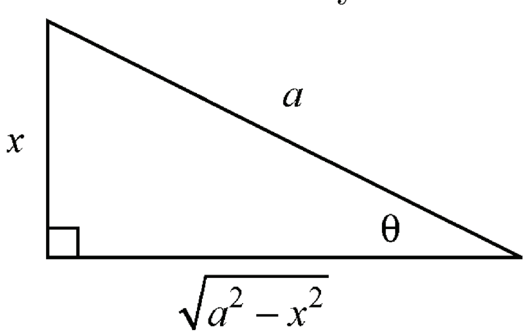
Table of Trigonometric Substitutions

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a \sin\theta, \quad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a \tan\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a \sec\theta, \quad 0 \leqslant \theta < \frac{\pi}{2}$ or $\pi \leqslant \theta < \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$

$$\int \sqrt{a^2 - x^2}\, dx$$

Let $x = a\sin\theta$, so $\sin\theta = \frac{x}{a}$

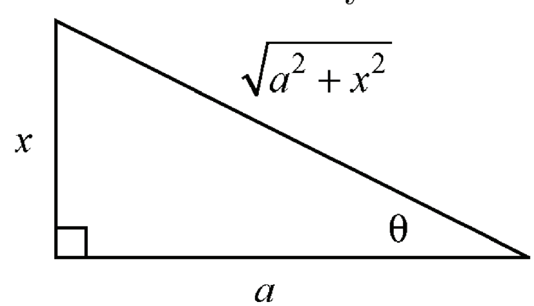
and use the identity $1 - \sin^2\theta = \cos^2\theta$



$$\int \sqrt{a^2 + x^2}\, dx$$

Let $x = a\tan\theta$, so $\tan\theta = \frac{x}{a}$

and use the identity $\tan^2\theta + 1 = \sec^2\theta$



$$\int \sqrt{x^2 - a^2}\, dx$$

Let $x = a\sec\theta$, so $\sec\theta = \frac{x}{a}$

and use the identity $\sec^2\theta - 1 = \tan^2\theta$