Physics 1 HW #8: Chapter 3

Problems 26-29, 31, 32, 49, 54.
For EVERY one of the problems, draw vector diagrams, labeling the vectors and write out the vector equation! Have fun!

3.26 We use the following notation:
\[ \vec{v}_{bs} = \text{velocity of boat relative to the shore}\]
\[ \vec{v}_{bw} = \text{velocity of boat relative to the water}, \]
and \[ \vec{v}_{ws} = \text{velocity of water relative to the shore}. \]

If we take downstream as the positive direction, then \( \vec{v}_{ws} = +1.5 \text{ m/s} \) for both parts of the trip. Also, \( \vec{v}_{bw} = +10 \text{ m/s} \) while going downstream and \( \vec{v}_{bw} = -10 \text{ m/s} \) for the upstream part of the trip.

The velocity of the boat relative to the shore is given by \[ \vec{v}_{bs} = \vec{v}_{bw} + \vec{v}_{ws} \]

While going downstream, \( v_{bs} = 10 \text{ m/s} + 1.5 \text{ m/s} \) and the time to go 300 m downstream is
\[ t_{down} = \frac{300 \text{ m}}{(10+1.5) \text{ m/s}} = 26 \text{ s} \]

When going upstream, \( v_{bs} = -10 \text{ m/s} + 1.5 \text{ m/s} = -8.5 \text{ m/s} \) and the time required to move 300 m upstream is \[ t_{up} = \frac{-300 \text{ m}}{-8.5 \text{ m/s}} = 35 \text{ s} \]

The time for the round trip is \( t = t_{down} + t_{up} = (26+35) \text{ s} = 61 \text{ s} \)
Prior to the leap, the salmon swims upstream through water flowing at speed \( |\mathbf{v}_{\text{WE}}| = 1.50 \, \text{m/s} \) relative to Earth. The fish swims at \( |\mathbf{v}_{\text{FW}}| = 6.26 \, \text{m/s} \) relative to the water in such a direction to make its velocity relative to Earth, \( \mathbf{v}_{\text{FE}} \), vertical. Since \( \mathbf{v}_{\text{FE}} = \mathbf{v}_{\text{FW}} + \mathbf{v}_{\text{WE}} \) as shown in the diagram at the right, we find that

\[
\theta = \cos^{-1} \left( \frac{|\mathbf{v}_{\text{WE}}|}{|\mathbf{v}_{\text{FW}}|} \right) = \cos^{-1} \left( \frac{1.50 \, \text{m/s}}{6.26 \, \text{m/s}} \right) = 76.1^\circ
\]

and the vertical velocity of the fish as it leaves the water is

\[
v_{0y} = |\mathbf{v}_{\text{FE}}| = |\mathbf{v}_{\text{FW}}| \sin \theta = (6.26 \, \text{m/s}) \sin 76.1^\circ = 6.08 \, \text{m/s}
\]

The height of the salmon above the water at the top of its leap (that is, when \( v_y = 0 \)) is given by

\[
\Delta y = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (6.08 \, \text{m/s})^2}{2(-9.80 \, \text{m/s}^2)} = 1.88 \, \text{m}
\]

3.28 \( \mathbf{v}_{\text{BW}} = 10 \, \text{m/s} \), directed northward, is the velocity of the boat relative to the water.

\( \mathbf{v}_{\text{WS}} = 1.5 \, \text{m/s} \), directed eastward, is the velocity of the water relative to shore.

\( \mathbf{v}_{\text{BS}} \) is the velocity of the boat relative to shore, and directed at an angle of \( \theta \), relative to the northward direction as shown.

\[
\mathbf{v}_{\text{BS}} = \mathbf{v}_{\text{BW}} + \mathbf{v}_{\text{WS}}
\]

The northward component of \( \mathbf{v}_{\text{BS}} \) is \( v_{\text{BS}} \cos \theta = v_{\text{BW}} = 10 \, \text{m/s} \) \hspace{1cm} (1)

The eastward component is \( v_{\text{BS}} \sin \theta = v_{\text{WS}} = 1.5 \, \text{m/s} \) \hspace{1cm} (2)

(a) Dividing equation (2) by equation (1) gives

\[
\theta = \tan^{-1} \left( \frac{v_{\text{WS}}}{v_{\text{BW}}} \right) = \tan^{-1} \left( \frac{1.50}{10.0} \right) = 8.53^\circ
\]

From equation (1), \( v_{\text{BS}} = \frac{10 \, \text{m/s}}{\cos 8.53^\circ} = 10.1 \, \text{m/s} \)

Therefore, \( \mathbf{v}_{\text{BS}} = 10.1 \, \text{m/s} \) at 8.53° E of N
(b) The time to cross the river is \( t = \frac{300 \text{ m}}{v_{\text{BS}} \cos \theta} = \frac{300 \text{ m}}{10.0 \text{ m/s}} = 30.0 \text{ s} \) and the downstream drift of the boat during this crossing is

\[
drift = (v_{\text{BS}} \sin \theta)t = (1.50 \text{ m/s})(30.0 \text{ s}) = 45.0 \text{ m}
\]

3.29 \( \vec{v}_{\text{BW}} \) = velocity of boat relative to the water,

\( \vec{v}_{\text{WS}} \) = velocity of water relative to the shore

and \( \vec{v}_{\text{BS}} \) = velocity of boat relative to the shore.

\( \vec{v}_{\text{BS}} = \vec{v}_{\text{BW}} + \vec{v}_{\text{WS}} \) as shown in the diagram.

The northward (that is, cross-stream) component of \( \vec{v}_{\text{BS}} \) is

\[
(\vec{v}_{\text{BS}})_{\text{north}} = (v_{\text{BW}}) \sin 62.5^\circ + 0 = (3.30 \text{ mi/h}) \sin 62.5^\circ + 0 = 2.93 \text{ mi/h}
\]

The time required to cross the stream is then \( t = \frac{0.505 \text{ mi}}{2.93 \text{ mi/h}} = 0.173 \text{ h} \)

The eastward (that is, downstream) component of \( \vec{v}_{\text{BS}} \) is

\[
(\vec{v}_{\text{BS}})_{\text{east}} = -(v_{\text{BW}}) \cos 62.5^\circ + v_{\text{WS}}
\]

\[
= -(3.30 \text{ mi/h}) \cos 62.5^\circ + 1.25 \text{ mi/h} = -0.274 \text{ mi/h}
\]

Since the last result is negative, it is seen that the boat moves upstream as it crosses the river. The distance it moves upstream is

\[
d = |(\vec{v}_{\text{BS}})_{\text{east}}|t = (0.274 \text{ mi/h})(0.173 \text{ h}) = 4.72 \times 10^{-2} \text{ mi} \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right) = 249 \text{ ft}
\]
Choose the positive direction to be the direction of each car’s motion relative to Earth. The velocity of the faster car relative to the slower car is given by $\vec{v}_{FS} = \vec{v}_{FE} + \vec{v}_{ES}$, where $\vec{v}_{FE} = +60.0 \text{ km/h}$ is the velocity of the faster car relative to Earth and $\vec{v}_{ES} = -\vec{v}_{SE} = -40.0 \text{ km/h}$ is the velocity of Earth relative to the slower car.

Thus, $\vec{v}_{FS} = +60.0 \text{ km/h} - 40.0 \text{ km/h} = +20.0 \text{ km/h}$ and the time required for the faster car to move 100 m (0.100 km) closer to the slower car is

$$t = \frac{d}{v_{FS}} = \frac{0.100 \text{ km}}{20.0 \text{ km/h}} = 5.00 \times 10^{-3} \text{ h} \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 18.0 \text{ s}$$

3.32 $\vec{v}_{bc} = \text{ the velocity of the ball relative to the car}$

$\vec{v}_{ce} = \text{ velocity of the car relative to Earth} = 10 \text{ m/s}$

$\vec{v}_{be} = \text{ the velocity of the ball relative to Earth}$

These velocities are related by the equation $\vec{v}_{be} = \vec{v}_{bc} + \vec{v}_{ce}$ as illustrated in the diagram.

Considering the horizontal components, we see that

$$v_{be} \cos 60.0^\circ = v_{ce} \quad \text{or} \quad v_{be} = \frac{v_{ce}}{\cos 60.0^\circ} = \frac{10.0 \text{ m/s}}{\cos 60.0^\circ} = 20.0 \text{ m/s}$$

From the vertical components, the initial velocity of the ball relative to Earth is

$$v_{be} = v_{bc} \sin 60.0^\circ = 17.3 \text{ m/s}$$

Using $v_y^2 = v_{0y}^2 + 2a_y (\Delta y)$, with $v_y = 0$ when the ball is at maximum height, we find

$$\left( \Delta y \right)_{max} = \frac{0 - v_{0y}^2}{2a_y} = \frac{0 - v_{be}^2}{2(-g)} = \frac{(17.3 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 15.3 \text{ m}$$

as the maximum height the ball rises.
3.49 The velocity of the boat relative to the shore is \( \mathbf{v}_{bs} = \mathbf{v}_{bw} + \mathbf{v}_{ws} \), where \( \mathbf{v}_{bw} \) is the velocity of the boat relative to the water and \( \mathbf{v}_{ws} \) is the velocity of the water relative to shore.

In order to cross the river (flowing parallel to the banks) in minimum time, the velocity of the boat relative to the water must be perpendicular to the banks. That is, \( \mathbf{v}_{bw} \) must be perpendicular to \( \mathbf{v}_{ws} \). Hence, the velocity of the boat relative to the shore must be

\[
\mathbf{v}_{bs} = \sqrt{v_{bw}^2 + v_{ws}^2} = \sqrt{(12 \text{ km/h})^2 + (5.0 \text{ km/h})^2} = 13 \text{ km/h}
\]

at \( \theta = \tan^{-1} \left( \frac{v_{bw}}{v_{ws}} \right) = \tan^{-1} \left( \frac{12 \text{ km/h}}{5.0 \text{ km/h}} \right) = 67^\circ \) to the direction of the current in the river (which is the same as the line of the riverbank).

The minimum time to cross the river is

\[
t = \frac{\text{width of river}}{v_{bw}} = \frac{1.5 \text{ km}}{12 \text{ km/h}} \left( \frac{60 \text{ min}}{1 \text{ h}} \right) = \boxed{7.5 \text{ min}}
\]

During this time, the boat drifts downstream a distance of

\[
d = v_{ws} t = (5.0 \text{ km/h}) \left(7.5 \text{ min}\right) \left( \frac{1 \text{ h}}{60 \text{ min}} \right) \left( \frac{10^3 \text{ m}}{1 \text{ km}} \right) = 6.3 \times 10^2 \text{ m}
\]

3.54 We are given that:

\( \mathbf{v}_{be} = 20 \text{ knots due north} \) (velocity of boat relative to Earth)

and

\( \mathbf{v}_{we} = 17 \text{ knots due east} \) (velocity of wind relative to Earth)
The velocity of the wind relative to the boat is

$$\vec{v}_{WB} = \vec{v}_{WE} + \vec{v}_{EB}$$

where $\vec{v}_{EB} = -\vec{v}_{BE} = 20 \text{ knots south}$ is the velocity of Earth relative to the boat. The vector diagram above shows this vector addition.

Since the vector triangle is a $90^\circ$ triangle, we find the magnitude of $\vec{v}_{WB}$ to be

$$v_{WB} = \sqrt{v_{WE}^2 + v_{EB}^2} = \sqrt{(17 \text{ knots})^2 + (20 \text{ knots})^2} = 26 \text{ knots}$$

and the direction is given by

$$\theta = \tan^{-1}\left(\frac{v_{EB}}{v_{WE}}\right) = \tan^{-1}\left(\frac{20 \text{ knots}}{17 \text{ knots}}\right) = 50^\circ$$

Thus, $\vec{v}_{WB} = 26 \text{ knots at } 50^\circ \text{ south of east}$

From the vector diagram above, the component of this velocity parallel to the motion of the boat (that is, parallel to a north-south line) is seen to be $\vec{v}_{EB} = -\vec{v}_{BE} = 20 \text{ knots south}$