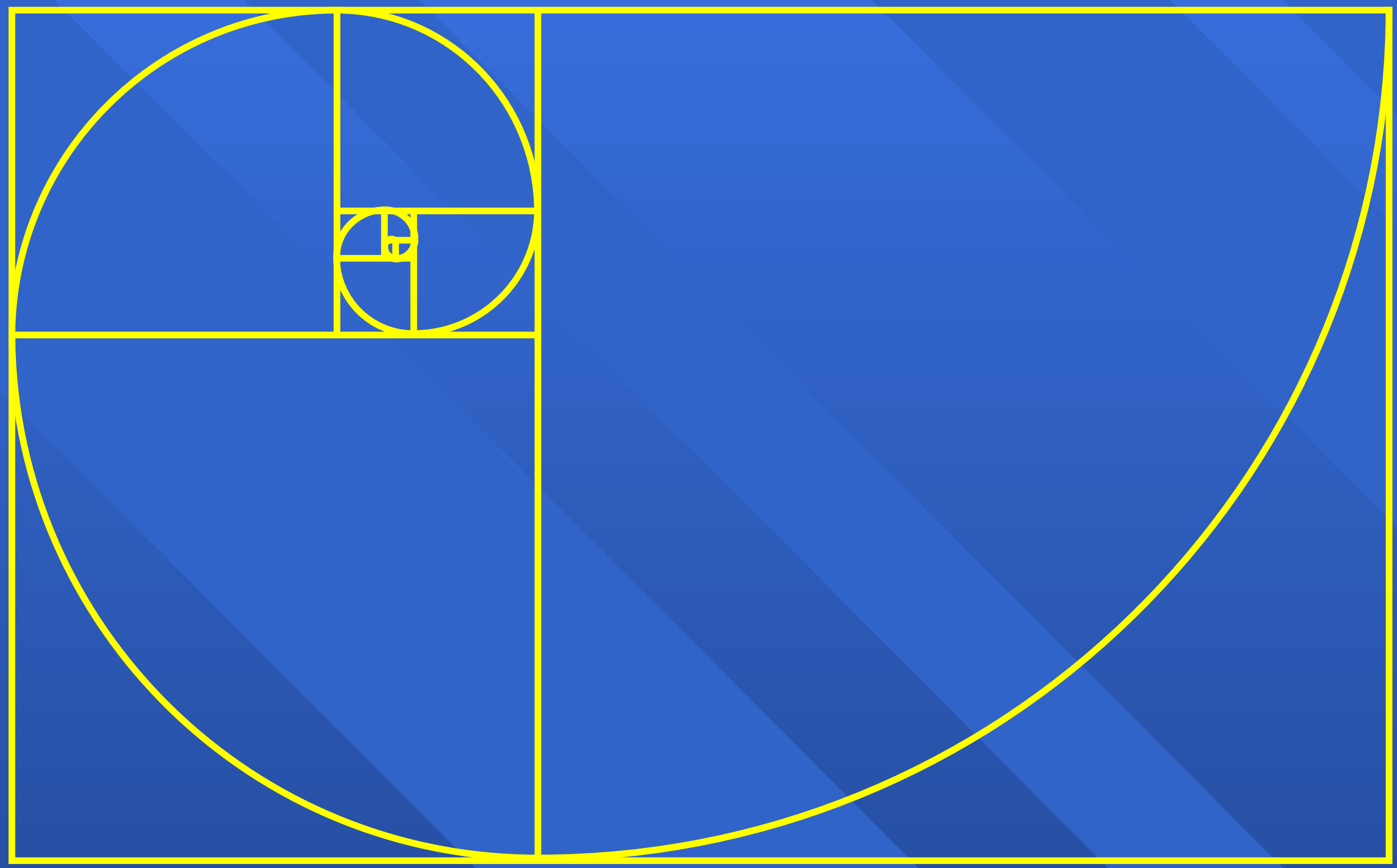
The Great Pyramid:

Why Was it Built and Who Built It?

John Taylor, 1859

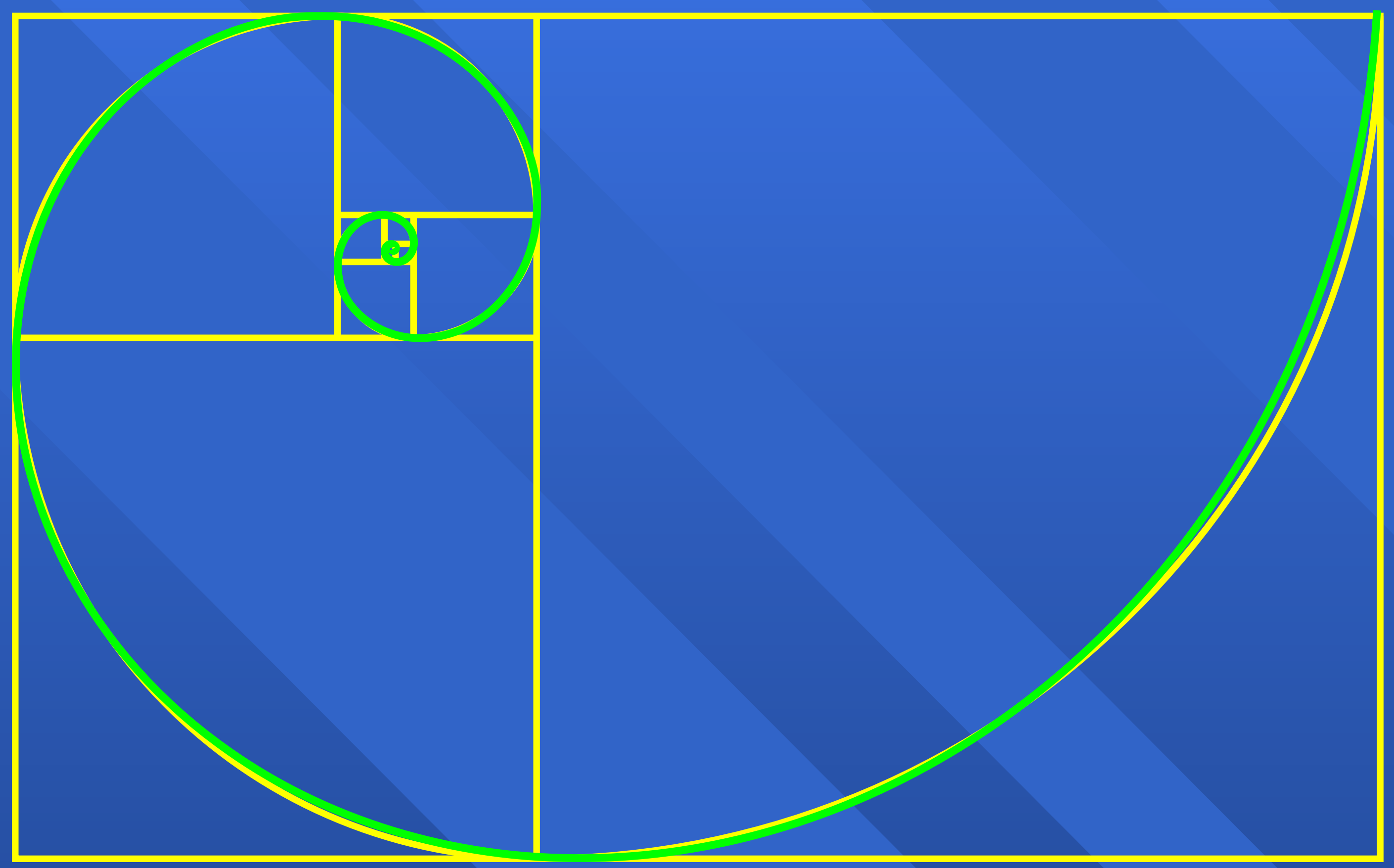


Spirals



The Golden Spiral

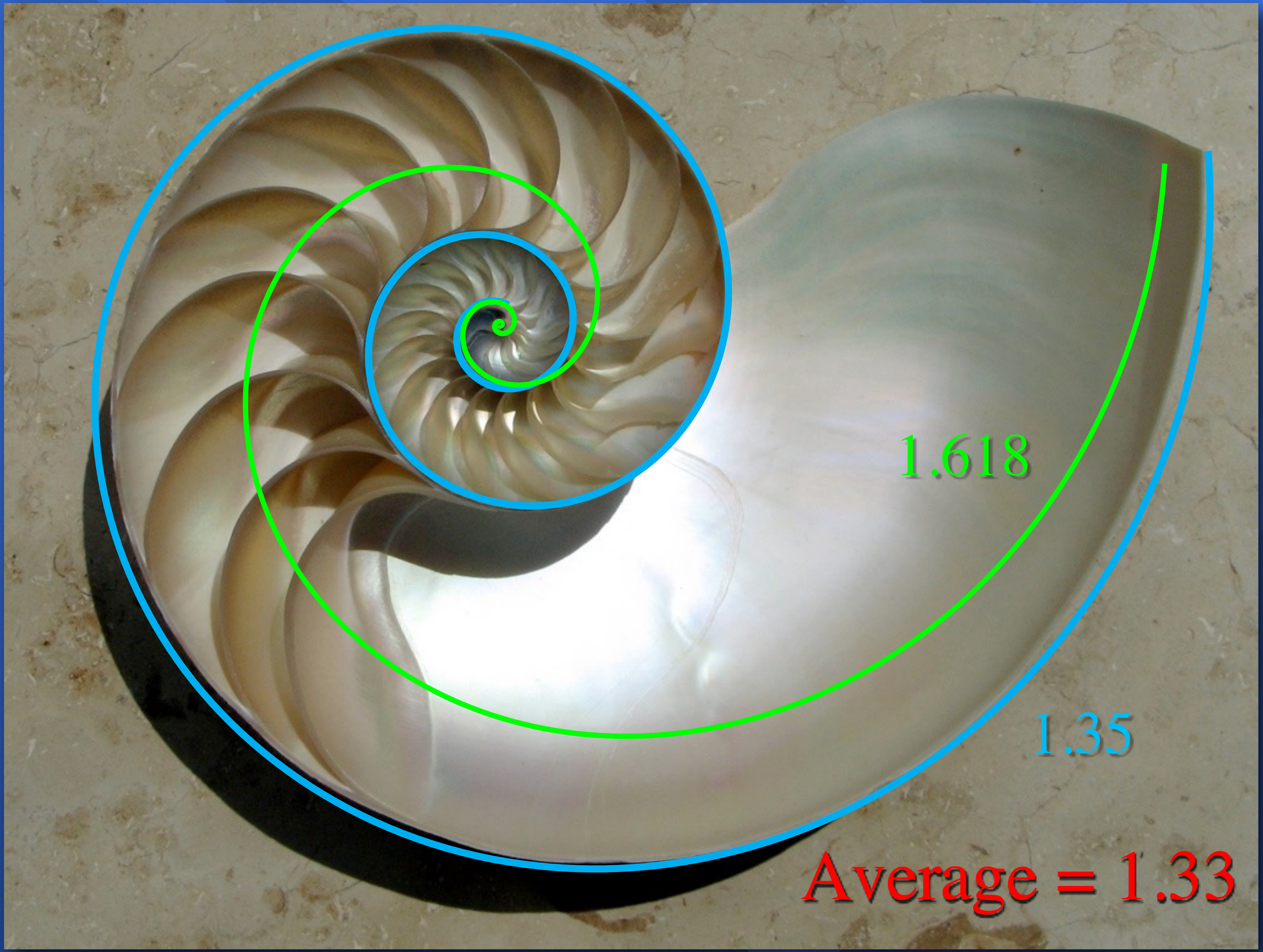
Spirals



The Golden Spiral

Logarithmic Spiral *Spira Mirabilis*

The Chambered Nautilus



Nautilus pompilius

Nested Radicals

$$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}} = ?$$

$$x = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}$$

$$x^2 = 1+\boxed{\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}}$$

$$x^2 = 1+x$$

$$x^2-x-1 = 0 \longrightarrow x = \Phi$$

Nested Radicals

$$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}} = \Phi$$

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$$x^2-x-1 = 0 \longrightarrow x = \Phi$$

Continued Fraction

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}}} = ?$$

$$x = 1+\cfrac{1}{\boxed{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}}}}$$

$$x = 1+\frac{1}{x}$$

$$x^2-x-1 = 0$$

$$x = \Phi$$

Continued Fraction

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}}} = \Phi$$

$$x = 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}}}$$

$$x = 1+\frac{1}{x}$$

$$x^2-x-1 = 0$$

$$x = \Phi$$

Golden Ratio Surprises

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}}} = \Phi$$

$$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}} = \Phi$$

Powers of the Golden Ratio

$\Phi^2 = \Phi + 1$

$\Phi^3 = 2\Phi + 1$

$\Phi^4 = 3\Phi + 2$

$\Phi^5 = 5\Phi + 3$

$\Phi^6 = 8\Phi + 5$

$\Phi^7 = 13\Phi + 8$

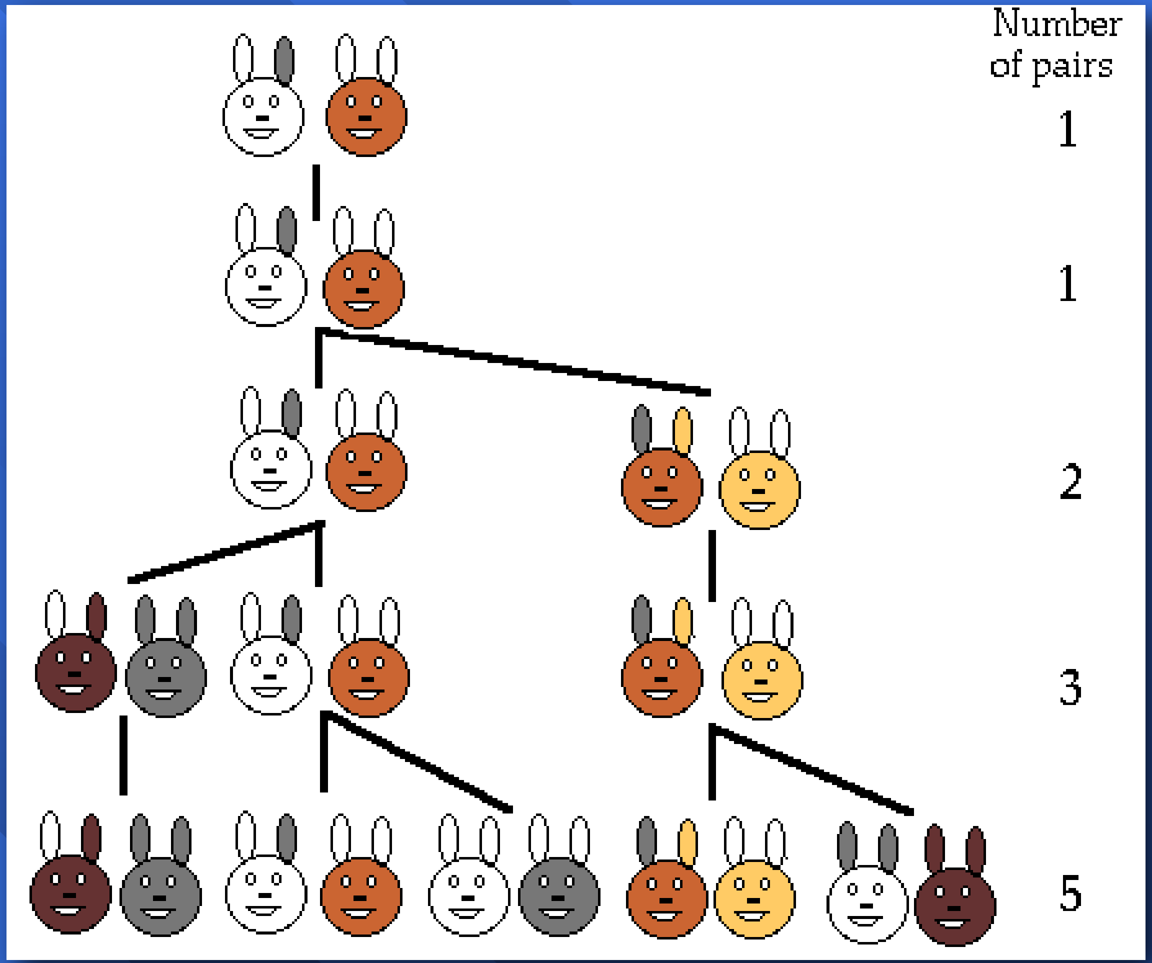
$\Phi^8 = 21\Phi + 13$

Apparently

$$\boxed{\Phi^n = a\Phi + b}$$

Leonardo of Pisa (Fibonacci)

A certain man put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair which from the second month on becomes productive?



1, 1, 2, 3, 5, 8, 13, 21, 34, ...

The Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

F_n represents the *n*th Fibonacci number

$F_1 = 1$ $F_2 = 1$ $F_3 = 2$

$$F_n = F_{n-1} + F_{n-2}\ , \text{ where } n \geq 3$$

$$F_2 = F_1 = 1$$

The *recursive definition* of the Fibonacci sequence.


FoxTrot
by Bill Amend
ONE CHEESY TORTILLA CHIP...
ONE CHEESY TORTILLA CHIP...
TWO CHEESY TORTILLA CHIPS...
THREE CHEESY TORTILLA CHIPS...
FIVE CHEESY TORTILLA CHIPS...
EIGHT CHEESY TORTILLA CHIPS...
MATH GEEKS SHOULDN'T BE ALLOWED ANYWHERE NEAR CERTAIN FOODS.
WHAT'S WRONG WITH FIBONACHOS?
THERE'RE ONLY 12 LEFT...NOW WHAT?

The Fibonacci Metric Converter

$F_1 = 1$
$F_2 = 1$
$F_3 = 2$
$F_4 = 3$
$F_5 = 5$
$F_6 = 8$
$F_7 = 13$
$F_8 = 21$
$F_9 = 34$
$F_{10} = 55$
$F_{11} = 89$
$F_{12} = 144$
$F_{13} = 233$
$F_{14} = 377$

Santa Rosa ⟶ Arcata ~228 miles

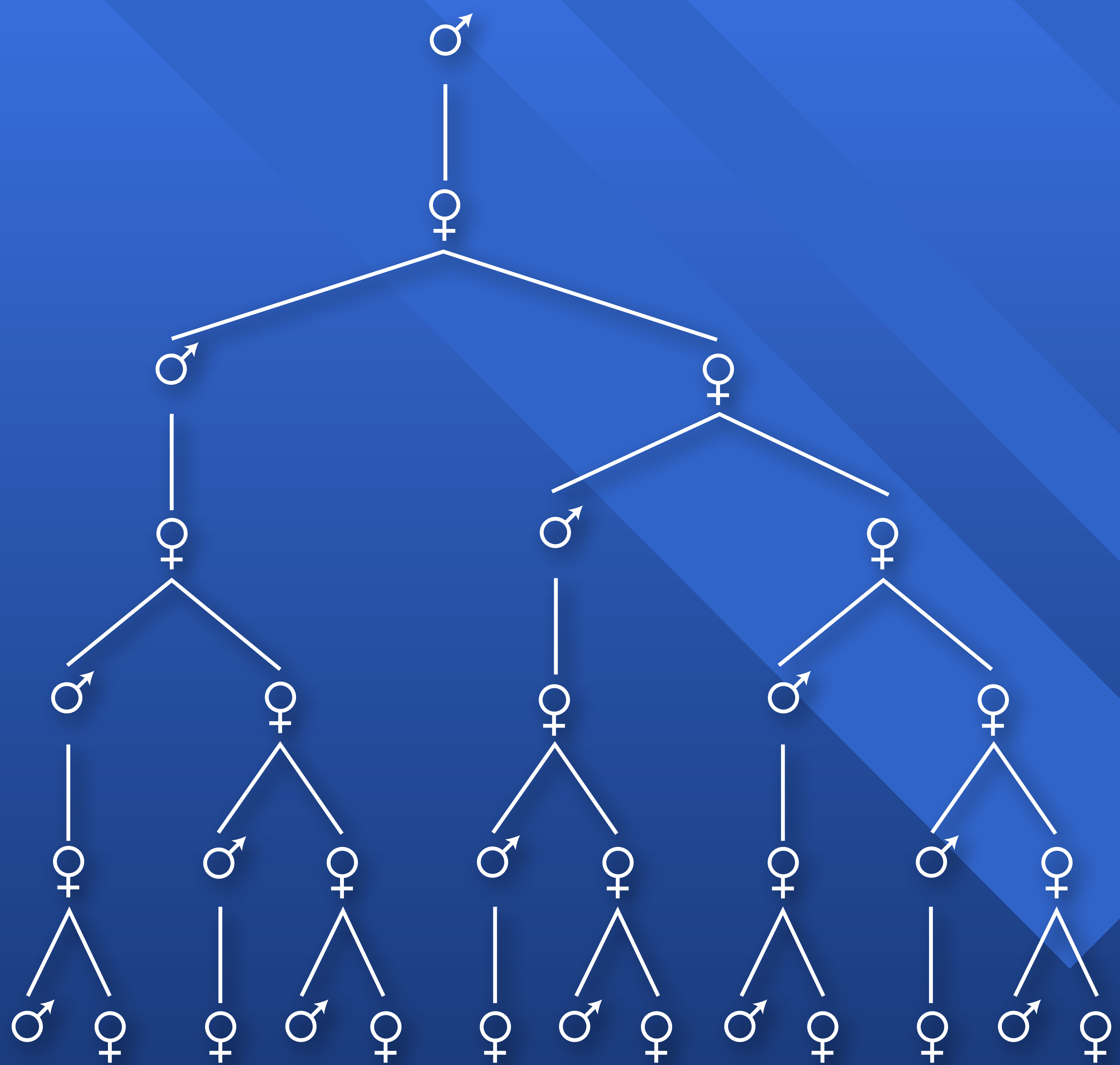
$$228 \text{ mi} = 144 + 55 + 21 + 8$$
$$= F_{12} + F_{10} + F_8 + F_6$$

To convert to kilometers

$$F_{13} + F_{11} + F_9 + F_7 = 233 + 89 + 34 + 13$$
$$= 369 \text{ km}$$

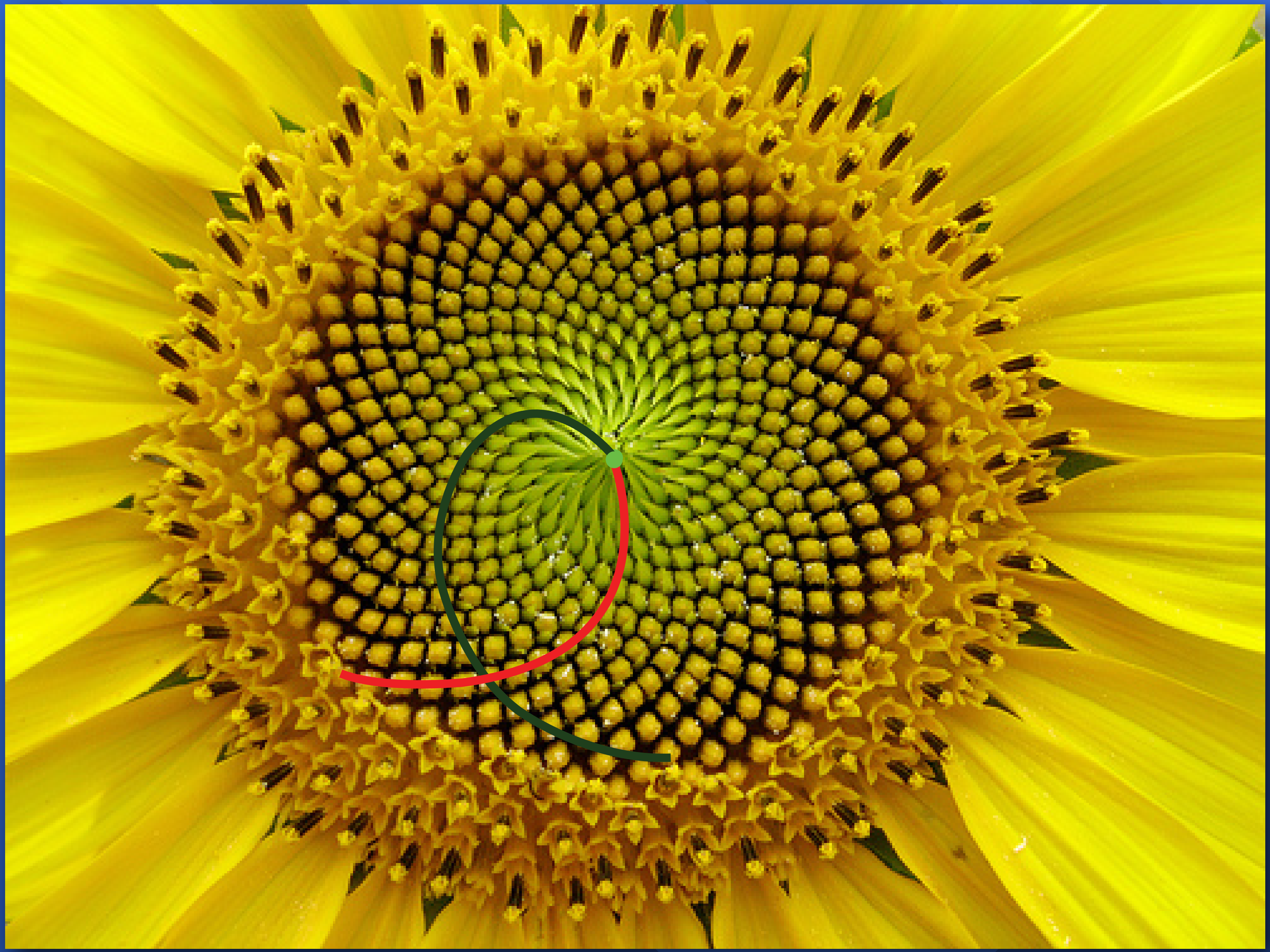
Actually, 228 mi = 367 km

Family Tree of a Male Honey Bee



Female Bees	Male Bees	Total Bees
0	1	1
1	0	1
1	1	2
2	1	3
3	2	5
5	3	8
8	5	13

Sunflower Spirals



21 Green and 34 Red

An Explicit Formula for F_n

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

$$F_{100} = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{100} - \left(\frac{1-\sqrt{5}}{2}\right)^{100}\right]$$

$$= 354{,}224{,}848{,}179{,}261{,}915{,}075$$

An Explicit Formula for F_n

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

Binet's Formula
1786 – 1856

L. Euler – 1765

A. de Moivre – 1730

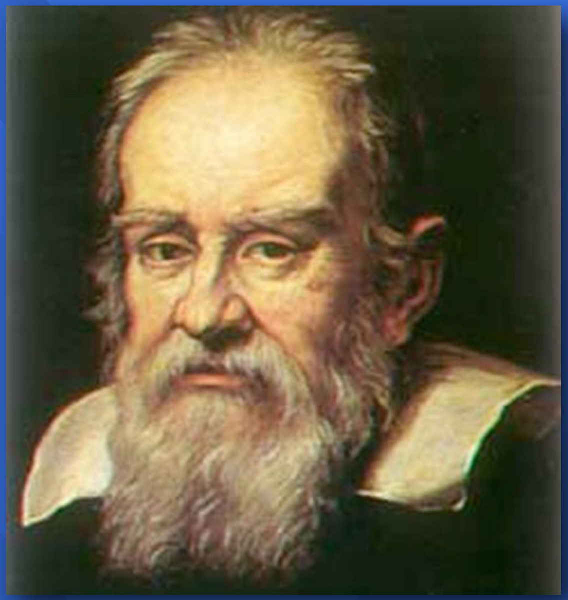
An Explicit Formula for F_n

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

$$F_n = \frac{1}{\sqrt{5}}\left[\Phi^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

$$F_n = \frac{1}{\sqrt{5}}\left[\Phi^n - \left(\frac{-1}{\Phi}\right)^n\right]$$

The Golden Ratio



Galileo Galilei

1564 – 1642

“The great book of nature lies ever open before our eyes and the truths of science are written in it ...

But we cannot read it unless we have first learned the language and the characters in which it is written ...

It is written in mathematical language and the characters are triangles, circles, and other geometrical figures; without whose help it is humanly impossible to understand a single word of it and without which we wander about in a dark maze.”

The Golden Ratio

$$\frac{AC}{CB} = \frac{AB}{AC}$$

$$\Phi = \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\dots}}}}$$