41.1 The Hydrogen Atom: Angular Momentum and Energy

41.2 The Hydrogen Atom: Wave Functions and Probabilities

1. List all possible states of a hydrogen atom that have \( E = -1.51 \) eV.

   \[
   \begin{array}{ccc}
   n & l & m \\
   3 & 0 & 0 \\
   3 & 1 & -1 \\
   3 & 2 & 0 \\
   3 & 2 & 1 \\
   3 & 2 & 2 \\
   3 & 2 & 0 \\
   3 & 2 & 1 \\
   3 & 2 & 2 \\
   \end{array}
   \]

2. What are the \( n \) and \( l \) values of the following states of a hydrogen atom?

   - State = 4d  \( n = 4 \)  \( l = 2 \)
   - State = 5f  \( n = 5 \)  \( l = 3 \)
   - State = 6s  \( n = 6 \)  \( l = 0 \)

3. How would you label the hydrogen-atom states with the following quantum numbers?

   - \( (n, l, m) = (4, 3, 0) \)  Label = 4f
   - \( (n, l, m) = (3, 2, 1) \)  Label = 3d
   - \( (n, l, m) = (3, 2, -1) \)  Label = 3d

4. Consider the two hydrogen-atom states 5d and 4f. Which has the higher energy? Explain.

   The 5d state has the higher energy.

   The energy of an atom depends only on the principal quantum number \( n \).

   \[
   E_n = \frac{-13.6}{n^2} \text{ eV}
   \]

   \[
   E_5 = \frac{-13.6}{5^2} = -0.544 \text{ eV}, \quad E_4 = \frac{-13.6}{4^2} = -0.850 \text{ eV}
   \]
5. a. As a multiple of \( \hbar \), what is the angular momentum of a \( d \) electron? \[ \sqrt{6} \hbar \]

b. What is the maximum \( z \)-component of angular momentum of a \( d \) electron? \[ 2 \hbar \]

c. Is \( (L_z)_{max} \) greater than, less than, or equal to \( L \)? Less than

d. What is the significance of your answer to part c?

The \( z \)-component of a vector will never be greater than the vector itself. The \( z \)-component would be equal to the vector only when the tilt angle of the orbit is zero.

6. Draw a picture similar to Figure 41.3 showing all possible orientations of the angular momentum vector for a \( p \) electron.

7. What is the difference between the probability density and the radial probability density?

In 3-dimensional space, the probability density is the likelihood of finding the electron in a volume around the nucleus. The radial probability density is the likelihood of finding the electron at the distance \( r \) from the nucleus.

8. Consider a \( 2s \) electron, as portrayed in Figures 41.5 and 41.6. In your own words, describe how these figures suggest a "shell structure" of electrons around the nucleus.

In the \( 2s \) state the probability of finding an electron near the origin is high and also the probability of finding the electron in a shell surrounding the origin is also high. But there is a lower probability density in the space between these two "shells."
41.3 The Electron’s Spin

9. The figure shows a spinning ball of negative charge. Does the magnetic moment of this spinning charge point up, point down, or point to the right? Explain.

If negative charge is rotating counter-clockwise looking down from the top, then the conventional current I has direction that is clockwise. Then, by the right-hand rule, the magnetic moment vector \( \vec{\mu} \) points down.

10. A bar magnet is moving to the right through a nonuniform magnetic field. The field is weaker toward the bottom of the page and stronger toward the top of the page.

a. Is there a net force on the magnet? If so, in which direction? Explain.

Down. Atoms with the south pole up experience a larger downward force on the south pole and a smaller upward force on the north pole.

b. Will the magnet be deflected by the field? If so, in which direction?

Yes. Down.

c. Would the magnet be deflected by a uniform magnetic field? Explain why or why not.

No. A uniform magnetic field would exert forces of equal strengths on both the north and south poles.

11. The figure shows the outcome of a Stern-Gerlach experiment with atoms of element X.

a. Do the peaks in a Stern-Gerlach experiment represent different values of the atom’s total angular momentum or different values of the \( z \)-component of its angular momentum? Explain.

These are different values of \( L_z \) for the same \( l \) value. The deflection depends on the \( z \)-component of the magnitude moment \( M_z \) which provides information about \( L_z \).

b. What angular-momentum quantum number characterizes these atoms? Explain.

The \( l=1 \) angular momentum quantum number—because it has three values of \( L_z \). These three values have \( m \) values of \(-1, 0, 1\).
41.4 Multielectron Atoms

41.5 The Periodic Table of the Elements

12. Do the following figures represent a possible electron configuration of an element? If so:
   i. Identify the element, and
   ii. Determine if this is the ground state or an excited state.
   If not, why not?
   a. 2\textsuperscript{p} \underline{\uparrow\uparrow} \\
   1\textsuperscript{s} \underline{\uparrow}
   Yes. This is the ground state of Nitrogen.
   b. 2\textsuperscript{p} \underline{\uparrow\uparrow} \\
   2\textsuperscript{s} 
   No. Only 2 electrons are allowed in the 2s state.
   c. 2\textsuperscript{p} \underline{\uparrow\uparrow} \\
   2\textsuperscript{s} 
   No. The 2s state would fill before the 2p state.

13. Do the following electron configurations represent a possible state of an element? If so:
   i. Identify the element, and
   ii. Determine if this is the ground state or an excited state.
   If not, why not?
   a. 1s\textsuperscript{2} 2s\textsuperscript{2} 2p\textsuperscript{6} 3s\textsuperscript{2}
      Yes. This is the ground state of Magnesium.
   b. 1s\textsuperscript{2} 2s\textsuperscript{2} 2p\textsuperscript{7} 3s
      No. The 2p state would fill with 6 electrons.
   c. 1s\textsuperscript{2} 2s\textsuperscript{2} 2p\textsuperscript{4} 3s\textsuperscript{2} 3p\textsuperscript{2}
      No. The 2p state would fill with 6 electrons before the 3p state. In excited states, the valence electrons would move to a higher energy state.

14. Why is the section of the periodic table labeled as “transition elements” exactly 10 elements wide in all rows?

There had been no previous d states in the first three rows of the periodic table. In the fourth row the 3d subshell “splits open” the periodic table to add 10 more transition elements corresponding to the 10 possible electrons in the 3d subshell.
41.6 Excited States and Spectra

15. The figure shows the energy levels of a hypothetical atom.
   a. What is the atom’s ionization energy?
      
      \[
      q_e \text{ eV}
      \]

   b. In the space below, draw the energy-level diagram as it would appear if the ground state were chosen as the zero of energy. Label each level and the ionization limit with the appropriate energy.

   \[
   \begin{array}{c}
   \text{Ground state} \\
   6s \\
   6p \\
   7s \\
   7p \\
   7d \\
   \text{9 eV}
   \end{array}
   \]

16. The figure shows the energy levels of a hypothetical atom.
   a. What *minimum* kinetic energy (in eV) must an electron have to collisionally excite this atom and cause the emission of a 620 nm photon? Explain.
      
      5 eV. The electron must excite the atom from its ground state up to the \((n+1)s\) state.

   b. Can an electron with \(K = 6\) eV cause the emission of 620 nm light? If so, what is the final kinetic energy of the electron? If not, why not?
      
      Yes. The final kinetic energy of the electron is \(6eV - 5eV = 1eV\).

   c. Can a 6 eV photon cause the emission of 620 nm light from this atom? Why or why not?
      
      No. In absorption of a photon, all of the photon’s energy must be transferred to the atom. The photon would need to have exactly 5 eV.

   d. Can 7 eV photons cause the emission of 620 nm light from these atoms? Why or why not?
      
      Yes. After absorption of the 7 eV photon, the atom is excited from the ground state to the \((n+1)p\) state. Then, a transition from \((n+1)p \rightarrow (n+1)s\) could occur. Next, a transition from \((n+1)s \rightarrow np\) could occur causing the emission of 620 nm light.
17. Seven possible transitions are identified on the energy-level diagram. For each, is this an allowed transition?

i. If allowed, is it an emission or an absorption transition?

ii. Is the photon infrared, visible, or ultraviolet?

iii. If not allowed, why not?

Transition 1:

No. The selection rule $\Delta l = 1$ does not allow a jump from an $s$ to an $s$ state.

Transition 2:

Yes. Emission. $4p \rightarrow 3s$. 330 nm. Ultraviolet.

Transition 3:

No. The selection rule $\Delta l = 1$ is violated. $3s \rightarrow 4d$

Transition 4:

No. The selection rule $\Delta l = 1$ is violated. $3p \rightarrow 6p$

Transition 5:

Yes. Emission. $6p \rightarrow 5s$. 2.1 $\mu$m = 2100 nm. Infrared.

Transition 6:

Yes. Emission. $6p \rightarrow 3d$. 1.2 $\mu$m = 1200 nm. Infrared.

Transition 7:

Yes. Emission. $5f \rightarrow 4d$. 4.0 $\mu$m = 4000 nm. Infrared.
41.7 Lifetimes of Excited States

18. A sample of atoms is excited at $t = 0$ ns. The graph shows the number of excited atoms as a function of time after $t = 0$ ns. Each atom decays back to the ground state by emitting a photon.
   a. How many atoms were initially excited?
   
   $200$

   b. At what time have half of the excited atoms decayed?
   
   $10$ ns

   c. How many photons are emitted in the first 20 ns?
   
   $150$

   d. At what time will there be 25 atoms remaining in the excited state? Explain.
   
   $29$ ns
   $N_{\text{exc}} = N_0 e^{-t/\tau}$
   $25 = 200 e^{-t/14 \text{ ns}}$
   $t = 29$ ns

   e. As best as you can determine from this graph, what is the lifetime of this excited state? Explain how you determined it.
   
   The lifetime $\tau \approx 14$ ns. That is the point in time at which the curve has decayed to 36.8% of its initial value. $0.368 \times 200 = 73.6$

19. At $t = 0$ ns, 1000 atoms are excited to a state that has a lifetime of 5 ns.
   a. Sketch a graph of the number of excited atoms $N_{\text{exc}}$ still present at time $t$. Include a numerical scale on the vertical axis.

   b. Sketch a graph of the total number of photons $N_{\text{photons}}$ that have been emitted by time $t$. Include a numerical scale on the vertical axis.
41.8 Stimulated Emission and Lasers

20. A photon with energy 2.0 eV is incident on an atom in the \( p \)-state. Does the atom undergo an absorption transition, a stimulated emission transition, or neither? Explain.

A stimulated emission transition. The absorbed photon frequency must exactly match the \( E_3 - E_1 \) energy difference of the atom. It does. Both are equal to 2.0 eV. Two identical photons will be emitted, each with 2.0 eV.

21. A glass tube contains \( 2 \times 10^{11} \) atoms, some of which are in the ground state and some of which are excited. The populations are shown for the atom’s three energy levels. Is it possible for these atoms to be a laser? If so, on which transition would laser action occur? If not, why not?

Yes. This can be a laser. The laser action would occur on the Level 3 \( s \)-state to Level 2 \( p \)-state transition because we have a population inversion with \( N_3 > N_2 \).