Fringe Field of Parallel Plate Capacitor

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Introduction

A parallel plate capacitor with variable separation is the standard apparatus used in physics labs to demonstrate the effect of the capacitor's geometry (plate area and plate separation) on capacitance. The theoretical equation for the parallel plate capacitor,

$$C_0 = \varepsilon_0 A/d$$  \hspace{1cm} 1

where $A$ represents the area and $d$ the separation distance between two plates, suggests a simple $1/x$ curvature for the plot of the measured capacitance versus separation distance. However, after a few millimeters of separation distance, the measured capacitance starts to deviate from the theoretical value. This deviation can reach as high as 10 times the theoretical value for a separation comparable to the dimensions of the plates. Obviously, this deviation is due to the fringing effect of the field between the edges of two plates.

There are several complicated approaches for including the fringing effect $^{1,2,3}$, which are beyond the scope of practicality in a sophomore Physics lab. The simplest formula to account for the fringing effect of a circular disk parallel plate capacitor is $^4$:

$$C' = \varepsilon_0\left[(\pi r^2/d) + r\ln(16\pi r/d-1)\right]$$  \hspace{1cm} 2

In this equation, $r$ and $d$ represent the radius and the separation distance for a circular disk parallel plate capacitor, respectively, and $\varepsilon_0$ is the susceptibility of the free space and equal to $8.85 \times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)$.

Although this equation improves the calculated capacitance, it is far from being even close to the measured value.

An empirical approach is used to equate the measured capacitance to its theoretical value by replacing $A$, the plate area, with $A_{\text{effective}}$, the effective area of the plate in the form of:

$$C'' = \varepsilon_0 A_{\text{effective}}/d.$$  \hspace{1cm} 3

This is similar to the correction being used for the length of a tube in the standing wave experiments with sound.
**Procedure**

A Pasco model 9043 circular disk parallel plate capacitor (PPC) was used in the configuration shown here. The resonant frequency of the RLC circuit was used in the equation: \( LC = \frac{1}{(2\pi f)^2} \) to find the capacitance \( C \). The inductance value, \( L \), can accurately be measured using a Z-meter. In this circuit, channel A of the oscilloscope was sensing the current in the circuit by measuring the voltage across the resistor. Channel B on the other hand was measuring the voltage across the entire RLC circuit. The oscilloscope was set in AB mode to create a Lissajous pattern, which becomes a straight line at the resonant frequency.

At each separation distance \( d \), the frequency of the function generator was changed until the resonant frequency of the circuit was found.

**Results**

Using equation 3 above, the measured values of the capacitance, \( C \), as a function of the separation distance were used to generate \( A_{\text{effective}} \) values. After plotting the \( A_{\text{effective}} / A \) ratio against \( d \), a linear fit to the data yielded an empirical equation relating the \( A_{\text{effective}} \) to the actual area of the plate, \( A \) and the separation distance \( d \) according to:

\[
A_{\text{effective}} = A(74.5d + 0.82)
\]

in the M.K.S. system of units (see Figure 1). Notice that the ratio of the \( A_{\text{effective}} / A \) is simply the ratio of the measured capacitance, \( C \), to the theoretical capacitance, \( C_0 \).
Plots of $C$ (measured capacitance), $C_0$ (calculated from equation 1), $C'$ (calculated from equation 2) and $C''$ (calculated from equation 3) versus the separation distance for a PPC with a plate radius of 0.1 m is shown in figure 2.

To test equation 4 for PPC of a different $A$, the PPC that had a radius of 0.1m was modified to have plates with a radius of 0.07m. The result of the experiment for this unit is shown in Figure 3.

As the plots in figures 2 and 3 suggest, the empirical correction of equation 4 is a good approximation for accounting for the fringing field of PPC and can be used as an added step in the PPC experiment.

References

Linear fit to the $A_{\text{effective}}/A$ vs distance plot

$A_{\text{effective}}/A = 74.5d + 0.82$

Figure 1: Linear curve fit to the plot of $A_{\text{effective}}/A$ ratio against separation distance $d$
Figure 2: Plots of capacitances from measured values (C), Theoretical values from equation 1 (C0), corrected values from equation 2 (C') and Empirically corrected values from equation 3 (C'') against separation distance d for the first plate with a radius of 0.1 m.
Figure 2: Plots of capacitances from measured values (C), Theoretical values from equation 1 (C0), corrected values from equation 2 (C’), and Empirically corrected values from equation 3 (C’’) against separation distance d for the second plate with a radius of 0.07 m.