Physics 42 Chapter 32 Homework

Problems: 6, 11, 13, 16, 17, 32, 46, 51, 55

6. An emf of 24.0 mV is induced in a 500-turn coil at an instant when the current is 4.00 A and is changing at the rate of 10.0 A/s. What is the magnetic flux through each turn of the coil?

P32.6 From \( |\Delta \phi| = L \left( \frac{\Delta I}{\Delta t} \right) \), we have

\[
L = \frac{\epsilon}{(\Delta I/\Delta t)} = \frac{24 \times 10^{-3} \text{ V}}{10 \text{ A/s}} = 2.40 \times 10^{-3} \text{ H}.
\]

From \( L = \frac{N \Phi_B}{I} \), we have \( \Phi_B = LI = \left( \frac{2.40 \times 10^{-3} \text{ H}}{4.00 \text{ A}} \right)(4.00 \text{ A}) = 19.2 \mu \text{Tm}^2 \).

11. A piece of copper wire with thin insulation, 200 m long and 1.00 mm in diameter, is wound onto a plastic tube to form a long solenoid. This coil has a circular cross section and consists of tightly wound turns in one layer. If the current in the solenoid drops linearly from 1.80 A to zero in 0.120 seconds, an emf of 80.0 mV is induced in the coil. What is the length of the solenoid, measured along its axis?

*P32.11 We can directly find the self inductance of the solenoid:

\[
\epsilon = -L \frac{dI}{dt} = +0.08 \text{ V} = -L \frac{0-1.8 \text{ A}}{0.12 \text{ s}}
\]

\[
L = 5.33 \times 10^{-3} \text{ V A} = \frac{\mu_0 N^2 A}{\ell}.
\]

Here \( A = \pi \ell^2 \), \( 200 \text{ m} = N 2\pi \ell \), and \( \ell = N \left( 10^{-3} \text{ m} \right) \). Eliminating extra unknowns step by step, we have

\[
5.33 \times 10^{-3} \text{ V A} = \frac{\mu_0 N^2 \pi \ell^2}{\ell} = \frac{\mu_0 N^2 \pi}{\ell} \left( \frac{200 \text{ m}}{2\pi N} \right)^2 = \frac{\mu_0 40000 \text{ m}^2}{4\pi \ell} = 10^{-7} \left( 40000 \text{ m}^2 \right) \text{Tm} \]

\[
\ell = 4 \times 10^{-3} \text{ W bm A} \approx \frac{0.750 \text{ m}}{5.33 \times 10^{-3} \text{ A V s}} = 0.750 \text{ m}
\]

13. A self-induced emf in a solenoid of inductance \( L \) changes in time as \( \epsilon = Ec e^{-kt} \). Find the total charge that passes through the solenoid, assuming the charge is finite.

P32.13 \( \epsilon = \epsilon_0 e^{-kt} = -L \frac{dI}{dt} \quad dI = -\frac{\epsilon_0}{L} e^{-kt}dt \)

If we require \( I \rightarrow 0 \) as \( t \rightarrow \infty \), the solution is

\[
I = \frac{\epsilon_0}{kL} e^{-kt} = \frac{\Delta q}{dt}
\]

\[
Q = \int_0^\infty L \epsilon_0 e^{-kt} dt = -\frac{\epsilon_0}{k} \frac{1}{k' L}
\]

\[
|Q| = \frac{\epsilon_0}{k' L}
\]
16. Show that \( I = I_0 e^{-t/\tau} \) is a solution of the differential equation

\[
IR + L \frac{dI}{dt} = 0
\]

where \( \tau = L/R \) and \( I_0 \) is the current at \( t = 0 \).

**P32.16** Taking \( \tau = \frac{L}{R} \), \( I = I_0 e^{-rt/\tau} : \frac{dI}{dt} = I_0 e^{-rt/\tau} \left( -\frac{1}{\tau} \right) \)

\[ R + L \frac{dI}{dt} = 0 \] will be true if

\[ I_0 Re^{-rt/\tau} + L \left( I_0 e^{-rt/\tau} \right) \left( -\frac{1}{\tau} \right) = 0. \]

Because \( \tau = \frac{L}{R} \), we have agreement with \( 0 = 0 \).

17. Consider the circuit in Figure P32.17, taking \( E = 6.00 \text{ V}, L = 8.00 \text{ mH}, \) and \( R = 4.00 \Omega \). (a) What is the inductive time constant of the circuit? (b) Calculate the current in the circuit 250 \( \mu \text{s} \) after the switch is closed. (c) What is the value of the final steady-state current? (d) How long does it take the current to reach 80.0% of its maximum value?

**P32.17**

(a) \( \tau = \frac{L}{R} = 2.00 \times 10^{-3} \text{ s} = 2.00 \text{ m s} \)

(b) \( I = I_{\text{max}} \left( 1 - e^{-\frac{rt}{\tau}} \right) = \left( \frac{6.00 \text{ V}}{4.00 \Omega} \right) \left( 1 - e^{-0.250/2.00} \right) = 0.176 \text{ A} \)

(c) \( I_{\text{max}} = \frac{E}{R} = \frac{6.00 \text{ V}}{4.00 \Omega} = 1.50 \text{ A} \)

(d) \( 0.800 = 1 - e^{-\frac{2.00}{2.00} \text{ m s}} \rightarrow t = (2.00 \text{ m s}) \ln(0.200) = 3.22 \text{ m s} \)

32. At \( t = 0 \), an emf of 500 V is applied to a coil that has an inductance of 0.800 H and a resistance of 30.0 \( \Omega \). (a) Find the energy stored in the magnetic field when the current reaches half its maximum value. (b) After the emf is connected, how long does it take the current to reach this value?

**P32.32**

(a) \( U = \frac{1}{2} L I^2 = \frac{1}{2} L \left( \frac{E}{2R} \right)^2 = \frac{L e^2}{8R^2} = \frac{(0.800)(500)^2}{8(30.0)^2} = 27.8 \text{ J} \)

(b) \( I = \left( \frac{E}{R} \right) \left[ 1 - e^{- \left( \frac{E}{R} \right) t} \right] \)

so

\[ \frac{E}{2R} = \left( \frac{E}{R} \right) \left[ 1 - e^{- \left( \frac{E}{R} \right) t} \right] \rightarrow e^{- \left( \frac{E}{R} \right) t} = \frac{1}{2} \]

\[ \frac{R}{L} t = \ln 2 \] so

\[ t = \frac{L}{R} \ln 2 = \frac{0.800}{30.0} \ln 2 = 18.5 \text{ m s} \]
46. A 1.00-μF capacitor is charged by a 40.0-V power supply. The fully charged capacitor is then discharged through a 10.0-mH inductor. Find the maximum current in the resulting oscillations.

\[ I_{max} = \sqrt{\frac{C}{L}} (\Delta V)_{max} = \sqrt{\frac{1.00 \times 10^{-6} \text{ F}}{10.0 \times 10^{-3} \text{ H}}} (40.0 \text{ V}) = 0.400 \text{ A}. \]

51. An LC circuit like the one in Figure 32.16 contains an 82.0-mH inductor and a 17.0-μF capacitor that initially carries a 180-μC charge. The switch is open for \( t < 0 \) and then closed at \( t = 0 \). (a) Find the frequency (in hertz) of the resulting oscillations. At \( t = 1.00 \text{ ms} \), find (b) the charge on the capacitor and (c) the current in the circuit.

\[ f = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{(0.0820 \text{ H})(17.0 \times 10^{-6} \text{ F})}} = 135 \text{ Hz} \]

(b) \[ Q = Q_{max} \cos \omega t = (180 \mu \text{C}) \cos (847 \times 0.001 \text{ Hz}) = 119 \mu \text{C} \]

(c) \[ \frac{dQ}{dt} = -\omega Q_{max} \sin \omega t = -(847)(180) \sin (847) = -114 \text{ mA} \]

55. Consider an LC circuit in which \( L = 500 \text{ mH} \) and \( C = 0.100 \mu \text{F} \). (a) What is the resonance frequency \( \omega_0 \)? (b) If a resistance of 1.00 kΩ is introduced into this circuit, what is the frequency of the (damped) oscillations? (c) What is the percent difference between the two frequencies?

\[ \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.500)(0.100 \times 10^{-6})}} = 4.47 \text{ krad/s} \]

(b) \[ \omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = 4.36 \text{ krad/s} \]

(c) \[ \frac{\Delta \omega}{\omega_0} = 2.53\% \text{ lower} \]