Modern Physics:
Quantum Physics & Relativity

c circular orbit
e elliptical orbit
u unbound orbit
You can’t get to Modern Physics without doing Classical Physics!

The fundamental laws and principles of Classical Physics are the basis Modern Physics.
Isaac Newton
(1642 -1727)

In *Principia* (1687)
Newton
• Invented Calculus
• 3 Laws of Motion
• Universal Law of Gravity

The force of gravity is *Universal*: The same force that makes an apple fall to Earth, causes the moon to fall around the Earth and the planets to orbit the Sun.
James Clerk Maxwell 1860s

Light is wave. The medium is the Ether.

The Speed is constant, c.

Maxwell’s Equations

\[ \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0} \]  
Gauss’s law

\[ \oint \vec{B} \cdot d\vec{A} = 0 \]  
Gauss’s law for magnetism

\[ \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt} \]  
Faraday’s law

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} + \varepsilon_0 \mu_0 \frac{d\Phi_e}{dt} \]  
Ampère-Maxwell law

\[ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.0 \times 10^8 \text{ m/s} \]
Michelson-Morely Experiment
1887
Measure the Speed of the Ether Wind
Light should travel faster with the wind and slower against it during the year.
Rotate arms to produce interference fringes and find different speeds of light caused by the Ether Wind, due to Galilean Relativity: light should travel slower against the Ether Wind. From that you can find the speed of the wind.

https://www.youtube.com/watch?v=7qJoRNseyLQ
Michelson-Morely Experiment 1887

The speed of light is independent of the motion and is always $c$. The speed of the Ether wind is zero.

OR: Lorentz Contraction

If there is an ether but we can’t detect it because the apparatus shrinks by a factor:

$$\sqrt{1 - v^2 / c^2}$$
A Problem with Electrodynamics

The force on a moving charge depends on the Frame.

Charge Rest Frame
(moving with charge)

\[ F = 0 \]

Wire Rest Frame
(At rest with wire)

\[ F = qvB \sin \theta \]

Stationary charge feels no force, since wire is uncharged

Moving charge feels force \( qv \times B \), since wire creates a magnetic field \( B \).

Magnetic field \( B \) "circulates" around wire
The Laws of Physics must be the same in all reference frames!!
Einstein realized this inconsistency and could have chosen either:

- Keep Maxwell's Laws of Electromagnetism, and abandon Galileo's Spacetime
- or, keep Galileo's Space-time, and abandon the Maxwell Laws.
Lorentz Contraction in Wire

Moving charges in the wire cause a Lorentz contraction in the distance between the moving charges in the wire so that from the rest frame of the charge outside the wire (or at rest in the wire) the moving charges bunch up and thus give a net charge to the wire.

Rest frame of wire: spacing between stationary metal ions equals Lorentz-contracted spacing between moving electrons. Wire is electrically neutral.

Rest frame of electrons: Lorentz-contracted spacing between moving metal ions is smaller than spacing between stationary electrons. Wire has net positive charge.
Einstein Saves Maxwell!

The force on a moving charge does NOT depend on the Frame.

From the rest frame of the charge, the wire is charged and it feels a Coulomb force. The forces in each frame are equal though they are due to different causes!
Magnetic Fields are SR effects of Electric Fields!

So if there were a positive charged, err... positively charged cat nearby, it would experience
On the Electrodynamics of Moving Bodies

1905
ON THE ELECTRODYNAMICS OF MOVING BODIES

BY A. EINSTEIN

June 30, 1905

It is known that Maxwell’s electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion. For if the magnet is in motion and the conductor at rest, there arises in the neighbourhood of the magnet an electric field with a certain definite energy, producing a current at the places where parts of the conductor are situated. But if the magnet is stationary and the conductor in motion, no electric field arises in the neighbourhood of the magnet. In the conductor, however, we find an electromotive force, to which in itself there is no corresponding energy, but which gives rise—assuming equality of relative motion in the two cases discussed—to electric currents of the same path and intensity as those produced by the electric forces in the former case.

Examples of this sort, together with the unsuccessful attempts to discover any motion of the earth relatively to the “light medium,” suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather that, as has already been shown to the first order of small quantities, the same laws of electrodynamics and optics will be valid for all frames of reference for which the
Einstein’s Principle of Relativity

**Principle of relativity** All the laws of physics are the same in all inertial reference frames.

- Maxwell’s equations are true in all inertial reference frames.
- Maxwell’s equations predict that electromagnetic waves, including light, travel at speed \( c = 3.00 \times 10^8 \text{ m/s} = 300,000 \text{ km/s} = 300 \text{ m/µs} \).
- Therefore, **light travels at speed \( c \) in all inertial reference frames.**

Every experiment has found that light travels at \( 3.00 \times 10^8 \text{ m/s} \) in every inertial reference frame, regardless of how the reference frames are moving with respect to each other.
Special Theory: 
**Inertial Frames:**
Frames do not accelerate relative to each other; Flat Spacetime – No Gravity. They are moving on inertia alone.

General Theory: 
**Noninertial Frames:**
Frames accelerate: Curved Spacetime, Gravity & acceleration.
Albert Einstein
1916

The General Theory of Relativity
Postulates of Special Relativity
1905

1. The laws of physics are the same in all inertial reference frames.

2. The speed of light in a vacuum is constant in all inertial reference frames, independent of the relative motion of source and observer. We assume a vacuum for this class.

3. Months later......\( E = mc^2 \)

\[
c = 3.0 \times 10^8 \text{ m} / \text{s} = 300 \text{ m} / \mu\text{s}
\]
Light slows down in a medium like glass as a function of wavelength. Who slows down more, red or blue in glass?
Faster than Light?

Cherenkov radiation is electromagnetic radiation emitted when a charged particle passes through an insulator at a speed greater than the speed of light in that medium. The characteristic "blue glow" of nuclear reactors is due to Cherenkov radiation.
Newton’s Principia in 1687.

Galilean Relativity

Velocities add: \[ V = U + V' \]
Relative Velocity

- Two observers moving relative to each other generally do not agree on the outcome of an experiment.
- For example, observers A and B below see different paths for the ball and measure different velocities:

\[ \mathbf{v}_{bB} = \mathbf{v}_{bA} + \mathbf{v}_{AB} \]

Velocity of ball relative to observer B

Velocity of ball relative to observer A

Velocity of A relative to observer B
What if instead of a ball, it is a light wave? Do velocities add according to Galilean Relativity?
Light travels in a vacuum at \( c \) in all inertial reference frames, regardless of the relative motion.

This light wave leaves Amy at speed \( c \) relative to Amy. It approaches Cathy at speed \( c \) relative to Cathy.

This light wave leaves Bill at speed \( c \) relative to Bill. It approaches Cathy at speed \( c \) relative to Cathy.
Clocks slow down and rulers shrink in order to keep the speed of light the same for all observers!
Time is Relative!
Space is Relative!
Only the SPEED OF LIGHT is Absolute!
Time
Dilation
Time Dilation

Spaceship Frame

\[ L = \frac{cT}{2} \]

Earth Frame:

\[ D = \frac{cT}{2} \]
\[ \frac{X}{2} = \frac{UT}{2} \]
The smallest time interval $\Delta t_0$ between two events is measured in the reference frame in which both events occur at the same place (space ship) and is called the **proper time**. The “stretching out” of the time interval is called **time dilation**. In another frame moving at speed $v$ with respect to the first, the measured time interval is increased by a factor of gamma (Earth frame). The ‘event’ is the light bouncing in the spaceship.

\[
\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \Delta t'
\]

Space Ship Frame: Proper Time: $\Delta t'$
Earth Lab Frame: Dilated Time: $\Delta t = \gamma \Delta t'$
## Gamma Factor

### Approximate Values for $\gamma$ at Various Speeds

<table>
<thead>
<tr>
<th>$v/c$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0010</td>
<td>1.000 000 5</td>
</tr>
<tr>
<td>0.010</td>
<td>1.000 05</td>
</tr>
<tr>
<td>0.10</td>
<td>1.005</td>
</tr>
<tr>
<td>0.20</td>
<td>1.021</td>
</tr>
<tr>
<td>0.30</td>
<td>1.048</td>
</tr>
<tr>
<td>0.40</td>
<td>1.091</td>
</tr>
<tr>
<td>0.50</td>
<td>1.155</td>
</tr>
<tr>
<td>0.60</td>
<td>1.250</td>
</tr>
<tr>
<td>0.70</td>
<td>1.400</td>
</tr>
<tr>
<td>0.80</td>
<td>1.667</td>
</tr>
<tr>
<td>0.90</td>
<td>2.294</td>
</tr>
<tr>
<td>0.92</td>
<td>2.552</td>
</tr>
<tr>
<td>0.94</td>
<td>2.931</td>
</tr>
<tr>
<td>0.96</td>
<td>3.571</td>
</tr>
<tr>
<td>0.98</td>
<td>5.025</td>
</tr>
<tr>
<td>0.99</td>
<td>7.089</td>
</tr>
<tr>
<td>0.995</td>
<td>10.01</td>
</tr>
<tr>
<td>0.999</td>
<td>22.37</td>
</tr>
</tbody>
</table>

### Equations

- $\beta = \frac{v}{c}$, $\beta < 1$
- $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, $\gamma \geq 1$
Time Dilation – Generalization

• If a clock is moving with respect to you, the time interval between ticks of the moving clock is observed to be longer than the time interval between ticks of an identical clock in your reference frame.

• All physical processes are measured to slow down when these processes occur in a frame moving with respect to the observer.
  – These processes can be chemical and biological as well as physical.

The faster you move, the longer you live relative to people at rest! Relative to yourself, you age at a normal rate!
Time Dilation – Verification

- Time dilation is a very real phenomenon that has been verified by various experiments.
- These experiments include:
  - Airplane flights
  - Muon decay
Time Dilation Verification – Muon Decays

- Muons are unstable particles that have the same charge as an electron, but a mass 207 times more than an electron.
- Muons have a half-life of $\Delta t_p = 2.2 \, \mu s$ when measured in a reference frame at rest with respect to them (a).
- Relative to an observer on the Earth, muons should have a lifetime of $\gamma \Delta t_p$ (b).
- A CERN experiment measured lifetimes in agreement with the predictions of relativity.
Sample Problem

Superwoman can travel at 0.75c in her glass space ship. She has to fly to Beta Diva, 25.0 light years away as measured from Earth in the Earth frame, to battle alien evil guys.

a) What is the total time for the trip for Superwoman?
b) What is the total time you measure on Earth?

What is the event? Which frame measures the proper time?
As speed approach c, lengths contract.
Length Contraction: Distances

\[ L = \frac{L_p}{\gamma} = L_p \sqrt{1 - \frac{v^2}{c^2}} \]

Rest (Proper) Frame Distance
Earth Frame measures \( L_0 \)

Moving Frame Distance
Rocket Frame measures \( L \)
Length Contraction

The proper length of an object is longest in the reference frame in which it is at rest. In another frame moving parallel to the object, its length is shortened by a factor of gamma.

\[ L' = L_p / \gamma = L_p \sqrt{1 - \frac{v^2}{c^2}} \]

Ruler (Proper) Frame
Measures length \( L_p \)

Earth Frame
Measures length \( L \)
Sample Problem

Superwoman can travel at 0.75c in her glass space ship. She has to fly to Beta Diva, 25.0 light years away as measured from Earth in the Earth frame, to battle alien evil guys.

a) What is the total time for the trip for Superwoman?
b) What is the total time you measure on Earth?
c) How far is Beta Diva as measured by Superwoman?
d) As Superwoman leaves Earth, you measure her length as she flies overhead at 0.75c. What is her length you measure? At rest she measures 2.75 m tall.
Binomial Expansion Trick for low speeds.

- The binomial approximation is useful when we need to calculate a relativistic expression for a nonrelativistic velocity \( v \ll c \).

The binomial approximation

If \( x \ll 1 \), then \( (1 + x)^n \approx 1 + nx \).

The binomial approximation is useful when we need to calculate a relativistic expression for a nonrelativistic velocity \( v \ll c \).

If \( v \ll c \) :

\[
\begin{align*}
\sqrt{1 - \beta^2} &= (1 - \frac{v^2}{c^2})^{1/2} \approx 1 - \frac{1}{2} \frac{v^2}{c^2} \\
\gamma &= \frac{1}{\sqrt{1 - \beta^2}} = (1 - \frac{v^2}{c^2})^{-1/2} \approx 1 + \frac{1}{2} \frac{v^2}{c^2}
\end{align*}
\]
Example

An 8.0-m-long school bus drives past at 30 m/s. By how much is its length contracted?

**MODEL**  The school bus is at rest in an inertial reference frame S′ moving at velocity \( v = 30 \text{ m/s} \) relative to the ground frame S. The given length, 8.0 m, is the proper length \( \ell \) in frame S′.

**SOLVE**  In frame S, the school bus is length contracted to

\[
L = \sqrt{1 - \beta^2 \ell}
\]

The bus’s velocity \( v \) is much less than \( c \), so we can use the binomial approximation to write

\[
L \approx \left( 1 - \frac{1}{2} \frac{v^2}{c^2} \right) \ell = \ell - \frac{1}{2} \frac{v^2}{c^2} \ell
\]

The *amount* of the length contraction is

\[
\ell - L = \frac{1}{2} \frac{v^2}{c^2} \ell = \frac{1}{2} \left( \frac{30 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} \right)^2 (8.0 \text{ m})
\]

\[
= 4.0 \times 10^{-14} \text{ m} = 40 \text{ fm}
\]

where 1 fm = 1 femtometer = \( 10^{-15} \text{ m} \).
**ASSESS** The bus “shrinks” by only slightly more than the diameter of the nucleus of an atom. It’s no wonder that we’re not aware of length contraction in our everyday lives. If you had tried to calculate this number exactly, your calculator would have shown $\ell - L = 0$ because the difference between $\ell$ and $L$ shows up only in the 14th decimal place. A scientific calculator determines numbers to 10 or 12 decimal places, but that isn’t sufficient to show the difference. The binomial approximation provides an invaluable tool for finding the very tiny difference between two numbers that are nearly identical.
2-D Relative Motion Book Problem

A moving rod is observed to have a length of 2.00 m and to be oriented at an angle of 30.0° with respect to the direction of motion, as shown. The rod has a speed of 0.995c. (a) What is the proper length of the rod? (b) What is the orientation angle in the proper frame?

Conceptual Check:
Is the proper length going to be shorter? Longer? The same?

Is the proper angle going to be smaller? Larger? The same?
Beth and Charles are at rest relative to each other. Anjay runs past at velocity $v$ while holding a long pole parallel to his motion. Anjay, Beth, and Charles each measure the length of the pole at the instant Anjay passes Beth. Rank in order, from largest to smallest, the three lengths $L_A$, $L_B$, and $L_C$. 

A. $L_A = L_B = L_C$
B. $L_B = L_C > L_A$
C. $L_A > L_B = L_C$
D. $L_A > L_B > L_C$
E. $L_B > L_C > L_A$
Beth and Charles are at rest relative to each other. Anjay runs past at velocity $v$ while holding a long pole parallel to his motion. Anjay, Beth, and Charles each measure the length of the pole at the instant Anjay passes Beth. Rank in order, from largest to smallest, the three lengths $L_A$, $L_B$, and $L_C$.

A. $L_A = L_B = L_C$
B. $L_B = L_C > L_A$
C. $L_A > L_B = L_C$

[Correct Answer] 

D. $L_A > L_B > L_C$
E. $L_B > L_C > L_A$
Last time....

Time Dilation

\[ \Delta t = \gamma \Delta t_p = \sqrt{\Delta t'^2 - \frac{v^2}{c^2}} \]

Length Contraction

\[ L' = \frac{L_p}{\gamma} = L_p \sqrt{1 - \frac{v^2}{c^2}} \]

\[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \gamma \geq 1 \]

\[ \beta = \frac{v}{c}, \quad \beta < 1 \]
Length Contraction Paradox?

Strange but not a paradox. They are not both true at the same time to each other. They are true relative to their own frame!
Strange but not a paradox. Helen undergoes accelerations and the situation is not symmetric. Special Relativity applies only to inertial, non-accelerating motion!!! Helen is younger when she returns because she is the one who was traveling, stopping, turning around, returning.
Quiz Twin Problem

The identical twins Speedo and Goslo join a migration from the Earth to Planet X. It is 20.0 ly away in a reference frame in which both planets are at rest. The twins, of the same age, depart at the same time on different spacecraft. Speedo’s craft travels steadily at 0.950c, and Goslo’s at 0.750c. Calculate the time difference between the twins after Goslo’s spacecraft lands on Planet X. Which twin is the older?
Addition of Velocities
Addition of Velocities

Galilean Relativity

- Observer on curb
  - $u' = 25 \text{ m/s}$
  - $u = 45 \text{ m/s}$

- Observer on curb
  - $u' = -25 \text{ m/s}$
  - $u = -5 \text{ m/s}$
The Lorentz Velocity Transformations

Consider two reference frames $S$ and $S'$. An object moves at velocity $u$ along the $x$-axis as measured in $S$, and at velocity $u'$ as measured in $S'$. Reference frame $S'$ moves with velocity $v$ relative to $S$, also along the $x$-axis.

The Lorentz velocity transformations are:

\[
\begin{align*}
    u' &= \frac{u - v}{1 - uv/c^2} \quad \text{and} \quad u = \frac{u' + v}{1 + u'v/c^2}
\end{align*}
\]

**NOTE:** It is important to distinguish carefully between $v$, which is the relative velocity between two reference frames, and $u$ and $u'$ which are the velocities of an object as measured in the two different reference frames.
Addition of Velocities

\[ u = \frac{v + u'}{1 + \frac{vu'}{c^2}} = \frac{.5c + c}{1 + \frac{.5cc}{c^2}} = c \]

If \( v = .5c \)
Problem

Superwoman can travel at 0.75c in her glass space ship. She has to fly to Beta Diva, 25.0 light years away as measured from Earth in the Earth frame, to battle alien evil guys.

a) What is the total time for the trip for Superwoman?
b) What is the total time you measure on Earth?
c) How far is Beta Diva as measured by Superwoman?
d) As Superwoman leaves Earth, you measure her length as she flies overhead at 0.75c. What is her length you measure? At rest she measures 2.75 m tall.
e) If she launches a space pod to Beta Diva while en route at 0.4c, what is the speed at which it approaches Beta Diva as measured by Beta Diva?

\[ u = \frac{v + u'}{\frac{v u'}{c^2} + 1} \]
You still can't send a message faster than light
Or teleport
light - it's just an image.

How to break the speed of light

https://www.youtube.com/watch?v=lR4tJr7sMPM&list=PL908547EAA7E4AE74&index=19
The Lorentz Transformations

Consider two reference frames $S$ and $S'$. An event occurs at coordinates $x, y, z, t$ as measured in $S$, and the same event occurs at $x', y', z', t'$ as measured in $S'$. Reference frame $S'$ moves with velocity $v$ relative to $S$, along the $x$-axis. The **Lorentz transformations** for the coordinates of one event are:

$$S: (x, y, x, t) \quad S': (x', y', x', t')$$

$$x' = \gamma(x - vt) \quad \quad x = \gamma(x' + vt')$$
$$y' = y \quad \quad y = y'$$
$$z' = z \quad \quad z = z'$$
$$t' = \gamma(t - vx/c^2) \quad \quad t = \gamma(t' + vx'/c^2)$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}}$$
Lorentz Transformations, Pairs of Events

The Lorentz transformations can be written in a form suitable for describing pairs of events

For S to S'

\[ \Delta x' = \gamma (\Delta x - v \Delta t) \]
\[ \Delta t' = \gamma \left( \Delta t - \frac{v}{c^2} \Delta x \right) \]

For S' to S

\[ \Delta x = \gamma (\Delta x' + v \Delta t') \]
\[ \Delta t = \gamma \left( \Delta t' + \frac{v}{c^2} \Delta x' \right) \]
Relativity of Simultaneity

https://www.youtube.com/watch?v=wteiuxyqtoM
Relativity of Simultaneity

- Two events occurring simultaneously in one reference frame are not simultaneous in any reference frame moving relative to it. Neither reference frame is ‘correct’: There are no preferred or special reference frames. *It’s all relative baby!*

- Any two observers at rest in the same frame MUST measure simultaneous events to be simultaneous if even if they don’t SEE it that way. That means that they measure the time difference between them to be zero. Use the Lorentz transformation

*Seeing is NOT the same as Being!*  
The time when an event is SEEN is not necessarily when the event actually happens!
Simultaneity

For S to S’
\[ \Delta x’ = \gamma (\Delta x - v \Delta t) \]
\[ \Delta t’ = \gamma \left( \Delta t - \frac{v}{c^2} \Delta x \right) \]

For S’ to S
\[ \Delta x = \gamma (\Delta x’ + v \Delta t’) \]
\[ \Delta t = \gamma \left( \Delta t’ + \frac{v}{c^2} \Delta x’ \right) \]

Fireworks go off at the same time according to Earth clocks in two cities, Alum and Boron, that are 300 km apart. The people in a spaceship that is flying in a straight line from Alum to Boron at 0.8c also observe the fireworks when the spaceship is directly over Alum. Do they see the fireworks in the two cities simultaneously? If the people in the spaceship say the fireworks were not simultaneous in Alum and Boron, how long before or after the fireworks flashed at Alum did the fireworks flash at Boron according to their calculations? Sketch the situation.
The Spacetime Interval in Invariant

- Consider two events that are separated in time by an interval \( \Delta t \), and are separated in space by an interval \( \Delta x \).

- Let us define the **spacetime interval** \( s \) between the two events to be:

\[
 s^2 = c^2(\Delta t)^2 - (\Delta x)^2
\]

- The spacetime interval \( s \) has the same value in all inertial reference frames.

- That is, the spacetime interval between two events is an **invariant**.
**Example 36.7** Using the spacetime interval

A firecracker explodes at the origin of an inertial reference frame. Then, 2.0 μs later, a second firecracker explodes 300 m away. Astronauts in a passing rocket measure the distance between the explosions to be 200 m. According to the astronauts, how much time elapses between the two explosions?

**Solve** The spacetime interval (or, rather, its square) in frame S is

\[ s^2 = c^2(\Delta t)^2 - (\Delta x)^2 = (600 \text{ m})^2 - (300 \text{ m})^2 = 270,000 \text{ m}^2 \]

where we used \( c = 300 \text{ m/μs} \) to determine that \( c \Delta t = 600 \text{ m} \). The spacetime interval has the same value in frame \( S' \). Thus

\[
 s^2 = 270,000 \text{ m}^2 = c^2(\Delta t')^2 - (\Delta x')^2 = c^2(\Delta t')^2 - (200 \text{ m})^2
\]

This is easily solved to give \( \Delta t' = 1.85 \text{ μs} \).

**Assess** The two events are closer together in both space and time in the rocket’s reference frame than in the reference frame of the ground.
E = MC^2
Invariant Mass

Invariant mass is independent of frame of reference and is the mass as measured in the proper frame:

Rest Energy: \[ E_0 = m_0 c^2 \]

Invariant mass: \[ m_0 = \frac{E_0}{c^2} \]

If a mass is moving, then the total energy increases by a gamma factor:

Total Energy: \[ E = \gamma mc^2 \]
\[ \Delta m = \frac{E}{c^2} \]

Mass is bound energy. \( c^2 \) is the conversion factor – the magnitude is 90 quadrillion joules per kilogram. Even a speck of matter with a mass of only 1 milligram has a rest energy of 90 billion Joules!!!

\[ c^2 = 9 \times 10^{16} \text{ J/kg} \]

\[ 1 \text{ GeV/c}^2 = 1.783 \times 10^{-27} \text{ kg} \]

The rest mass of a proton is 0.938 GeV/c²
Proof: $E = mc^2$

Irène and Frédéric Joliot-Curie 1933

Pure energy converted into particles: pair production
Proof of $E=mc^2$

https://www.youtube.com/watch?v=hW7DW9NIO9M&index=42&list=PL908547EAA7E4AE74
Relativistic Energy

The total energy $E$ of a particle is

$$E = \gamma_p mc^2 = E_0 + K = \text{rest energy} + \text{kinetic energy}$$

This total energy consists of a rest energy

$$E_0 = mc^2$$

and a relativistic expression for the kinetic energy

$$K = (\gamma_p - 1)mc^2 = (\gamma_p - 1)E_0$$

This expression for the kinetic energy is very nearly $\frac{1}{2}mu^2$ when $u \ll c$. 
Kinetic Energy

The kinetic energy is the total energy less the rest energy:

$$KE = E - E_0 = (1 - \gamma)m_0c^2$$
Relativistic Momentum & Mass

Imagine you are at rest and a bullet shoots past approaching c. What is the momentum of the particle relative to you?

\[ p = \gamma mu \]

The speed is bounded, is the momentum? NO!

As the bullet’s speed increases, the momentum and mass increase:

“Relativistic” Mass: \[ m = \gamma m_0 \]

rest energy: \[ E_0 = m_0 c^2 \]

total energy \[ E = \gamma m_0 c^2 \]

\[ m = \gamma m_0 \]

Note: We don’t use this notation!
Total Energy

\[ E = \gamma mc^2 \]
\[ E_0 = mc^2 \]
\[ p = \gamma mu \]

\[ pc = \gamma muc = \gamma m\frac{u}{c^2} = \frac{u}{c} (\gamma mc^2) = \beta E \]

\[ E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} \rightarrow E^2 (1-\frac{u^2}{c^2}) = (mc^2)^2 \rightarrow E^2 - \frac{u^2}{c^2} E^2 = E_0^2 \rightarrow E^2 = \beta^2 E^2 + E_0^2 \]

\[ E^2 = (pc)^2 + E_0^2 \]

If \( m = 0 \) (photon) \( E^2 = (pc)^2 + (m_0c^2)^2 = \) \( \text{?} \)

photon momentum: \[ p = \frac{E}{c} \]
If a photon has momentum, does it have mass?

A New Limit on Photon Mass

A new limit on photon mass, less than $10^{-51}$ grams or $7 \times 10^{-19}$ electron volts, has been established by an experiment in which light is aimed at a sensitive torsion balance; if light had mass, the rotating balance would suffer an additional tiny torque. This represents a 20-fold improvement over previous limits on photon mass.

Photon mass is expected to be zero by most physicists, but this is an assumption which must be checked experimentally. A nonzero mass would make trouble for special relativity, Maxwell's equations, and for Coulomb's inverse-square law for electrical attraction.

The work was carried out by Jun Luo and his colleagues at Huazhong University of Science and Technology in Wuhan, China (junluo@mail.hust.edu.cn, 86-27-8755-6653). They have also carried out a measurement of the universal gravitational constant $G$ (Luo et al., Physical Review D, 15 February 1999) and are currently measuring the force of gravity at the sub-millimeter range (a departure from Newton's inverse-square law might suggest the existence of extra spatial dimensions) and are studying the Casimir force, a quantum effect in which nearby parallel plates are drawn together. (Luo et al., Physical Review Letters, 28 February 2003)
Physicists Say They've Created a Device That Generates 'Negative Mass'

Say what?
MIKE MCRAE  14 JAN 2018

Physicists have created what they say is the first device that's capable of generating particles that behave as if they have negative mass.

The device generates a strange particle that's half-light/half-matter, and as if that isn't cool enough, it could also be the foundation for a new kind of laser that could operate on far less energy than current technologies.

Negative Mass Particles Created for the First Time by Researchers

For years, physicists have spent time and effort to find real-world examples of negative mass. But now, for the first time in the fields of quantum physics, a team...
You Try: Sample Problem

Superwoman can travel at 0.75c in her glass space ship. She has to fly to Beta Diva, 25.0 light years away as measured from Earth in the Earth frame, to battle alien evil guys.

f) If her rest mass is 65 kg, what is her rest Energy?
g) What is her total relativistic Energy you measure? What does she measure? Is it different?
h) What is her kinetic energy?

rest energy: \( E_0 = m_0 c^2 \)

total energy \( E = \gamma m_0 c^2 \)

\[ KE = E - E_0 = (\gamma - 1)m_0 c^2 \]
In problems with collisions or decays, 
Use conservation of Energy and momentum!

**Conservation of Energy:**

\[ E_i = E_f \]
\[ E_{\text{total}} = \gamma mc^2 \]

**Conservation of momentum:** \( \sum p_i = \sum p_f \)

\[ p = \gamma mu \]
43. An unstable particle at rest breaks into two fragments of unequal mass. The mass of the first fragment is $2.50 \times 10^{-28}$ kg, and that of the other is $1.67 \times 10^{-27}$ kg. If the lighter fragment has a speed of $0.893c$ after the breakup, what is the speed of the heavier fragment?

Relativistic momentum of the system of fragments must be conserved. For total momentum to be zero after as it was before, we must have, with subscript 2 referring to the heavier fragment, and subscript 1 to the lighter, $p_2 = p_1$.

$$\gamma_2 m_2 u_2 = \gamma_1 m_1 u_1 = \frac{2.50 \times 10^{-28} \text{ kg}}{\sqrt{1 - (0.893)^2}} \times (0.893c)$$

or

$$\frac{(1.67 \times 10^{-27} \text{ kg}) u_2}{\sqrt{1 - (u_2/c)^2}} = \left(4.960 \times 10^{-28} \text{ kg}\right)c$$

Proceeding to solve, we find

$$\left(\frac{1.67 \times 10^{-27}}{4.960 \times 10^{-28}} \frac{u_2}{c}\right)^2 = 1 - \frac{u_2^2}{c^2}$$

$$12.3 \frac{u_2^2}{c^2} = 1 \quad \text{and} \quad u_2 = 0.285c$$
62. An unstable particle with a mass of $3.34 \times 10^{-27}$ kg is initially at rest. The particle decays into two fragments that fly off along the $x$ axis with velocity components $0.987c$ and $-0.868c$. Find the masses of the fragments.

**Conservation of Energy:**

\[ E_i = E_f \]

\[ E_{total} = \gamma mc^2 \]

**Conservation of momentum:**

\[ \sum p_i = \sum p_f \]

\[ p = \gamma mu \]
Energy Released: The Mass Defect

Atomic Decay: Parent atoms have more mass than product atoms. The difference is released in the form of Kinetic energy.

\[ E = \Delta mc^2 \]

\[ \Delta m = \left( m_{\text{parents}} - m_{\text{products}} \right) \]

Mass is just a form of energy!
## Compare Reactions

<table>
<thead>
<tr>
<th>Reaction Type</th>
<th>Reaction</th>
<th>Energy Released per kg of Fuel (J/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemical @ 700K</td>
<td>( C + O_2 \rightarrow CO_2 )</td>
<td>( 3.3 \times 10^7 )</td>
</tr>
<tr>
<td>Fission @ 1000K</td>
<td>( n + U-235 \rightarrow Ba-143 + Kr-91 + 2 \text{ n} )</td>
<td>( 2.1 \times 10^{12} )</td>
</tr>
<tr>
<td>Fusion @ ( 10^8 \text{K} )</td>
<td>( H-2 + H-3 \rightarrow He-4 + \text{ n} )</td>
<td>( 3.4 \times 10^{14} )</td>
</tr>
</tbody>
</table>
A rechargeable AA battery with a mass of 25.0 g can supply a power of 1.20 W for 50.0 min. (a) What is the difference in mass between a charged and an uncharged battery? (b) What fraction of the total mass is this mass difference?

\[
a) \quad \Delta E = P t = \left(1.20 \text{ J/s}\right)\left(50 \text{ min}\right)\left(60 \text{ s/min}\right) = 3600 \text{ J}
\]

\[
\Delta m = \frac{\Delta E}{c^2} = \frac{3600 \text{ J}}{\left(3 \times 10^8 \text{ m/s}\right)^2} = 4.00 \times 10^{-14} \text{ kg}
\]

\[
b) \quad \frac{\Delta m}{m} = \frac{4.00 \times 10^{-14} \text{ kg}}{25 \times 10^{-3} \text{ kg}} = 1.60 \times 10^{-12}
\]
\[ E = \Delta m c^2 \]

Solar Flux:

\[ P = 3.77 \times 10^{26} \text{W} \]

\[ \frac{dm}{dt} = 4.19 \times 10^9 \text{kg/s} \]

\[ {}_1^2 \text{H} + {}_1^3 \text{H} \rightarrow {}_2^4 \text{He} + {}_0^1 \text{n} + 17.6 \text{MeV} \]
Creating Matter From Pure Energy

Matter-AntiMatter

\[ E = \Delta mc^2 \]
Atomic Mass Units

1u = 1/12 mass of Carbon-12

\[ 1u = 1.6605 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV} / c^2 \]

\[
\begin{align*}
\Delta m &= (238.0508 - 234.0436 - 4.0026)u = 0.0046u \\
E &= \Delta mc^2 = 0.0043 \times 931.5 \text{ MeV} / c^2 = 4.3 \text{ MeV}
\end{align*}
\]

Most of this energy is Kinetic!
Electron Volts

1 eV is the energy an electron gains across a 1 volt potential.

\[ E = qV \]

\[ 1eV = (1.6 \times 10^{-19} \, C)(1J / C) \]

\[ 1eV = 1.6 \times 10^{-19} \, J \quad OR \quad 1J \sim 10^{19} \, eV \]

The Tevatron: 1 Trillion (Tera) Electron Volts

Mass is given as GeV/c^2

p-restmass: 0.938 GeV / c^2  \quad e-restmass: 0.511 MeV / c^2, 1 GeV/c^2 = 1.783 \times 10^{-27} \, kg
The Tevatron: Fermi Lab $E = \Delta mc^2$
The protons will each have an energy of 7 TeV, giving a total collision energy of 14 TeV. It will take around 90 μs for an individual proton to travel once around the collider.
The International Linear Collider is a proposed future international particle accelerator. It would create high-energy particle collisions between electrons and positrons, their antimatter counterparts. The ILC would provide a tool for scientists to address many of the most compelling questions of the 21st century—questions about dark matter, dark energy, extra dimensions and the fundamental nature of matter, energy, space and time.

http://www.slac.stanford.edu/
In the ILC's design, two facing linear accelerators, each 20 kilometers long, hurl beams of electrons and positrons toward each other at 99.9999999998 percent of the speed of light (with a few TeV energy). Each beam contains ten billion electrons or positrons compressed down to a minuscule three nanometer thickness. As the particles hurtle down the collider, superconducting accelerating cavities operating at temperatures near absolute zero pump more and more energy into them. The beams collide 2000 times every second in a blazing crossfire that creates a firework of new particles. (Computer animation of the field inside a superconducting accelerator resonator.)
Thousands of particles explode from the collision point of two relativistic (100 GeV per ion) gold ions in the STAR detector of the Relativistic Heavy Ion Collider (Brookhaven National Laboratory). Electrically charged particles are discernible by the curves they trace in the detector's magnetic field.

\[ E = \Delta mc^2 \]
In grand unification theories (GUT), unification of the three forces occurs around $10^{16}$ GeV. To unify gravity with the other forces, energies of $10^{19}$ GeV, known as Planck Mass, ($\sim 2.17645 \times 10^{-8}$ kg) must be achieved in particle detectors.
Relativistic Doppler Effect
Relativistic Doppler Effect

- Another consequence of time dilation is the shift in frequency found for light emitted by atoms in motion as opposed to light emitted by atoms at rest.
- If a light source and an observer approach each other with a relative speed, $v$, the frequency is shifted HIGHER (wavelength shifted SHORTER – Blue) and is measured by the observer to be:

$$f_{\text{obs}} = \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}} f_{\text{source}}$$

- If a light source and an observer move apart from each other with a relative speed, $v$, the frequency is shifted LOWER (wavelength shifted LONGER – Red) and is measured by the observer to be:

$$f_{\text{obs}} = \frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}} f_{\text{source}}$$
Relativistic Doppler Effect

**Blue Shift**: Moving Toward

**Red Shift**: Moving Away

\[ f_{\text{obs}} = \frac{\sqrt{1 \pm v/c}}{\sqrt{1 - mv/c}} f_{\text{source}} \]

\[ c = \lambda f \]

\[ f = \frac{c}{\lambda} \]

\[ f = \frac{1}{T} \]

This observer sees blueshift

This observer sees redshift
Spectral lines shift due to the relative motion between the source and the observer

- Cosmological Redshift: Expanding Universe
- Stellar Motions: Rotations and Radial Motions
- Solar Physics: Surface Studies and Rotations
- Gravitational Redshift: Black Holes & Lensing
- Exosolar Planets via Doppler Wobble
Extra Solar Planets: Gravitational Doppler Wobble

The newly discovered Neptune-sized extrasolar planet circling the star Gliese 436.
Cosmological Red Shift

\[ f_{\text{obs}} = \frac{\sqrt{1 - \frac{v}{c}}}{\sqrt{1 + \frac{v}{c}}} f_{\text{source}} \]
Cosmological redshift is caused by the expansion of spacetime.
Hubble

Cepheids

Andromeda Galaxy

$V = H_o \cdot d$
Hubble Time: $T_0 = 1/H_0$

$T_0 = 1/70 \text{ km/s/Mpc}$

$\sim 14 \text{ Billion Years}$

(assuming constant expansion rate)
The diagram illustrates the evolution of the universe from the Big Bang to the modern universe. It shows the following stages:

- **Inflation**: 0 to $10^{-32}$ seconds
- **Quark Soup**: 1 second
- **Big Freeze Out**: 300,000 years
- **Parting Company**: 1 billion years
- **First Galaxies**: 12-15 billion years
- **Modern Universe**: 12-15 billion years

The diagram also shows the age of the universe at different stages: 0, 10^{-32} seconds, 1 second, 300,000 years, 1 billion years, and 12-15 billion years.
Possible Models of the Expanding Universe

A decelerating universe reaches its current size in the least amount of time. The universe could eventually contract and collapse into a "big crunch" or expand indefinitely. A coasting universe (center) is older than a decelerating universe because it takes more time to reach its present size, and expands forever. An accelerating universe (right) is older still. The rate of expansion actually increases because of a repulsive force that pushes galaxies apart.
An alien civilization occupies a brown dwarf, nearly stationary relative to the Sun, several lightyears away. The extraterrestrials have come to love original broadcasts of *I Love Lucy*, on our television channel 2, at carrier frequency 57.0 MHz. Their line of sight to us is in the plane of the Earth’s orbit. Find the difference between the highest and lowest frequencies they receive due to the Earth’s orbital motion around the Sun.

\[
型 f_{\text{obs}} = \frac{\sqrt{1 \pm v/c}}{\sqrt{1 - v^2/c^2}} f_{\text{source}}
\]
Quick Quiz

You are observing a spacecraft moving away from you. You measure it to be shorter than when it was at rest on the ground next to you. You also see a clock through the spacecraft window, and you observe that the passage of time on the clock is measured to be slower than that of the watch on your wrist. Compared to when the spacecraft was on the ground, what do you measure if the spacecraft turns around and comes toward you at the same speed?

(a) The spacecraft is measured to be longer and the clock runs faster.
(b) The spacecraft is measured to be longer and the clock runs slower.
(c) The spacecraft is measured to be shorter and the clock runs faster.
(d) The spacecraft is measured to be shorter and the clock runs slower.
Quick Quiz

Answer: (d). Time dilation and length contraction depend only on the relative speed of one observer relative to another, not on whether the observers are receding or approaching each other.
Albert Einstein
1916

The *General* Theory of Relativity
Equivalence of Gravitational and Inertial Mass

In space, acceleration cannot be distinguished from gravity. The equivalence of gravitational and inertial mass is demonstrated through these images:

(a) At rest on Earth
(b) In space, acceleration
(c) In space, acceleration
(d) At rest on Earth

Can’t tell difference
Principle of Equivalence

• An gravity free accelerated frame of reference is equivalent to an inertial frame in a gravitational field.

• No local experiment can distinguish between the two frames.

• Einstein proposed that a beam of light should be bent downward by a gravitational field
  – The bending would be small
  – A laser would fall less than 1 cm from the horizontal after traveling 6000 km
Curvature of Spacetime

The curvature of space-time completely replaces Newton’s gravitational theory. According to Einstein there is no such thing as a gravitational field.

Einstein specified a certain quantity, the curvature of time-space, that describes the gravitational effect at every point.

The Field Equation:

\[ G_{\mu \nu} = 8\pi T_{\mu \nu} \]

- Specifies the curvature of spacetime
- Specifies the mass/energy content of spacetime
$$R_{\eta\eta} = -\frac{2a^2}{\delta \psi} \frac{\delta^2 \psi}{\delta \psi^2} \cot \theta + \frac{2ac}{\delta \psi} \frac{\delta^2 \psi}{\delta \psi^2} \cot \theta + \frac{a}{\delta} \frac{\delta \psi}{\delta d} \cot \theta - \frac{a}{2\delta} \frac{\delta \psi}{\delta \psi^2}$$

$$-\frac{2a^2}{\delta \psi^2} \frac{(\delta^2 \psi)^2}{\delta \psi^2} + \frac{4ac}{\delta \psi} \frac{\delta^2 \psi}{\delta \psi^2} - \frac{a^2}{\delta d} \frac{\delta \psi}{\delta \psi^2} + \frac{ac}{\delta d} \frac{\delta \psi}{\delta \psi^2} + \frac{2a}{\delta \psi} \frac{\delta \psi}{\delta \psi^2} - \frac{a}{\delta \psi} \frac{\delta \psi}{\delta \psi^2}$$

$$-\frac{3a}{\delta \psi} \frac{\delta \psi}{\delta \psi^2} + \frac{2a^2 c}{\delta \psi} \frac{\delta \psi}{\delta \psi^2} + \frac{2a^2}{\delta \psi} \frac{\delta^2 \psi}{\delta \psi^2} \frac{\delta^2 \psi}{\delta \psi^2} - \frac{a^2}{\delta d} \frac{\delta \psi}{\delta \psi^2} - \frac{a}{\delta d} \frac{\delta \psi}{\delta \psi^2} + \frac{a}{\delta \psi} \frac{\delta \psi}{\delta \psi^2} + \frac{a^3}{\delta \psi} \frac{\delta \psi}{\delta \psi^2}$$

$$+ \frac{a \delta \psi}{\delta d} \frac{\delta \psi}{\delta \psi^2} - \frac{a \delta \psi}{\delta d} \frac{\delta \psi}{\delta \psi^2} - \frac{a}{\delta \psi} \frac{\delta \psi}{\delta \psi^2} + \frac{a}{\delta \psi} \frac{\delta \psi}{\delta \psi^2} + \frac{a}{\delta \psi} \frac{\delta \psi}{\delta \psi^2} + \frac{a}{\delta \psi} \frac{\delta \psi}{\delta \psi^2}$$

$$+ \frac{2a^2 b}{\delta \psi} \frac{\delta \psi}{\delta \psi^2} - \frac{2a c \delta \psi}{\delta \psi^2} \frac{\delta \psi}{\delta \psi^2} - \frac{a^2 c}{\delta \psi} \frac{\delta \psi}{\delta \psi^2} - \frac{a^2 b}{\delta \psi} \frac{\delta \psi}{\delta \psi^2} + \frac{a \delta \psi}{\delta \psi^2}$$

$$+ \frac{a \delta \psi}{\delta \psi^2} - \frac{a \delta \psi}{\delta \psi^2} - \frac{a \delta \psi}{\delta \psi^2} + \frac{a \delta \psi}{\delta \psi^2} + \frac{a \delta \psi}{\delta \psi^2} + \frac{a \delta \psi}{\delta \psi^2}$$

$$+ \frac{\delta d}{4 \delta d} - \frac{a \delta \psi}{\delta \psi^2} - \frac{a \delta \psi}{\delta \psi^2} - \frac{a \delta \psi}{\delta \psi^2} + \frac{a \delta \psi}{\delta \psi^2} + \frac{a \delta \psi}{\delta \psi^2} + \frac{a \delta \psi}{\delta \psi^2}$$

$$+ \frac{\delta d}{4 \delta d} + \frac{\delta d}{4 \delta d} + \frac{\delta d}{\delta \psi} - \frac{a \delta \psi}{\delta \psi^2} - \frac{a \delta \psi}{\delta \psi^2} + \frac{a \delta \psi}{\delta \psi^2} + \frac{a \delta \psi}{\delta \psi^2} + \frac{a \delta \psi}{\delta \psi^2} + \frac{a \delta \psi}{\delta \psi^2}$$

$$+ \frac{\delta d}{4 \delta d} + \frac{\delta d}{4 \delta d} + \frac{\delta d}{\delta \psi} - \frac{a \delta \psi}{\delta \psi^2} - \frac{a \delta \psi}{\delta \psi^2} + \frac{a \delta \psi}{\delta \psi^2} + \frac{a \delta \psi}{\delta \psi^2} + \frac{a \delta \psi}{\delta \psi^2} + \frac{a \delta \psi}{\delta \psi^2}$$
\[
\begin{align*}
    R_{n\theta} &= \frac{-2ac}{\delta^2} \frac{\partial \varphi}{\partial \psi} \cot \theta + \frac{2ab}{\delta^2} \frac{\partial \varphi}{\partial \eta} \cot \theta - \frac{2}{\delta} \frac{\partial \varphi}{\partial \psi} \cot \theta - \frac{\partial \varphi}{\delta^2} \frac{\partial \varphi}{\partial \psi} \frac{\partial \psi}{\partial \eta} + \frac{2}{\delta^2} \frac{\partial \varphi}{\partial \eta} \frac{\partial \psi}{\partial \eta} \frac{\partial \psi}{\partial \psi} \\
    &\quad - \frac{2a}{\delta^2} \frac{\partial^2 \varphi}{\partial \psi^2} - \frac{2ac}{\delta^2} \frac{\partial \varphi}{\partial \psi} \frac{\partial \varphi}{\partial \psi} + \frac{4ab}{\delta^2} \frac{\partial \varphi}{\partial \eta} \frac{\partial \varphi}{\partial \eta} - \frac{2}{\delta^2} \frac{\partial \varphi}{\partial \psi} \frac{\partial \psi}{\partial \eta} + \frac{2}{\delta^2} \frac{\partial \varphi}{\partial \psi} \frac{\partial \psi}{\partial \eta} - \frac{2}{\delta^2} \frac{\partial \varphi}{\partial \psi} \frac{\partial \psi}{\partial \psi}
\end{align*}
\]
\[ + \frac{ab \frac{\partial b}{\partial \psi} c \frac{\partial \psi}{\partial \psi}}{\delta^2 \psi} + \frac{ab \frac{\partial b}{\partial \theta} c \frac{\partial \theta}{\partial \theta}}{\delta^2 \psi} - \frac{a^2 b \frac{\partial b}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{\delta^2 \psi} - \frac{a \frac{\partial a}{\partial \theta} b^2 \frac{\partial \psi}{\partial \psi}}{\delta^2 \psi} + \frac{\frac{\partial d}{\partial \theta} \frac{\partial d}{\partial \theta}}{4 \delta^2} - \frac{\frac{\partial a}{\partial \theta} \frac{\partial \psi}{\partial \theta}}{4 \delta^2} \]

\[ + \frac{ab \frac{\partial a}{\partial \theta} \frac{\partial \psi}{\partial \theta}}{4 \delta \psi} - \frac{\frac{\partial^2 d}{\partial \theta \partial \theta}}{2 \delta} - \frac{\frac{\partial b}{\partial \theta} \frac{\partial c}{\partial \theta}}{4 \delta \psi} + \frac{\frac{\partial a}{\partial \theta} \frac{\partial b}{\partial \theta}}{4 \delta \psi} - \frac{\frac{\partial c}{\partial \theta} \frac{\partial c}{\partial \theta}}{2 \delta} + \frac{\frac{\partial a}{\partial \theta} \frac{\partial a}{\partial \theta}}{2 \delta} \]

\[ + \frac{c \frac{\partial^2 c}{\partial \theta \partial \theta}}{\delta} + \frac{\frac{\partial b}{\partial \theta} \frac{\partial c}{\partial \theta}}{2 \delta} - \frac{\frac{\partial b}{\partial \theta} \frac{\partial b}{\partial \theta}}{2 \delta} - \frac{\frac{\partial b}{\partial \theta} \frac{\partial c}{\partial \theta}}{2 \delta} - \frac{\frac{\partial a}{\partial \theta} \frac{\partial c}{\partial \theta}}{4 \delta} + \frac{\frac{\partial a}{\partial \theta} \frac{\partial a}{\partial \theta}}{4 \delta} \]

\[ + \frac{ab \frac{\partial a}{\partial \theta} \frac{\partial \theta}{\partial \theta}}{\delta^2} - \frac{\frac{\partial a}{\partial \theta} \frac{\partial b}{\partial \theta} \frac{\partial c}{\partial \theta}}{2 \delta^2} - \frac{\frac{\partial a}{\partial \theta} \frac{\partial b}{\partial \theta} \frac{\partial c}{\partial \theta}}{2 \delta^2} - \frac{\frac{\partial a}{\partial \theta} \frac{\partial c}{\partial \theta}}{2 \delta^2} - \frac{\frac{\partial a}{\partial \theta} \frac{\partial b}{\partial \theta}}{2 \delta^2} + \frac{\frac{\partial a}{\partial \theta} \frac{\partial b}{\partial \theta}}{4 \delta^2} \]

\[ + \frac{a \left( \frac{\partial \psi}{\partial \theta} \right)^2 c}{4 \delta^2} + \frac{\frac{\partial a}{\partial \theta} \frac{\partial b}{\partial \theta} \frac{\partial c}{\partial \theta}}{4 \delta^2} + \frac{\left( \frac{\partial \psi}{\partial \theta} \right)^2 b c}{4 \delta^2} + \frac{\frac{\partial a}{\partial \theta} \frac{\partial b}{\partial \theta} \frac{\partial c}{\partial \theta}}{4 \delta^2} - \frac{\frac{\partial a}{\partial \theta} \frac{\partial b}{\partial \theta}}{4 \delta^2} \]

\[ R_{\theta \theta} = -\frac{2ab \frac{\partial \psi}{\partial \psi} \cot \theta}{\delta \psi} + \frac{2bc \frac{\partial \psi}{\partial \psi} \cot \theta}{\delta \psi} - \frac{\frac{\partial d}{\partial \psi} \cot \theta}{d} - \frac{c \frac{\partial \psi}{\partial \psi} \cot \theta}{\delta} + \frac{\frac{\partial b}{\partial \psi} \frac{\partial c}{\partial \psi}}{2 \delta} + \frac{a \frac{\partial a}{\partial \psi} \cot \theta}{2 \delta} \]

\[ -\frac{2ab \frac{\partial^2 \psi}{\partial \psi^2}}{\delta \psi} - \frac{2\frac{\partial^3 \psi}{\partial \psi^3}}{\psi} - \frac{2ab \left( \frac{\partial \psi}{\partial \psi} \right)^2}{\delta \psi^2} + \frac{6 \left( \frac{\partial \psi}{\partial \psi} \right)^2}{\psi^2} + \frac{4bc \frac{\partial \psi}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{\delta \psi^2} - \frac{ab \frac{\partial a}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{\delta \psi} \]
Mass WARPS Space-time
Mass grips space by telling it how to curve and space grips mass by telling it how to move!
- John Wheeler
Shape of Spacetime

https://www.youtube.com/watch?v=oCK5oGmRtxQ
Testing General Relativity

- General relativity predicts that a light ray passing near the Sun should be deflected in the curved space-time created by the Sun’s mass.
- The prediction was confirmed by astronomers during a total solar eclipse in 1919 by Sir Arthur Eddington.
Confirmation of The General Theory: Precession of the Perihelion of Mercury
Einstein proposed that a beam of light should be bent downward by a gravitational field. The bending would be small. A laser would fall less than 1 cm from the horizontal after traveling 6000 km.

Einstein’s cross. The four bright spots are images of the same galaxy that have been bent around a massive object located between the galaxy and the Earth. The massive object acts like a lens, causing the rays of light that were diverging from the distant galaxy to converge on the Earth. (If the intervening massive object had a uniform mass distribution, we would see a bright ring instead of four spots.)
In the cluster Abell 2218, distant blue galaxies behind the large cluster of galaxies are "squished" into a circular shape around the middle of the foreground cluster. By measuring the amount of distortion in the more distant blue galaxies, we can determine the mass of the cluster. In fact, we can even measure how much mass there is that we can't see -- this galaxy cluster happens to have nearly 400 trillion times the sun's mass in "dark" matter.
Gravitational Redshift
Gravity Waves

LISA
LIGO

End-station @ 4 km

Mid-station @ 2 km

4 km (2.5 mi)

Gravitational Wave Observatories

Operational
Under Construction
Planned

LIGO Hanford
LIGO Livingston
VIRGO

GEO600

XAGRA

LIGO India
LIGO again detects gravitational waves

https://www.youtube.com/watch?v=biwlfcljx9Q
Gravity Probe B
Gravitational Frame Dragging
Space-Time Twist
Gyroscopes on Gravity Probe B

~ millionth degree
Black Holes & Worm Holes

Wormholes are tunnels that connect two areas of space. Can wormholes lead to time travel?
A decelerating universe reaches its current size in the least amount of time. The universe could eventually contract and collapse into a "big crunch" or expand indefinitely. A coasting universe (center) is older than a decelerating universe because it takes more time to reach its present size, and expands forever. An accelerating universe (right) is older still. The rate of expansion actually increases because of a repulsive force that pushes galaxies apart.