7.1

$$\int \sin^n x\,dx$$

$$= \int \underbrace{\sin^{n-1} x}_{u} \cdot \underbrace{\sin x\,dx}_{dv}$$

let

$u = \sin^{n-1} x$

$du = (n-1)\cdot \sin^{n-2} x \cdot \cos x\,dx$

$dv = \sin x\,dx$

$v = -\cos x$

$$u \cdot v - \int v\,du$$

$$= -\sin^{n-1} x \cdot \cos x + (n-1)\int \sin^{n-2} x \cdot \cos^2 x\,dx$$

$$= -\sin^{n-1} x \cdot \cos x + (n-1)\int \sin^{n-2} x\,(1-\sin^2 x)\,dx$$

$$= -\sin^{n-1} x \cdot \cos x + (n-1)\int \sin^{n-2} x\,dx - (n-1)\int \sin^n x\,dx$$

$$(n-1)\int \sin^n x\,dx + \int \sin^n x\,dx = -\sin^{n-1} x \cdot \cos x + (n-1)\int \sin^{n-2} x\,dx$$

$$n\cdot \int \sin^n x\,dx = -\sin^{n-1} x \cdot \cos x + (n-1)\cdot \int \sin^{n-2} x\,dx$$

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x \cdot \cos x + \frac{n-1}{n}\cdot \int \sin^{n-2} x\,dx$$

7.1

(3)

$$\int \cos^n x \, dx$$

$$\int \underbrace{\cos^{n-1} x}_{u} \cdot \underbrace{\cos x \, dx}_{dv}$$

Let

$u = \cos^{n-1} x$

$du = (n-1)\cos^{n-2} x \cdot (-\sin x)$

$dv = \cos x \, dx$

$v = \sin x$

$$= \cos^{n-1} x \cdot \sin x + (n-1) \int \cos^{n-2} x \cdot \sin^2 x \, dx$$

$$= \cos^{n-1} x \cdot \sin x + (n-1) \cdot \int \cos^{n-2} x \left(1 - \cos^2 x\right) dx$$

$$= \cos^{n-1} x \cdot \sin x + (n-1) \cdot \int \cos^{n-2} x \, dx - (n-1) \int \cos^n x \, dx$$

$$(n-1) \int \cos^n x \, dx + \int \cos^n x \, dx = \cos^{n-1} x \cdot \sin x + (n-1) \int \cos^{n-2} x \, dx$$

$$n \int \cos^n x \, dx = \cos^{n-1} x \cdot \sin x + (n-1) \cdot \int \cos^{n-2} x \, dx$$

$$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \cdot \sin x + \frac{n-1}{n} \cdot \int \cos^{n-2} x \, dx$$

7.1

(4)

$$\int \sec^n x\,dx$$

$$= \int \underbrace{\sec^{n-2} x}_{u} \cdot \underbrace{\sec^2 x\,dx}_{dv}$$

$$u = \sec^{n-2} x$$

$$du = (n-2)\sec^{n-3} x \cdot \sec x \cdot \tan x\,dx$$

$$du = (n-2)\sec^{n-2} x \cdot \tan x\,dx$$

$$dv = \sec^2 x\,dx$$

$$v = \tan x$$

$$u \cdot v - \int v\,du$$

$$= \sec^{n-2} x \cdot \tan x - (n-2)\int \sec^{n-2} x \cdot \tan^2 x\,dx$$

$$= \sec^{n-2} x \cdot \tan x - (n-2)\int \sec^{n-2} x \cdot (\sec^2 x - 1)\,dx$$

$$= \sec^{n-2} x \cdot \tan x - (n-2)\int \sec^n x\,dx + (n-2)\int \sec^{n-2} x\,dx$$

$$\int \sec^n x\,dx + (n-2)\int \sec^n x\,dx = \sec^{n-2}x \cdot \tan x + (n-2)\int \sec^{n-2}x\,dx$$

$$(n-1)\int \sec^n x\,dx = \sec^{n-2}\cdot \tan x + (n-2)\int \sec^{n-2}x\,dx$$

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2}\cdot \tan x + \frac{n-2}{n-1}\cdot \int \sec^{n-2}x\,dx$$

$n \geq 2$