Problems 4.7.8.9.14.20.22.24.29.33.35.38.40.45.

4. How fast must a meter stick be moving if its length is measured to shrink to 0.500 m?

\[ L = L_p \sqrt{1 - \frac{v^2}{c^2}} \]
\[ v = c \sqrt{1 - \left( \frac{L}{L_p} \right)^2} \]

Taking \( L = \frac{L_p}{2} \) where \( L_p = 1.00 \text{ m} \) gives
\[ v = c \sqrt{1 - \frac{1}{4}} = 0.866c \]

7. An atomic clock moves at 1 000 km/h for 1.00 h as measured by an identical clock on the Earth. How many nanoseconds slow will the moving clock be compared with the Earth clock, at the end of the 1.00-h interval?

\[ \Delta t = \gamma \Delta t_p = \frac{\Delta t_p}{\sqrt{1 - \frac{v^2}{c^2}}} \]
\[ \Delta t_p = \left( \sqrt{1 - \frac{v^2}{c^2}} \right) \Delta t \approx \left( 1 - \frac{v^2}{2c^2} \right) \Delta t \]

\[ \Delta t - \Delta t_p = \left( \frac{v^2}{2c^2} \right) \Delta t \]

If
\[ v = 1 000 \text{ km/h} = \frac{1.00 \times 10^6}{3 600} \text{ m/s} = 277.8 \text{ m/s} \]

Then
\[ \frac{v}{c} = 9.26 \times 10^{-7} \]

And
\[ (\Delta t - \Delta t_p) = \left( 4.28 \times 10^{-13} \right) (3 600 \text{ s}) = 1.54 \times 10^{-9} \text{ s} = 1.54 \text{ ns} \]

8. A muon formed high in the Earth’s atmosphere travels at speed \( v = 0.990c \) for a distance of 4.60 km before it decays into an electron, a neutrino, and an antineutrino \((\mu^- \rightarrow e^- + \nu + \bar{\nu})\). (a) How long does the muon live, as measured in its reference frame? (b) How far does the muon travel, as measured in its frame?

\[ \frac{v}{c} = 0.990, \quad \gamma = 7.09 \]

(a) The muon’s lifetime as measured in the Earth’s rest frame is
\[ \Delta t = \frac{4.60 \text{ km}}{0.990c} \]

And the lifetime measured in the muon’s rest frame is
\[ \Delta t_p = \frac{\Delta t}{\gamma} = \frac{1}{7.09} \left[ \frac{4.60 \times 10^3 \text{ m}}{0.990 \times 3.00 \times 10^8 \text{ m/s}} \right] = 2.18 \mu s \]

(b) \[ L = L_p \sqrt{1 - \left( \frac{v}{c} \right)^2} = L_p \frac{1}{\gamma} = \frac{4.60 \times 10^3 \text{ m}}{7.09} = 649 \text{ m} \]
9. A spacecraft with a proper length of 300 m takes 0.750 μs to pass an Earth observer. Determine the speed of the spacecraft as measured by the Earth observer.

P39.9 The spaceship is measured by the Earth observer to be length-contracted to
\[ L = L_p \sqrt{1 - \frac{v^2}{c^2}} \quad \text{or} \quad L^2 = L_p^2 \left(1 - \frac{v^2}{c^2}\right) \]

Also, the contracted length is related to the time required to pass overhead by:
\[ L = vt \quad \text{or} \quad L^2 = \frac{v^2 t^2}{c^2} = \left(\frac{v}{c}\right)\frac{c^2}{c^2} \]

Equating these two expressions gives
\[ L_p^2 - L_p^2 \left(\frac{v}{c}\right)^2 = \left(\frac{v}{c}\right)^2 \]

or
\[ \left[ L_p^2 + \left(\frac{v}{c}\right)^2 \right] \frac{c^2}{c^2} = L_p^2 \]

Using the given values: \( L_p = 300 \text{ m} \) and \( t = 7.50 \times 10^{-7} \text{ s} \)

this becomes \( \left(1.41 \times 10^5 \text{ m}^2\right) \frac{c^2}{c^2} = 9.00 \times 10^4 \text{ m}^2 \)

giving \( v = \boxed{0.800c} \)

14. The identical twins Speedo and Goslo join a migration from the Earth to Planet X. It is 20.0 ly away in a reference frame in which both planets are at rest. The twins, of the same age, depart at the same time on different spacecraft. Speedo’s craft travels steadily at 0.950c, and Goslo’s at 0.750c. Calculate the age difference between the twins after Goslo’s spacecraft lands on Planet X. Which twin is the older?

P39.14 In the Earth frame, Speedo’s trip lasts for a time
\[ \Delta t = \frac{\Delta x}{v} = \frac{20.0 \text{ ly}}{0.950 \text{ ly/yr}} = 21.05 \text{ yr} \]

Speedo’s age advances only by the proper time interval
\[ \Delta t_p = \frac{\Delta t}{\gamma} = 21.05 \text{ yr} \sqrt{1 - 0.95^2} = 6.574 \text{ yr} \text{ during his trip} \]

Similarly for Goslo,
\[ \Delta t_p = \frac{\Delta x}{v} \sqrt{1 - \frac{v^2}{c^2}} = \frac{20.0 \text{ ly}}{0.750 \text{ ly/yr}} \sqrt{1 - 0.75^2} = 17.64 \text{ yr} \]

While Speedo has landed on Planet X and is waiting for his brother, he ages by
\[ \frac{20.0 \text{ ly}}{0.750 \text{ ly/yr}} - \frac{20.0 \text{ ly}}{0.950 \text{ ly/yr}} = 5.614 \text{ yr} \]

Then \( \boxed{Goslo} \) ends up older by 17.64 yr – (6.574 yr + 5.614 yr) = 5.45 yr.
20. A moving rod is observed to have a length of 2.00 m and to be oriented at an angle of 30.0° with respect to the direction of motion, as shown in Figure P39.23. The rod has a speed of 0.995c. (a) What is the proper length of the rod? (b) What is the orientation angle in the proper frame?

\[ \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.995^2}} = 10.0 \]

We are also given: \( L_1 = 2.00 \text{ m} \), and \( \theta = 30.0^\circ \) (both measured in a reference frame moving relative to the rod).

Thus,
\[ L_{1x} = L_1 \cos \theta_1 = (2.00 \text{ m})(0.867) = 1.73 \text{ m} \]

and
\[ L_{1y} = L_1 \sin \theta_1 = (2.00 \text{ m})(0.500) = 1.00 \text{ m} \]

L_{2x} is a proper length, related to \( L_{1x} \) by
\[ L_{1x} = \frac{L_{2x}}{\gamma} \]

Therefore,
\[ L_{2x} = 10.0L_{1x} = 17.3 \text{ m} \]

and
\[ L_{2y} = L_{1y} = 1.00 \text{ m} \]

(Lengths perpendicular to the motion are unchanged).

(a) \[ L_2 = \sqrt{(L_{2x})^2 + (L_{2y})^2} \]
\[ L_2 = \sqrt{(17.3 \text{ m})^2 + (1.00 \text{ m})^2} \]
\[ L_2 = 17.4 \text{ m} \]

(b) \[ \theta_2 = \tan^{-1} \frac{L_{2y}}{L_{2x}} \]
\[ \theta_2 = \tan^{-1} \frac{1.00 \text{ m}}{17.3 \text{ m}} \]
\[ \theta_2 = 3.30^\circ \]

22 A red light flashes at position \( x_R = 3.00 \text{ m} \) and time \( t_R = 1.00 \times 10^{-9} \text{ s} \), and a blue light flashes at \( x_B = 5.00 \text{ m} \) and \( t_B = 9.00 \times 10^{-9} \text{ s} \), all measured in the S reference frame. Reference frame S’ has its origin at the same point as S at \( t = t' = 0 \); frame S’ moves uniformly to the right. Both flashes are observed to occur at the same place in S’. (a) Find the relative speed between S and S’. (b) Find the location of the two flashes in frame S’. (c) At what time does the red flash occur in the S’ frame?

\[ \gamma = \frac{\gamma}{\sqrt{1 - \left( \frac{2.50 \times 10^8 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} \right)^2}} = 1.81 \]

\[ \gamma = \left[ 2.00 \text{ m} - \gamma \left( 8.00 \times 10^{-9} \text{ s} \right) \right] \]

(b) \[ x = \gamma \left( \gamma - \frac{v}{c^2} \right) : \]
\[ x' = 1.81 \left[ 3.00 \text{ m} - \left( 2.50 \times 10^8 \text{ m/s} \right) \left( 1.00 \times 10^{-9} \text{ s} \right) \right] \]
\[ x' = 4.97 \text{ m} \]

(c) \[ t' = \gamma \left( t - \frac{v}{c^2} \right) : \]
\[ t' = 1.81 \left[ 1.00 \times 10^{-9} \text{ s} - \left( \frac{2.50 \times 10^8 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} \right) \left( 3.00 \text{ m} \right) \right] \]
\[ t' = -1.33 \times 10^{-8} \text{ s} \]
24. A Klingon spacecraft moves away from the Earth at a speed of 0.800c (Fig. P39.26). The starship Enterprise pursues at a speed of 0.900c relative to the Earth. Observers on the Earth see the Enterprise overtaking the Klingon craft at a relative speed of 0.100c. With what speed is the Enterprise overtaking the Klingon craft as seen by the crew of the Enterprise?

\[ u'_x = \frac{u_x - v}{\sqrt{1 - u_x v/c^2}} = \frac{0.900c - 0.800c}{\sqrt{1 - (0.900)(0.800)}} = 0.357c \]

29. An unstable particle at rest breaks into two fragments of unequal mass. The mass of the first fragment is \(2.50 \times 10^{-28}\) kg, and that of the other is \(1.67 \times 10^{-27}\) kg. If the lighter fragment has a speed of 0.893c after the breakup, what is the speed of the heavier fragment?

Relativistic momentum of the system of fragments must be conserved. For total momentum to be zero after as it was before, we must have, with subscript 2 referring to the heavier fragment, and subscript 1 to the lighter, \(p_2 = p_1\)

\[ \gamma_2 m_2 u_2 = \gamma_1 m_1 u_1 = \frac{2.50 \times 10^{-26} \text{ kg}}{\sqrt{1 - (0.893c)^2}} \times (0.893c) \]

or

\[ \frac{(1.67 \times 10^{-27} \text{ kg}) u_2}{\sqrt{1 - (u_2/c)^2}} = (4.960 \times 10^{-28} \text{ kg})c \]

Proceeding to solve, we find

\[ \left( \frac{1.67 \times 10^{-27} u_2}{4.960 \times 10^{-28} c} \right)^2 = 1 - \frac{u_2^2}{c^2} \]

\[ 12.3 \frac{u_2^2}{c^2} = 1 \text{ and } u_2 = 0.285c \]

33. Find the momentum of a proton in MeV/c units assuming its total energy is twice its rest energy.

\[ E = \gamma m c^2 = 2mc^2 \text{ or } \gamma = 2 \]

Thus,

\[ \frac{u}{c} = \sqrt{1 - \left( \frac{1}{\gamma} \right)^2} = \frac{\sqrt{3}}{2} \text{ or } u = \frac{c\sqrt{3}}{2} \]

The momentum is then

\[ p = \gamma mu = 2m \left( \frac{c\sqrt{3}}{2} \right) = \left( \frac{mc}{c} \right) \sqrt{3} \]

\[ p = \left( \frac{938.3 \text{ MeV}}{c} \right) \sqrt{3} = 1.63 \times 10^3 \text{ MeV/c} \]
35. A proton moves at 0.950c. Calculate its (a) rest energy, (b) total energy, and (c) kinetic energy.

\[ E_R = mc^2 = (1.67 \times 10^{-27} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 = 1.50 \times 10^{-10} \text{ J} = 938 \text{ MeV} \]

\[ E = \gamma mc^2 = \frac{1.50 \times 10^{-10} \text{ J}}{\left[1 - (0.950c/c)^2\right]^{1/2}} = 4.81 \times 10^{-10} \text{ J} = 3.00 \times 10^3 \text{ MeV} \]

\[ K = E - mc^2 = 4.81 \times 10^{-10} \text{ J} - 1.50 \times 10^{-10} \text{ J} = 3.31 \times 10^{-10} \text{ J} = 2.07 \times 10^3 \text{ MeV} \]

38. In a typical color television picture tube, the electrons are accelerated through a potential difference of 25 000 V. (a) What speed do the electrons have when they strike the screen? (b) What is their kinetic energy in joules?

\[ q(\Delta V) = K = (\gamma - 1)mc^2 \]

Thus, \[ \gamma = \frac{1}{\sqrt{1 - (u/c)^2}} = 1 + \frac{q(\Delta V)}{mc^2} = 1 + \frac{25 000 \text{ eV}}{511 000 \text{ eV}} = 1.0489 \]

so \[ 1 - (u/c)^2 = 0.9089 \] and \[ u = 0.302c \]

\[ K = (\gamma - 1)mc^2 = q(\Delta V) = \left(1.60 \times 10^{-19} \text{ C}\right)(2.50 \times 10^4 \text{ J/C}) = 4.00 \times 10^{-15} \text{ J} \]

40. Consider electrons accelerated to an energy of 20.0 GeV in the 3.00-km-long Stanford Linear Accelerator. (a) What is the \( \gamma \) factor for the electrons? (b) What is their speed? (c) How long does the accelerator appear to them?

\[ E = \gamma mc^2 = 20.0 \text{ GeV} \]

Thus, \[ \gamma = \frac{20.0 \times 10^9 \text{ eV}}{0.511 \times 10^6 \text{ eV}} = 3.91 \times 10^4 \]

(b) \[ \gamma = \frac{1}{\sqrt{1 - (u/c)^2}} = 3.91 \times 10^4 \] from which \[ u = 0.999999997c \]

(c) \[ L = L_p \sqrt{1 - \left(\frac{u}{c}\right)^2} = \frac{L_p}{\gamma} = \frac{3.00 \times 10^3 \text{ m}}{3.91 \times 10^4} = 7.67 \times 10^{-2} \text{ m} = 7.67 \text{ cm} \]

45. The power output of the Sun is \( 3.77 \times 10^{26} \text{ W} \). How much mass is converted to energy in the Sun each second?

\[ P = \frac{dE}{dt} = \frac{d(mc^2)}{dt} = c^2 \frac{dm}{dt} = 3.85 \times 10^{26} \text{ W} \]

Thus, \[ \frac{dm}{dt} = \frac{3.85 \times 10^{26} \text{ J/s}}{(3.00 \times 10^8 \text{ m/s})^2} = 4.28 \times 10^9 \text{ kg/s} \]